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Quality Cost Modeling Process for Production Systems

İnci Şentarlı

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1. Introduction

Minimizing the sum of quality costs related to limited resources forms one of the main determinants in a company's success. In production systems, resources such as labors inspecting items produced, labors training personnel to prevent items from erroneous results, labors reworking on erroneous items , labors dealing with complaints about erroneous products etc. are limited. Hence, it is important to find out the most suitable mix of conformance and nonconformance costs of products in allocation of the scarce resources.

In literature, the cost of poor quality as a result of imperfections of a vendor's incoming input materials and the errors of the manufacturing process were investigated (Tagaras, 1996). An optimization model including appointment of the cost of quality between the two parties as buyers and suppliers was also analyzed (Baiman, 2000).

However, models optimizing total quality costs including each quality cost components has not been studied extensively. In this study, a generalized optimization model for a multistage production system based on quality costs as conformance and nonconformance costs has been developed.

2. The quality cost relationship

Total quality costs can be divided into two categories as conformance costs which are of two types as prevention and appraisal costs and nonconformance costs which consist of internal and external failure costs. An increase in prevention and appraisal costs results in an increase in conformance of products to the required quality level. On the contrary, as magnitude of the conformity of products to the desired level increases, money incurred because of the imperfections in the products decreases. In short, as costs of achieving good quality rises, costs of poor quality falls. Therefore in a quality cost model, a relationship exists between conformance and nonconformance costs (Juran, 1974; Feigenbaum,1983).

Increased conformance costs will lead to a reduction in nonconformance costs. On the other hand, excess spending in conformance costs will lead to high total quality costs because they will exceed the reduction in failure costs (Freeman, 1960; Masser,1957). As a result, this brings one to the conclusion that when the costs of conformance are balanced with the costs of nonconformance, total quality cost may pass through a near optimum point.

3. The new quality cost model

To formulate the optimization problem, it is assumed that objective function and the constraints are continuous functions and the feasible set is not empty. The other assumptions are as follows:

- The company is interested in the production of one product type and production resources are not shared by other systems.
- All costs are assumed to be linear
- Probability of defective incoming material, and probability of a defective output due to manufacturing process in a stage are assumed to be independent.
- If poor quality of the product is not detected internally in the company, it is detected externally by buyers.

The objective function of the proposed model can be stated as

$$\text{Min} \mid \text{f}_{\text{TPC}} + \text{f}_{\text{TAC}} - \text{f}_{\text{TIFC}} - \text{f}_{\text{TEFC}} \mid \quad (1)$$

where total prevention cost function is

$$f_{TPC}(S_j) = x_0 \sum_{j}^{J} S_J c_{pj} \quad (2)$$

In this equation, c_{pj} resembles prevention cost per unit j and S_J denotes fraction of units benefited from the prevention program in stage j and x_0 gives amount of input

Total appraisal cost function is given by

$$f_{TAC}(I_k) = x_0 \sum_{k}^{K} I_k c_{ak} \quad (3)$$

where c_{ak} is appraisal cost per unit k and I_k denotes fraction of units inspected in stage k.

The third quality cost component function can be decomposed into functions which yield total internal failure cost due to poor quality inputs (f $_{TIFIC}$), and total internal failure cost due to erroneous manufacturing (f $_{TIFMC}$).

$$f_{TIFC} = f_{TIFIC} + f_{TIFMC} \tag{4}$$

where total internal failure due to erroneous manufacturing is

$$f_{TIFMC} = \sum_{i=1}^{F} f_{TIFMC_i} \tag{5}$$

Total rework cost in the system f $_{TIFMC}$ takes the form

$$f_{TIFMC_1}(d_i) = x_0 d_1 r_1 c_{r1} + x_0 \sum_{j=2}^{n} d_j r_j c_{rj} \prod_{i=1}^{j-1} (1.0 - d_i + r_i d_i) \tag{6}$$

where d_i is poor quality % in stage i , r_i is % of erroneous units reworked in stage i, c_{ri} denotes rework cost after stage i and x_0 is the amount of input in the system. The derivation of this formula is presented in Appendix A.

Then, total scrap cost equation is

$$f_{TIFMC_2}(d_i) = x_0 d_1 (1.0 - r_1) c_{s1} + x_0 \sum_{j=2}^{n} d_j (1.0 - r_j) c_{sj} \prod_{i=1}^{j-1} (1.0 - d_i + r_i d_i) \tag{7}$$

where c_{sj} is scrap cost per item after stage j.

Cost of internal failures due to imperfections in inputs may be expressed as

$$f_{TIFIC}(q_i) = \sum_{i=1}^{K} z_i q_i c_{inp,i} \tag{8}$$

where q_i is probability of failure due to imperfections of input i, $c_{inp,i}$ is extra input cost per item, z_i denotes input component i and sum of each component gives the total amount of input (eqtn 9).

$$\sum_{i=1}^{Z} z_i = x_0 \tag{9}$$

Lastly, total external failure cost equation is

$$f_{TEFC}(P_b) = x_0 P_b c_{bsr} + x_0 P_{nb} c_{nbsr} \tag{10}$$

where P_b is probability of receiving a bad signal from a buyer, P_{nb} is probability of not receiving a bad signal from a buyer after an erronous product is delivered, c_{bsr} denotes cost per erroneous item delivered when bad signal received and c_{nbsr} is cost per erroneous item delivered when bad signal is not received. The first term in this equation gives external quality cost when a bad signal is received from the buyer, second term gives the case when no signal is received.

The objective function is subject to

$$S_n + d_n + P_b + P_{nb} = 1.0 \qquad (11)$$

$$0.0 \leq S_j \leq 1.0 \qquad j=1, \ldots J \qquad (12)$$

$$0.0 \leq d_i \leq 1.0 \qquad j=1, \ldots J \qquad (13)$$

$$0.0 \leq q_j \leq 1.0 \qquad j=1, \ldots J \qquad (14)$$

$$0.0 \leq P_b \leq 1.0 \qquad (15)$$

$$0.0 \leq P_{nb} \leq 1.0 \qquad (16)$$

Constraint defining workforce size:

$$\sum_{i=1}^{n} W_{Ai} + \sum_{i=1}^{n} W_{Pi} + \sum_{i=1}^{n} W_{IFi} + W_{EF} \leq W_t \qquad (17)$$

Constraints for the relationship between output and workforce are

$$x_0 S_i K_i \leq W_{Ai} \qquad (18)$$

$$x_0 I_i L_i \leq W_{Pi} \qquad (19)$$

$$x_0 d_i M_i \leq W_{IFi} \qquad (20)$$

$$x_o (P_b + P_{nb}) N_i \leq W_{EF} \qquad (21)$$

where K_i, L_i, M_i and N_i represent the worker hours needed per item.

The total quality cost (f_{TQC}) equation (eqtn 22) for a multistage production system developed in this study consists of total prevention cost, total appraisal cost, total internal failure cost and total external failure cost.

$$f_{TQC} = f_{TPC} + f_{TAC} + f_{TIFC} + f_{TEFC} \qquad (22)$$

4. Discussion of results for different cases

In this study, a computer code has been developed to find out the near optimum quality costs. The results of five different cases studied using this code are given below. Table I shows minimum values of the objective function obtained versus quality cost per items for a unique stage production system. Comparing these results reveals that the lowest objective function value belongs to fifth case. These examples highlight that as joint effects of different quality costs per item decrease, minimum objective function value also decreases.

Table I. Comparison of objective function minimum value ($) versus quality cost ($) per Items*

Cost per item / Min. Value	cp	ca	cr	cs	cbsr	cnbsr
136.0	4.0	2.0	3.0	5.0	5.0	7.0
58.5	1.5	0.5	1.0	2.0	2.0	3.0
14.67	0.5	0.5	1.5	2.0	2.0	4.0
8.9	0.5	0.5	1.0	3.0	4.0	5.0
0.14	0.15	0.2	1.5	2.2	2.6	2.9

*x0=100.0

Table II tabulates total quality cost values versus quality fraction of items for the studied cases. Comparison of these examples shows that the lowest total quality cost belongs to fifth case. While external failure fractions come out as less than 0.1, conformance fractions approaches to 0.9 in the fifth example. In particular, the examples illustrate that higher the fraction of units benefited from the prevention program, the lower the total quality cost.

Table II. Comparison of total quality cost ($) versus quality related fractions per items*.

