

Animation of Water Droplet Flow on Structured Surfaces

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Abstract

Several methods for rendering and modeling water have been made and a few of them address the natural phenomenon of water droplets flow. As far as we know, none of those methods have used bump maps in order to simulate the flow of a droplet on structured surfaces. The normals of the bump map, that describes the geometry of the micro structured surface, are used in the flow computation of the droplets. As a result, the water droplets will meander down on the surface as if it has a micro structure. Existing models were not suitable for this purpose. Therefore, a new model is proposed in this paper. The droplet will also leave a trail, which is produced by changing the background texture on the surface. This method will not present a physically correct simulation of water droplets flow on a structured surface. However, it will produce a physically plausible real-time animation.

1. Introduction

There is an endless ever-changing kingdom of phenomenon provided by the nature that is possible to model, animate and render. These phenomena offers, with their complexity and richness, a great challenge for every computer artist. Several natural phenomena, like fire, smoke, snow, clouds, waves, trees and plants, have with different success been modeled in computer graphics through the years. Several different methods that address the problems of rendering and modeling water and other similar fluids have been developed since the 1980's. Most of them concern animation of motion in water in forms of waves and other connected fluids and surfaces, i.e. whole bodies of water. For example have oceans waves^{10 5 13} and waves approaching and braking on a beach¹² been modeled. Realistic and practical animation of liquids^{2 3} has also been made. Only a few methods that have been proposed during the 1990's address the problems of the natural phenomenon of water droplets. Methods for simulating the flow of liquids were proposed to render a tear falling down a cheek⁴ and changes in appearances due to

weathering¹. Different methods for animation of the flow of water droplets running down a curved surface with⁷ or without obstacles on it⁸ have also been proposed. Different ways to create droplets have been used⁶, for example metaballs that are affected by the gravitation were used as one solution¹⁴. It is quite difficult to simulate the flow of water droplets for the purpose of high-precision engineering, due to the complicated process that the flow and the shape of the droplet represent. This process has many unknown factors that plays a big role. The shape and the motion of a water droplet on a surface depend on the gravity force that acts on the droplet, the respective surface tensions of the surface and the water droplet, and the inter-facial tension between them⁶. Shape and motion is also under the sway of other things like air resistance and evaporation. These effecting factors can be divided into two different groups. As an example, gravity and wind can be placed in the group of external forces. Factors like surface tension and inter-facial tension belongs to the group of internal forces. To be able to create an accurate physical simulation of the phenomenon of water droplets, a tremendous amount of forces and factors would

have to be taken into account. As mentioned above many of the dominant factors for water droplets are still unknown not only within computer graphics but also within physics. To the long list of effecting factors these ones can be added:

- Motion of the water within the droplet.
- The capillarity of the surface.
- The interaction forces between each point on the surface of the droplet and the solid surface.

1.1. Main Contribution

Trying to take all of these different factors into account would create an accuracy that goes far beyond what is possible to do in the scope of this paper. A method is proposed for generating an animation of the flow of water droplets on a structured surface. Instead of creating a structured surface with a huge amount of polygons, a bump mapped⁹ flat surface is used. Furthermore, the bump normal is used to control the motion of the droplets. To our knowledge, this has never been investigated before. Hence, the droplet will meander down the surface and move as if it actually was flowing on a structured surface. However, as mentioned earlier, all the different factors which have an influence on water droplets and their flow, have not been taken to account in the method. The aim of this paper is not to make a simulation that is physically correct at every point, but to make a plausible animation of droplets meandering down on a bump mapped surface.

2. Previous Research

There are at least four published papers about droplets and their flow that address similar problems as this paper.

2.1. Animation of Water Droplets on a Glass Plate

Kaneda et al⁶ propose a method for realistic animation of water droplets and their streams on a glass plate. The main purpose is to generate a realistic animation, taken into account gravity of water droplets, inter-facial tensions and merging of water. Those are the dominant parameters of dynamical systems. A high-speed rendering is also proposed, which takes reflection and refraction of light into account. Their method will reduce the calculation cost of animations that contains scenes seen through a rainy windshield or windowpane.

