

# EOOLT

**2007**

**Proceedings of the  
1st International Workshop on  
Equation-Based Object-Oriented  
Languages and Tools**

**Berlin, Germany, July 30, 2007,  
in conjunction with ECOOP**

**Editors**

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Linköping Electronic Conference Proceedings, No. 24  
Linköping University Electronic Press  
Linköping, Sweden, 2007

ISSN 1436-9915  
Forschungsberichte der Fakultät IV -Elektrotechnik und Informatik.  
Technische Universität Berlin  
Bericht Nr. 2007-11

ISSN 1650-3740 (online, Linköping University Electronic Press)  
<http://www.ep.liu.se/ecp/024/>  
ISSN 1650-3686 (print, Linköping University Electronic Press)

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## Preface

Computer aided modeling and simulation of complex systems, using components from multiple application domains, such as electrical, mechanical, hydraulic, control, etc., have in recent years witnessed a significant growth of interest. In the last decade, novel equation-based object-oriented (EOO) modeling languages, (e.g. Modelica, gPROMS, and VHDL-AMS) based on acausal modeling using equations have appeared. Using such languages, it has become possible to model complex systems covering multiple application domains at a high level of abstraction through reusable model components.

The interest in EOO languages and tools is rapidly growing in the industry because of their increasing importance in modeling, simulation, and specification of complex systems. There exist several different EOO language communities today that grew out of different application areas (multi-body system dynamics, electronic circuit simulation, chemical process engineering). The members of these disparate communities rarely talk to each other in spite of the similarities of their modeling and simulation needs.

The EOOLT workshop series aims at bringing these different communities together to discuss their common needs and goals as well as the algorithms and tools that best support them.

Despite the short deadlines and the fact that this is a new not very established workshop series, there was a good response to the call-for-papers. Thirteen papers and one presentation were accepted to the workshop program. All papers were subject to reviews by the program committee, and are present in these electronic proceedings. The workshop program started with a welcome and introduction to the area of equation-based object-oriented languages, followed by paper presentations and discussion sessions after presentations of each set of related papers.

On behalf of the program committee, the Program Chairmen would like to thank all those who submitted papers to EOOLT'2007. Special thanks go to David Broman who created the web page and helped with organization of the workshop. Many thanks to the program committee for reviewing the papers. EOOLT'2007 was hosted by the Technical University of Berlin, in conjunction with the ECOOP'2007 conference.

Berlin, July 2007

Peter Fritzson  
François Cellier  
Christoph Nytsch-Geusen



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# The use of the UML within the modelling process of Modelica-models

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**Abstract.** This paper presents the use of the Unified Modeling Language (UML) in the context of object-oriented modelling and simulation of hybrid systems with Modelica. The definition of a specialized version of UML for the graphical description and model based development of hybrid systems in Modelica – the UML<sup>H</sup> – was taken place in the GENSIM project [1, 2]. For a better support of the modelling process, an UML<sup>H</sup> editor with different views (class diagrams, statechart diagrams, collaboration diagrams) was implemented as a part of the Modelica simulation tool MOSILAB [3]. In the EOOLT-workshop the use of UML<sup>H</sup> and its semantics will be demonstrated by the development of a simplified model of a Pool-Billiard game in Modelica.

**Keywords:** UML<sup>H</sup>, modelling of hybrid systems, Modelica

## 1 Introduction

On the one hand, the Unified Modeling language (UML) is the established standard for the development and graphical description of object-oriented software systems [4]. The UML offers a couple of diagrams, which describe different views (e.g. class diagrams, statechart diagrams, collaboration diagrams) on object-oriented classes. On the other hand *Modelica* [5] is a pure textual simulation language, which means the program code of long and highly structured models might be often heavy to understand. Thus, the combination of UML and Modelica was taken place within the GENSIM project. An UML editor for the Modelica based simulation tool MOSILAB was developed, which can be used for describing and generating Modelica models in a graphical way [3].

In this paper a special forming of UML for the modelling process of hybrid systems, the UML<sup>H</sup>, will be presented. In a first step, the elements of the UML<sup>H</sup> and their semantics for the Modelica-language will be introduced. After that, the use of UML<sup>H</sup> will be illustrated by the example of a simplified version of a Pool-Billiard game.

## 2 UML<sup>H</sup> and Modelica

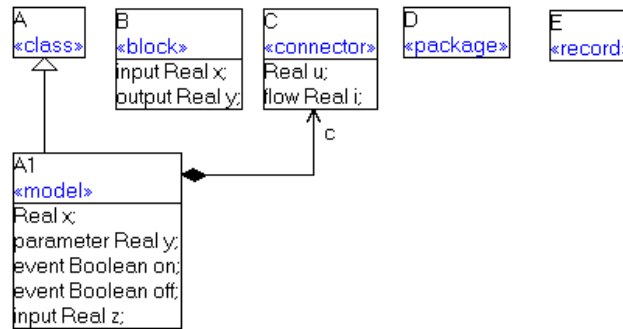
The development of the UML<sup>H</sup> was motivated by the following main reasons:

- to support the user within the modelling process of complex Modelica models in a easy manner,
- to have a method for the graphical documentation of the object-oriented construction of Modelica-models,
- to have a graphical analogy for the statechart extension of Modelica, which was introduced in the GENSIM project as a linguistic means of expression for model structural dynamics.

The UML<sup>H</sup> includes only a subset of the UML standard, which is necessary for the graphical description of Modelica models: the class diagram view, the statechart diagram view and the collaboration diagram view.

### 2.1 Class-diagrams

A class diagram in UML<sup>H</sup> is a rectangle, which contains in the upper part the class name and the Modelica class type. The optional lower part comprises the attributes (parameters, variables etc.) of the Modelica class. Inheritance and composition is expressed in the same way as in UML (compare with Fig. 1.)



**Fig. 1.** UML<sup>H</sup> class diagram

Starting from this graphical notation, the correspondent Modelica code can be generated automatically, e.g. with MOSILAB<sup>1</sup>. The following code shows the classes A, A1 and C, which are inner classes of the package *UML\_H*:

<sup>1</sup> In MOSILAB the UML<sup>H</sup> diagrams are directly integrated within the Modelica code by the use of specialized annotations.

```

package UML_H
  annotation(UMLH(ClassDiagram="<umlhclass><name>..."));

  class A
    annotation(UMLH(classPos=[31,53]));
  end A;

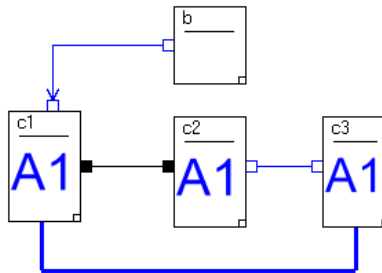
  model A1
    annotation(Icon(Text(extent=...,string="A1", ...));
    annotation(UMLH(classPos=[31,146]));
    extends A;
    event Boolean on;
    event Boolean off;
    Real x;
    input Real z;
    parameter Real y;
    C c;
    ...
  end A1;

  connector C annotation(UMLH(classPos=[192,54]));
    Real u;
    flow Real i;
  end C;
  ...
end UML_H;

```

## 2.2 Collaboration diagrams

Collaboration diagrams in UML<sup>H</sup> are also rectangles, which contain the object name and the type or the icon of the Modelica class, divided by a horizontal line. Four different connections types exist between the objects (see with Fig. 2.):



**Fig. 2.** UML<sup>H</sup> collaboration diagram.

- Type 1: connections of connector variables (thin black line with filled squares at the ends)

- Type 2: connections of scalar variables (thin blue line with unfilled squares at the ends)
- Type 3: connections of scalar input/output variables (thin blue line with an arrow and a unfilled square)
- Type 4: multi-connections as a mixture of different connection types, e.g. type 1 and type 2 (fat blue line)

The following Modelica-code expresses the collaboration-diagram of Fig. 2:

```

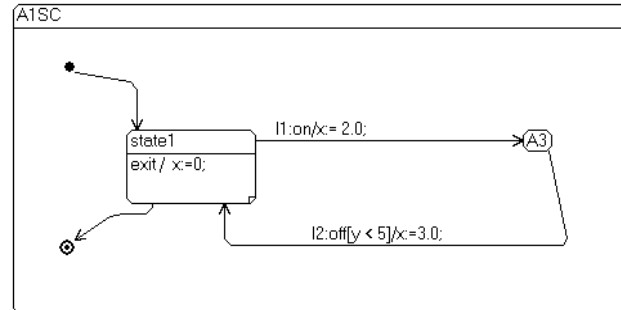
model System
  annotation(CompConnectors(CompConn(label="label2",
    points=[-81,52; -81,43; -24,43; -24,51])));
  UML_H.A1 c1 annotation(extent=[-87,72; -74,52]);
  UML_H.A1 c2 annotation(extent=[-57,71; -44,51]);
  UML_H.A1 c3 annotation(extent=[-30,71; -18,51]);
  UML_H.B b annotation(extent=[-57,91; -44,77]);
equation
  // connection type 1:
  connect(c1.c,c2.c)annotation(points=[-74,62;-57,62]);
  // connection type 2:
  c2.y=c3.y annotation(points=[-44,62; -30,62]);
  // connection type 4 (mixture of type 1 and 2):
  connect(c1.c,c3.c) annotation(label="label2");
  c1.x=c3.x annotation(label="label2");
  // connection type 3:
  b.y=c1.z annotation(points=[-57,84; -79,84; -79,72]);
end System;

```

### 2.3 Statechart diagrams

A statechart diagram in UML<sup>H</sup> is represented as a rectangle with round corners. In general, a statechart diagram contains several states and the transition definition between the states. Figure 3 shows four different types of States:

- Initial states, symbolized with a black filled circle,
- Final states, symbolized with a point in a unfilled circle,
- