Global Optimality Conditions for Discrete and Nonconvex Optimization, With Applications to Lagrangian Heuristics and Column Generation

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The well-known and established global optimality conditions based on the Lagrangian formulation of an optimization problem are consistent if and only if the duality gap is zero. We develop a set of global optimality conditions that are structurally similar but are consistent for any size of the duality gap. This system characterizes a primal-dual optimal solution by means of primal and dual feasibility, primal Lagrangian epsilon-optimality, and, in the presence of inequality constraints, delta-complementarity, that is, a relaxed complementarity condition. The total size epsilon + delta of those two perturbations equals the size of the duality gap at an optimal solution. The characterization is further equivalent to a near-saddle point condition which generalizes the classic saddle point characterization of a primal-dual optimal solution in convex programming.
The system developed can be used to explain, to a large degree,when and why Lagrangian heuristics for discrete optimization are successful in reaching near-optimal solutions. Further, experiments on a set covering problem illustrate how the new optimality conditions can be utilized as a foundation for the construction of Lagrangian heuristics. Finally, we outline possible uses of the optimality conditions in column generation algorithms and in the construction of core problems, and illustrate our findings on instances of the generalized assignment problem.