The route that the water a droplet takes as it meanders down on a glass plate is determined by impurities on the surface and inside the droplet itself. To be able to animate water droplets and their stream a discrete surface model is developed and the surface of the glass plate is divided into a mesh. Figure 1 shows a lattice that is used on a glass plate. To every lattice point on the glass plate an affinity, 0-1, for water is assigned in advance.

A water droplet begins to meander down a surface when

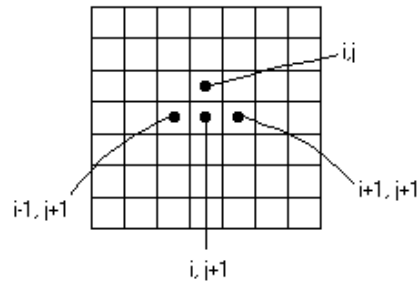


Figure 1: A discrete surface model, with the droplet at position (i, j)

the mass exceeds a static critical weight. To simulate the meandering the droplet at point (i, j) can move to one of three different points on the lattice, as shown in Figure 1. If some water exists on any of the three points, the droplet will move to the lattice point with the direction $(i, j + 1)$ has the highest priority. In case there is no water already existing on the different points, a value depending on for example the angle of inclination is used as a decision parameter. They claim that the speed of the droplet is not depending on the mass of the droplet. Instead it depends on the wetness and the angle of inclination of the glass plate. When two droplets collides and merges the speed of the new droplet is calculated by using equation law of conservation of the momentum. A meandering droplet that has no water ahead will decelerate and when the dynamic critical weight is larger than the mass of the droplet, it will finally stop.

2.2. Animation of Water Droplet Flow on Curved Surfaces

The previously proposed method is not able to simulate a water droplet on a curved surface, which is an important and necessary technique for drive simulators. Therefore an extended method for generating realistic animation of water droplets and their streams on curved surfaces is proposed by Kaneda et al⁸. The dynamics, such as gravity and inter-facial tension that acts on water droplets is also taken into account in this method. Two different rendering methods that takes refraction and reflection into account, is also proposed. One method pursues photo-reality with help of a high quality rendering. The other proposes a fast rendering method that uses a simple model of water droplets.

A discrete surface model is used to make it possible to simulate the flow of droplets running down the curved surface. The curved surface is divided into small quadrilateral meshes and may be specified by Beziér patches. It is converted to a discrete model, using a quadrilateral mesh with a normal vector at the center. Affinity contributes to the meander of the streams and to the wetting phenomenon. The degree of affinity for water is assigned to each mesh in advance.

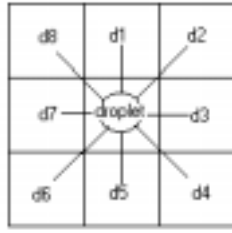


Figure 2: *The eight directions of movement*

This value describes the lack of uniformity on a surface, for example a glass plate. The uniformity can be impurities and small scratches.

The droplet is affected by gravity and wind. When these forces exceed a static critical force, the water droplet starts to meander down the surface. The critical force originates from the inter-facial tension between water and a surface and is the resistance that prevents the droplet from moving. The direction of movement is classified into eight different directions as shown in figure 2. The probabilities for each direction is calculated based on three different factors. The first one is the direction of movement under circumstances in which it obeys Newton's law of motion. The second factor is the degree of affinity for water on the meshes next to the droplet. The last one is the wet or dry condition of the eight neighboring meshes. The water droplet is moved to the next mesh when the direction of movement is determined and if the accumulated time exceeds a frame time, the droplet is moved to the next mesh.

A solution to the wetting phenomenon that appears when a droplet meander down a surface, as well as the problem with two droplets merging, is also addressed. Two different methods for rendering water droplets are proposed. The fast version use spheres. The more sophisticated use meta-balls.

2.3. Simulating the flow of liquid droplets

Fournier et al⁴ present a model that is oriented towards an efficient and visually satisfying simulation. It focuses on the simulation of large liquid droplets as they travel down a surface. The aim is to simulate the visual contour and shape of water droplets when it is affected by the underlying surface and other force fields.