Total Quality Cost / Fraction	S	I	d	Pb	Pnb
408.00	0.580	0.200	0.200	0.090	0.010
152.5	0.650	0.160	0.160	0.100	0.010
75.33	0.800	0.100	0.093	0.050	0.010
86.1	0.850	0.100	0.056	0.056	0.010
36.86	0.900	0.250	0.041	0.041	0.001

*x0=100 items

5. Conclusions

Using mathematical models is faster and cheaper than constructing and manipulating real systems in surveying affects of different parameters. Hence, the new quality cost model developed in this study may be used to find out the most economic total quality costs in a multistage production system. Optimum combination of quality cost components can be studied with respect to different parameters using the constrained optimization model proposed. The model designed will be useful in allocating scarce resources among competing quality based demands in a near optimum way.

Nomenclature

c_{ri} : rework cost after stage i

$c_{inp,i}$: extra input cost for input I

c_{pj} : prevention cost per unit j

c_{ak} : appraisal cost per unit k

c_{sj} : scrap cost per item after stage j

c_{bsr} : cost per erroneous item delivered when bad signal received

c_{nbsr} : cost per erroneous item delivered when bad signal is not received

d_i : poor quality % in stage i

f_{TQC} : Total quality cost

f_{TAC} : Total appraisal cost

f_{TPC} : Total prevention cost

f_{TIFMC} : Total internal failure cost due to manufacturing

f_{TIFIC} : Total internal failure cost due to inputs

f_{TEFC} : Total external failure cost

I_k : fraction of units inspected in stage k

P_b: probability of receiving a bad signal from a buyer

P_{nb}: probability of not receiving a bad signal from a buyer

q_i : % failure probability of input i

r_i : % of defective items reworked

S_J : fraction of units benefited from the prevention program in stage j

W_{Ai} : appraisal workforce size

W_{Pi} : prevention workforce size

W_{IFi} : internal failure workforce size

W_{EF} : external failure workforce size

W_t : total workforce size

x_0 : amount of input

z_i : input component i

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Appendix A

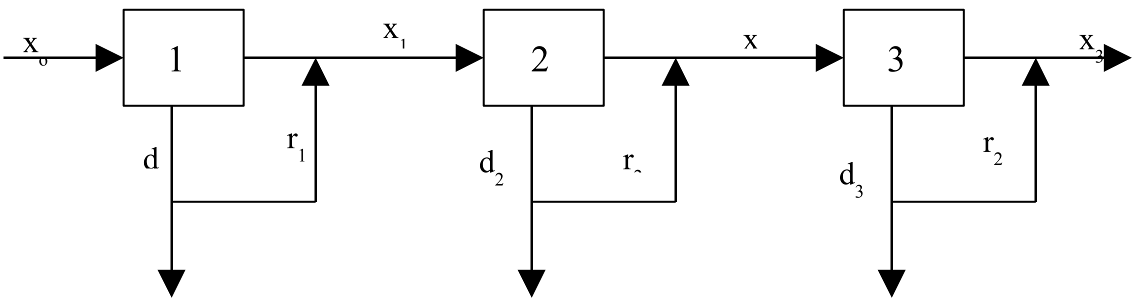

Figure A1. A multistage production system

$$x_1 = x_0(1.- d_1) + x_0 d_1 r_1 \qquad \text{A1}$$

or

$$x_1 = x_0(1.- d_1 + d_1 r_1) \qquad \text{A2}$$

and

Rework cost in stage 2: $x_1 d_2 r_2$ A3

inserting eqtn A2;

it is equal to $x_0 d_2 r_2 (1.- d_1 + d_1 r_1)$ A4

$$x_2 = x_1(1.- d_2) + x_1 d_2 r_2$$

or

$$x_2 = x_1(1.- d_2 + d_2 r_2) \qquad \text{A5}$$

so

$$x_2 = x_0(1. - d_1 + d_1 r_1)(1. - d_2 + d_2 r_2) \qquad \text{A6}$$

Rework cost in stage 3: $x_2 d_3 r_3 c_{r3}$

or is equal to $x_0 d_3 r_3 c_{r3} \prod_{i=1}^{2} (1.0 - d_i + r_i d_i)$ A7

Therefore, total rework cost is $x_0 d_1 r_1 c_{r1} + x_0 \sum_{j=2}^{n} d_j r_j c_{rj} \prod_{i=1}^{j-1} (1.0 - d_i + r_i d_i)$ A8