

# EXPERIMENTAL AND THEORETICAL INVESTIGATION OF COPLANAR-PLATE CAPACITORS ON LAYERED DIELECTRIC SUBSTRATES

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In this work we report the results of theoretical and experimental investigation of coplanar-plate capacitors on layered dielectric substrates. Analytic formulas are derived for the capacitance and compared with experimental results and results of Momentum simulation. The formulas are used for experimental characterization of thin ferroelectric films in a frequency range of 0.5 –50 GHz, where the film parameters (permittivity, loss tangent) are extracted from measured S-parameters.

## 1 Introduction

Coplanar patches sandwiched between dielectric layers, Fig.1, are used in capacitors, radiator arrays [1], as contact pads in ICs, calibration standard (open circuit) in on-chip microprobe measurements [2] etc. Experimental characterization of electric field dependences of thin film parameters [3] is another important area of application of this coplanar structure. Analysis of complex impedance (capacitance, losses) between the patches based on commercial tools, such as Momentum or similar, is time consuming and often not correct where substrate layers with permittivity several hundred to several thousand (as in the case of ferroelectrics [3]) are concerned.

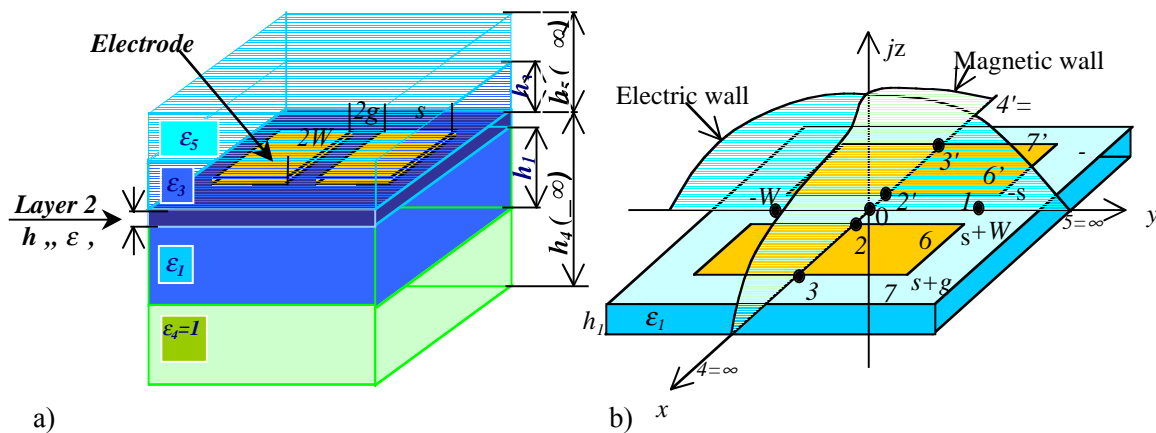


Fig.1 Multilayer substrate coplanar capacitor (a), and one its layers used in conformal transformations (b).

The analytic approximations for two layer substrate proposed in [4] are useful only for large aspect ratios  $W/g$ , Fig.1, since they do not take into account fringing fields at the both ends of the gap. In this work, we report closed form analytic approximations for the complex impedance (capacitance and losses) between symmetric rectangular patches sandwiched in a multilayer substrate using conformal mapping technique. It is assumed in derivations that the layers further from the strips have higher permittivity (losses). The formulas take into account

all fringing fields about the patches and are valid for large range of aspect ratios and permittivities.

## 2 Three layer substrate capacitor

For the sake of simplicity we take a capacitor on a three dielectric layer substrate, so that  $\epsilon_2 > \epsilon_1$ ,  $\epsilon_4 = \epsilon_5 = 1$ , and the thickness of layers 1, 2 and 3 ( $h_1, h_2, h_3$ ) are finite (Fig.1a). The condition  $\epsilon_2 > \epsilon_1$  is to ensure the magnetic wall approximation [3] at dielectric/dielectric interfaces (looking from the strips) used in evaluating partial capacitances. The capacitance of this structure is represented as a sum of partial capacitances of all layers:  $C = C_a + C_1 + C_2 + C_3$ . Electric and magnetic walls are assumed at the symmetry planes (as shown in Fig.1b for layer 1) to simplify the transformations. A sequence of conformal transformations in two mutually perpendicular planes [6] is employed to evaluate the partial capacitances. First, the capacitance between the plates,  $C_a$ , is evaluated assuming no dielectric layers. Then the capacitances associated with the dielectric layers 1, 2 and 3 are evaluated assuming dielectric layers with equivalent permittivity  $(\epsilon_1 - 1)$ , and  $(\epsilon_2 - \epsilon_1)$ . The original transformations used in capacitance evaluations will be presented in an extended paper. The resulting total capacitance is:

$$C = 2\epsilon_{e3}\epsilon_o \frac{K(k_a')}{K(k_a)} sK(k_o') \quad (1)$$

with effective dielectric constant

$$\epsilon_{e3} = 1 + (\epsilon_1 - 1)q_1 + (\epsilon_2 - \epsilon_1)q_2 + (\epsilon_3 - 1)q_3 \quad (2)$$

and filling factors

$$q_i = \frac{1}{2} \frac{K(k_{ii}')}{K(k_{ii})} \frac{K(k_a)}{K(k_a')} \frac{K(k_i')}{K(k_o')}; \quad i = 1, 2, 3 \quad (3)$$

where

$$k_i = \frac{\tanh\left(\frac{\pi g}{2h_i}\right)}{\tanh\left[\frac{\pi(s+g)}{2h_i}\right]}; \quad k_i' = \sqrt{1 - k_i^2}; \quad i = 1, 2, 3 \quad (4)$$

$$k_{ii} = \frac{1}{\cosh\left(\frac{\pi}{2} \frac{W}{sK(k_i)}\right)}; \quad k_{ii}' = \sqrt{1 - k_{ii}^2}; \quad i = 1, 2, 3 \quad (5)$$

$$k_a = \frac{1}{\cosh\left(\frac{\pi}{2} \frac{W}{sK(k_o)}\right)}; \quad k_a' = \sqrt{1 - k_a^2} \quad (6)$$

$$k_o = \frac{g}{s+g}; \quad k_o' = \sqrt{1 - k_o^2}$$

In case  $\epsilon_3 = 1$  one has a capacitor with plates on top of two layer substrate, while in case  $\epsilon_2 = \epsilon_1$  or  $\epsilon_1 = 1$  the plates of the capacitor are sandwiched between two dielectric layers 3 and 2 (or 1).

### 3 Losses

In general, there are losses associated with the partial capacitors (dielectric layers) discussed above. We will consider these losses using partial conductances  $G$ . Partial conductances are evaluated the same way as the partial capacitances using conductivities  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$  of corresponding layers. The total conductance of the capacitor is  $G=G_1+G_2+G_3$ , i.e. they are connected in parallel. By replacing in partial (not shown here) capacitances  $(\epsilon_1-1)\epsilon_0$ , by  $\sigma_1$ ,  $(\epsilon_2-\epsilon_1)\epsilon_0$ , by  $(\sigma_2-\sigma_1)$  and  $(\epsilon_3-1)\epsilon_0$  by  $\sigma_3$  we arrive at the following expression for the total conductance:

$$G = \sigma_1 s \left[ \frac{K(k'_{11})}{K(k_{11})} K(k'_1) - \frac{K(k'_{22})}{K(k_{22})} K(k'_2) \right] + \sigma_2 \frac{K(k'_{22})}{K(k_{22})} s K(k'_2) + \sigma_3 \frac{K(k'_{33})}{K(k_{33})} s K(k'_3) \quad (8)$$

In (8) we assumed the conductivity of layer 1 be larger than the conductivity of layer 2,  $\sigma_1 > \sigma_2$ . Formally, the loss tangent and conductivity of a dielectric (semiconductor) layer are related as  $\sigma_i = \epsilon_0 \epsilon \omega \tan \delta_i$ ,  $i=1, 2, 3$ . Using (1) and (8) one can compute the effective loss tangent and/or the Q-factor of the capacitor if the dimensions and parameters of the dielectric layer are specified:

$$\tan \delta_e = \frac{1}{Q} = \frac{G}{\omega C} \quad \omega = 2\pi f \quad \text{is frequency.} \quad (9)$$

### 4 Inverse problem - computations of the dielectric constants and losses from measured impedance

For a known (i.e. measured) capacitance  $C$  and Q-factor the dielectric constants and loss tangent of any of the layers are determined from (1), (2), and (9). For example, assuming the thickness of the layer 2 and parameters of all other layers known, the measured capacitance may be used to compute  $\epsilon_{e3}$  from (1), and the dielectric constant of layer 2 from (2):

$$\epsilon_2 = \epsilon_1 + \frac{\epsilon_{e3} - 1 - (\epsilon_1 - 1)q_1 - (\epsilon_3 - 1)q_3}{q_2} \quad (10)$$

Using the measured Q-factor ( $\tan \delta_e$ ), one can compute the loss tangent of layer 2:

$$\tan \delta_2 = \left[ \frac{C}{\epsilon_0 \epsilon_2 s K(k'_2) K(k'_{22})} \right] \tan \delta_e + \frac{\sigma_1}{\omega \epsilon_0 \epsilon_2} \left[ 1 - \frac{K(k'_{11})}{K(k_{11})} \frac{K(k_{22})}{K(k'_{22})} \frac{K(k'_1)}{K(k'_2)} \right] - \frac{\sigma_3}{\omega \epsilon_0 \epsilon_2} \frac{K(k'_{33})}{K(k_{33})} \frac{K(k_{22})}{K(k'_{22})} \frac{K(k'_3)}{K(k'_2)} \quad (11)$$

### 5 Results and discussions

First we compare the results obtained in this work with the capacitance model proposed in [4]. Depicted in Fig.2 are the dependences of capacitance of a two-layer substrate capacitor on the half-length  $W$  of the plates. In this simulation  $s=125 \mu\text{m}$ ,  $2g=4 \mu\text{m}$ ,  $\epsilon_2=100$ ,  $\epsilon_1=11.7$ ,  $h_1=500\mu\text{m}$ . The thickness of the plates is assumed to be  $t=0$ , and no losses are included. Extremely large discrepancy between the results is due to the fringing field capacitances at both ends of the gap, ignored in [4]. As it follows from comparison of Fig.2a and Fig.2b the error is larger for wider gapwidths with more fringing field. Only for very large aspect ratio,  $W/g$  ( $W \rightarrow \infty$  limit) and small gapwidth, the error becomes relatively small, decreasing slightly for larger thickness of layer 2.

A comparison with Momentum simulations is shown in Fig.3a. In these simulations  $2W=s=125 \mu\text{m}$ ,  $h_2=1.0 \mu\text{m}$ ,  $\epsilon_2=100$ ,  $\epsilon_1=11.7$ ,  $h_1=400 \mu\text{m}$ . Both the substrate 1 and film 2 are

assumed lossless. The strips are  $t=0$  thickness perfect conductors. The discrepancy is small for smaller gapwidths. However Momentum simulations “complain” about the high permittivity of the layer 2, and it is hard to judge where the discrepancy seen for larger gapwidths is coming from Momentum or conformal mapping results.

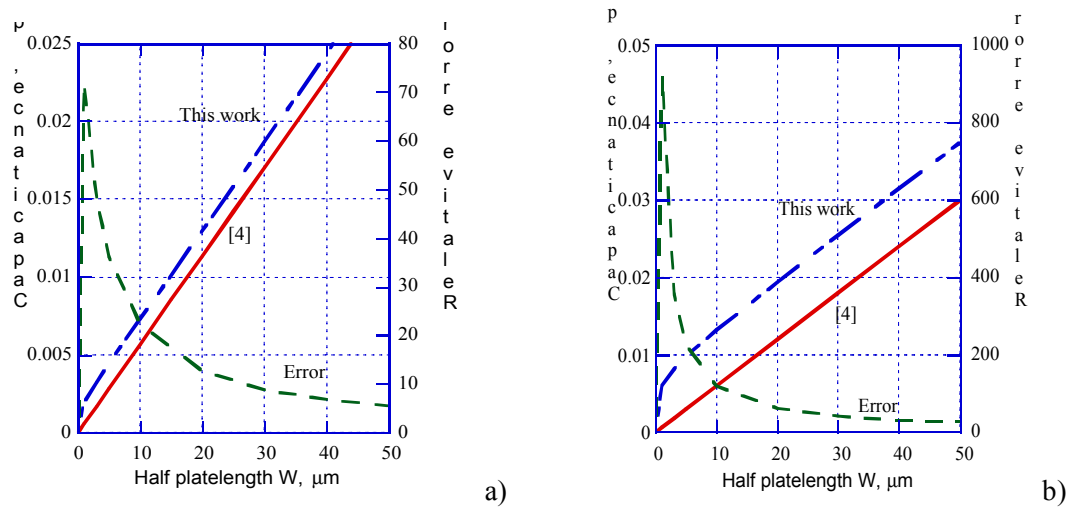


Fig.2 Capacitances and relative discrepancies vs. half-length  $W$  of the plates for  $h_2=0.5 \mu\text{m}$ ,  $2g=4 \mu\text{m}$  (a), and  $h_2=5.0 \mu\text{m}$ ,  $2g=20 \mu\text{m}$  (b)