The surface is defined as a mesh of triangles. At the beginning of the simulation a "neighborhood" graph is built. In this graph each triangle is linked to the triangles adjacent to itself. Through the entire simulation each triangle knows which droplets are over it as well as every droplet know which triangle it lies on at the moment. Adhesion and roughness is considered in this method. The adhesion is a force that works along the surface normal. A droplet will fall from a leaning surface if the adhesion force of the droplet

becomes smaller than the component of the droplets acceleration force that is normal to the surface. The roughness of the surface is assumed to only reduce the tangential force.

The motion of droplets is generated by a particle system, where droplet is represented by one particle each. This representation offers many advantages for simulations that have a wide spectrum of behaviors, because of the generality and flexibility such systems can offer. A droplet might travel over several triangles between two time steps. To ensure that the droplet is properly affected by the deformations on the surface it has traversed, the motion of the droplet over each individual triangle is computed. When a droplet travel from one triangle to another, the neighborhood graph is used to quickly identify which triangle the droplet moves to. The two forces gravity, and friction, which affects the water droplets, are assumed to be constant over a triangle.

2.4. Animation of Water Droplets Moving Down a Surface

Kaneda et al⁷ propose a method for generating an animation with water droplets that meander down a transparent surface. A large amount of droplets are used to generate a realistic and useful animation for drive simulators. Their method employs a particle system in which water droplets travel on a discrete surface model. The proposed method involves extensions of previously discussed papers^{6,8}. One of the main achievements is modeling of obstacles that act against water droplets, like the wiper on the windshield.

The curved surface is divided into small quadrilateral meshes and the droplets move from one mesh point to another under the influence of external forces and obstacles. The degree of affinity for water is assigned in advance to each mesh. Affinity describes the lack of uniformity on an object surface due to such things as small scratches and other impurities. The degree of affinity in most cases is assigned randomly based on a normal distribution in order to render the droplets meandering and wetting phenomenon.

By taking into account some dominant factors the direction of movement can be determined. The dominant factors that affects the meandering of water droplets that is mentioned in the paper is:

1. Direction of movement under circumstances in which it obeys Newton's law of motion.
2. Degree of affinity for water of the neighboring meshes.
3. The wet or dry condition of the neighboring meshes
4. Existence of obstacles on the neighboring meshes

A stochastic approach is taken for determining the direction of movement, because the route of the stream cannot be calculated deterministically. This is due to the many unknown factors that play a role. This means in other words that the direction of movement is classified into eight different directions, as done in an earlier mentioned paper⁸. The

probabilities of movement for every direction is calculated with the four dominate factors, described above, taken into account.

The method for rendering water droplets which is proposed in this paper is based on a method that is published by Kaneda et al⁶. The method uses environment mapping to generate realistic images of water droplets. Spheres are used to approximate the water droplets. The contact angle of the water on the surface is taken into account. This method has been extended further in this paper. Such factors as defocus and blur effects are added to generate more realistic images.

3. Droplet Flow Controlled by Bump maps

The different factors that have an affect on the flow of the water droplet are almost countless. Hence, a correct animation is more or less impossible to make. The goal of this paper is therefore to make a physically plausible animation that will produce a natural looking animation of the flow. A real wetting effect which will affect other droplets was not be implemented. Neither was a method for merging of droplets. A simple solid sphere was used to model the droplets. An animation was implemented using C++ and OpenGL. In the animation a flat surface is modeled using a texture and a bump map which is retrieved from the texture. An object oriented particle system was used where each droplet is a particle. This will make the animation easy to control. Furthermore, it is easy to add more droplets to the animation.

3.1. External and internal forces

There are different forces that acts on the water droplets as they meander on the surface. The different forces can be divided into two groups, the external forces, \mathbf{f}^{ext} , and the internal forces, \mathbf{f}^{int} . Kaneda et al⁸ set the external forces to be gravity and wind. However, we will set the external force to be gravity only, since no wind is applied in the proposed model. Nonetheless wind or any other external force could be added if applicable. Moreover, we will use the same denotation of vectors as used by Kaneda et al and also introduce some new vectors.

