

## Topology Optimization of the Navier-Stokes Equations

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We consider the problem of optimal design of flow domains for Navier-Stokes flows in order to minimize a given performance functional. We attack the problem using topology optimization techniques, or control in coefficients, which are widely known in structural optimization of solid structures for their flexibility, generality, and yet ease of use and integration with existing FEM software. Topology optimization rapidly finds its way into other areas of optimal design, yet until recently it has not been applied to problems in fluid mechanics. The success of topology optimization methods for the minimal drag design of domains for Stokes fluids (see the study of Borrvall and Petersson [Internat. J. Numer. Methods Fluids, vol. 41, no. 1 pp. 77-107, 2003]) has lead to attempts to use the same optimization model for designing domains for incompressible Navier-Stokes flows.

We show that the optimal control problem obtained as a result of such a straightforward generalization is ill-posed, at least if attacked by the direct method of calculus of variations. We illustrate the two key difficulties with simple numerical examples and propose changes in the optimization model that allow us to overcome these difficulties. Namely, to deal with impenetrable inner walls that may appear in the flow domain we slightly relax the incompressibility constraint as typically done in penalty methods for solving the incompressible Navier-Stokes equations.

In addition, to prevent discontinuous changes in the flow due to very small impenetrable parts of the domain that may disappear, we consider so-called filtered designs, that has become a "classic" tool in the topology optimization toolbox. Technically, however, our use of filters differs significantly from their use in the structural optimization problems in solid mechanics, owing to the very unlike design parametrizations in the two models. We rigorously establish the well-posedness of the proposed model and then discuss related computational issues.