Fig.3b shows experimental dependences of capacitances of two capacitors with identical plates fabricated on a silicon substrate with and without a ferroelectric film. The thickness of the ferroelectric film is  $0.5 \mu\text{m}$ , and the thickness of the silicon ( $\epsilon_f=11.7$ ) substrate is  $400 \mu\text{m}$ . Both capacitors are fabricated using the same mask, so that the dimensions of the plates are the same:  $2g=4 \mu\text{m}$ ,  $s=150 \mu\text{m}$ ,  $2W=125 \mu\text{m}$ . The dielectric constant of the ferroelectric film is very large, i.e.  $\epsilon_2 > 200$ , i.e.  $\epsilon_2 - \epsilon_f \approx \epsilon_2$ , hence the partial capacitance  $C_2$  due to the ferroelectric film may be approximated as a difference between measured capacitances with and without ferroelectric films. This capacitance is plotted in Fig.3b. The partial capacitance associated with the ferroelectric film computed using conformal mapping,  $C_2$ , also plotted in Fig.3b, is in a fairly good agreement with measured results. The measured results are used to compute the dielectric permittivity of the ferroelectric film (layer 2), which is plotted in Fig.3b. This results indicate that conformal mapping may be used both for correct modelling of the capacitor, and for the measurements of dielectric properties of the substrate layers. More examples of measurements will be reported in an extended paper.

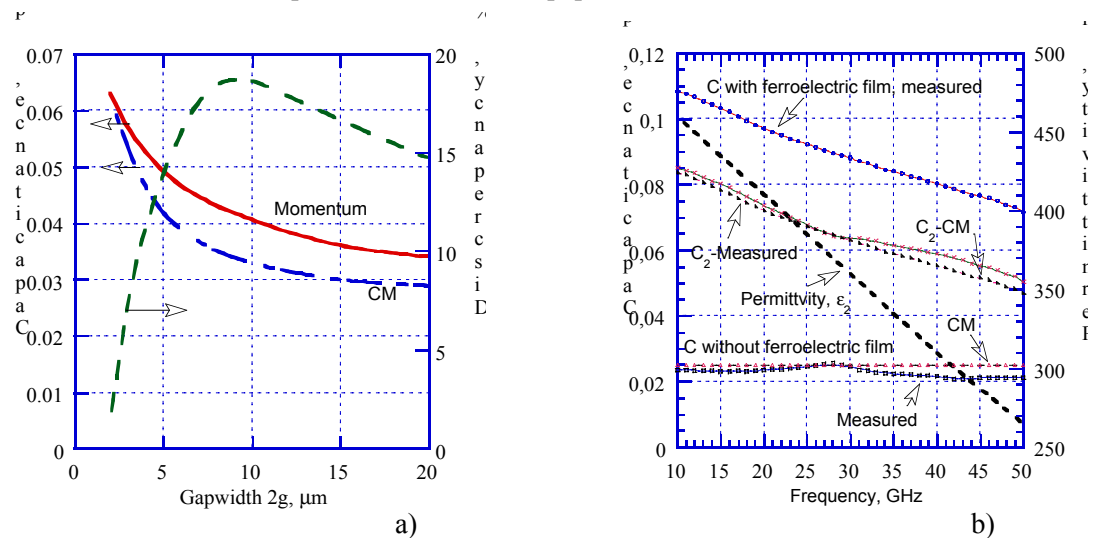


Fig.3 Comparisons with Momentum simulations (a), and experimental results (b) with conformal mapping (CM).

## 6 Conclusions

Simple analytic approximations for complex impedance of a three layer substrate coplanar capacitor are presented. The formulas may easily be extended to multilayer substrate capacitors provided the layers away from the strips have higher permittivity.

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