The internal force is a force of resistance and its direction is opposite to the direction of movement, \mathbf{d}_p :

$$\mathbf{f}^{int} = -\alpha \mathbf{d}_p. \quad (1)$$

The resistance originates from the inter-facial tension that exists between the water droplet and the surface. The affinity which is denoted α is in advance experimentally set to some value, which is assumed to be constant all over the surface for simplicity.

3.2. Direction of movement

The direction of movement can be computed by applying the Gram Schmidt orthogonalization algorithm¹¹ as shown

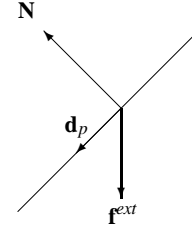


Figure 3: The direction of movement d_p for a bump with normal \mathbf{N} and gravity \mathbf{f}^{ext}

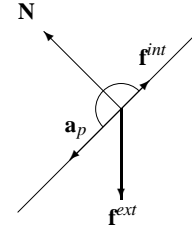


Figure 4: Forces acting on the droplet

in figure 3:

$$\mathbf{d}_p = \mathbf{f}^{ext} - (\mathbf{N} \cdot \mathbf{f}^{ext}) \mathbf{N}. \quad (2)$$

The normal vector \mathbf{N} is the unit length normal which is retrieved at every point from the bump map. This normal will affect the water droplets as they meander down the surface. It will appear as the droplets are directed in a natural way by the visual bumps on the surface underneath the droplet. Furthermore, the whole polygon has a main direction downwards or tangent \mathbf{T} , computed from the external force \mathbf{f}^{ext} and the normal of the polygon \mathbf{N}' :

$$\mathbf{T} = \mathbf{f}^{ext} - (\mathbf{N}' \cdot \mathbf{f}^{ext}) \mathbf{N}'. \quad (3)$$

The bi-normal of the plane is computed as:

$$\mathbf{B} = \mathbf{T} \times \mathbf{N}'. \quad (4)$$

In order to calculate the acceleration of the water droplet, the mass, m , and the forces that acts on the droplet, \mathbf{f}^{ext} and \mathbf{f}^{int} , are used. The acceleration \mathbf{a}_p shown in figure 4 is then decomposed into the component toward the direction of movement \mathbf{d}_p , by projecting it onto this vector⁸:

$$\mathbf{a}_p = \frac{(\mathbf{f}^{ext} + \mathbf{f}^{int}) \cdot \mathbf{d}_p}{m} \mathbf{d}_p. \quad (5)$$

The velocity \mathbf{v} of the droplet is computed by adding the acceleration \mathbf{a}_p to the velocity for each step. Similarly, the velocity is added to the position P . Furthermore, the velocity



Figure 5: One frame from the droplet animation. The trails show that the droplets are affected by the underlying bump mapped surface.

must be projected down onto the plane, in order to prevent the drop from leaving the surface, which of course is modeled in nature by other forces. Nevertheless, this will work for our purposes. This algorithm gives us the new position of the droplet and the droplet is moved to that point during one frame of animation. Hence, the following computations are necessary besides computing the acceleration:

$$\mathbf{v}_{i+1} = \mathbf{v}_i + \mathbf{a}_p, \quad (6)$$

$$\mathbf{v}_p = \mathbf{T}(\mathbf{T} \cdot \mathbf{v}_{i+1}) + \mathbf{B}(\mathbf{B} \cdot \mathbf{v}_{i+1}), \quad (7)$$

$$P_{i+1} = P_i + \mathbf{v}_p. \quad (8)$$

3.3. Speed Control

In nature, water meandering down a surface will not accelerate up to full speed, due to several of the forces mentioned in the introduction. Therefore, a speed controller was implemented, delimiting the speed in two ways. First a maximum speed was introduced. Secondly, the speed will be reduced on bumpy areas. Thus, letting the droplet flow rapidly on flat surfaces, but be slowed down considerably on bumpy areas. A bumpy area is defined as a position where:

$$\mathbf{N} \cdot \mathbf{N}' < 1 - \epsilon, \quad (9)$$

where ϵ is a threshold value that can be used to control how large the bumps should be in order to slow down the droplet more than usual bumpiness would.

3.3.1. The trail

In order to produce a natural looking trail on the surface which the droplet has traversed, a texture map is used. A snapshot from the animation is shown in figure 5. In the animation a texture map is used and the height map is derived from it. The trail is produced by altering the glossiness of the part of the texture that the droplet has passed. It can easily be confirmed by looking on a wall, on which water have been poured on, that the thin layer of water in the trail reflects light with a higher degree of specularity than the underlying surface has.

4. Discussion

The aim is to make an animation that look as natural and realistic as possible. Because of that and several physical factors that still are unknown for the flow of water droplets, there are lot of tampering that needs to be done with the different parameters. The only way to get a satisfying result is to experiment with the different values and see what is going to happen.

As shown earlier the velocity is projected down to the plain in order to prevent the droplet from leaving the surface. This is something that maybe can be controlled in a smoother way. For example, a factor that makes the droplet adhere to the surface would be one way to handle it.

The wetting effects that the flow of a droplet has on the traversed surface is only implemented as a change of the specular light in the trail after the water droplet. If the wetting phenomenon were to be more correct implemented, the droplet would for instance leave a small amount of water behind as it flows down the surface. This would reduce the size of the droplet and finally make it stop. The wetting phenomenon would also make other droplets that comes near a trail of water adhere to it. Subsequently it would flow almost strictly in the same trail as the droplet before. The problem with two merging droplets is not addressed in this paper.

Only one normal is retrieved from the map for each droplet. Nonetheless, it is also possible to use several normals for this computation. The droplet is after all covering more than one position in the height map. It turns out not to be an good idea to compute an average of the normals involved to use in the droplet computations, since the effect on the droplet will be diminished due to the averaging.

Another way to use more than one normal would be to define the bumpiness which should slow down the droplet as described earlier. If the average deviation of the normals from the mean normal, under the droplet, is larger than some threshold value, then the area is considered being bumpy.

Even though the animation is realistic, nature can sometimes surprise you. This is especially true for droplets on a structured surface. Sometimes droplets will not meander

straight down on a totally flat surface. Instead they will meander sideways. By making an experiment where two pictures are taken, one of the structured surface and one of a water droplet that flow over the surface, a comparison of the simulated result and the real thing could be done. The picture taken on the surface would subsequently be used as a texture and a bump map could be retrieved from it in order to produce an animation. The result would show how far from the real thing the animation is.

The proposed model will make the simulation of the flow of a water droplet on a structured surface considerable faster than if polygons were used to form the micro structure. The object itself will also be rendered faster.

5. Conclusions

A method for animation of water droplets flow on structured surfaces was proposed and the droplets in this method were affected by the underlying bump mapped surface. The proposed method will save time due to the use of a bump mapped surface instead of a larger amount of different triangles. Several parameters were used for the animation of the flow of a water droplets, like the gravity working on the droplet, affinity of the surface, and the mass of the droplet used in the proposed method. Moreover, is the algorithm simplified in such way that adhesion to the surface on bumpy areas is modeled by slowing down the drop. A maximum speed is also used for modeling adhesion. All this will make the animation of the individual droplets fast.

5.1. Future Work

There are several improvements that can be done to the proposed method. Moreover, there are various of extensions that can be employed to the present method. Some examples of possible improvements are, to make a better simulation of the wetting phenomenon, so that it will affect the droplets size and shape. Hence, droplets will become smaller as they leave a trail.

Another improvement would be to create the droplet with help of Beziér curves and let the control points be altered by the bump mapped normals. Different normals should affect the different parts of the droplet, making the droplet stretch and bend. The shape of the water droplet is something that overall should be improved.

If the affinity of the surface would depend on the bump map, then it would probably give the meandering of the water droplet a much more realistic and natural look. The method should be extended so that the droplet adheres to the surface and when the adhesion becomes small enough the droplet will depart from the surface.

Other proposals for extensions can be, to implement the enlargement effect that water droplets have on their underlying surface and how light are reflected in the water droplets.

Another extension is implement collision and merging of water droplets.

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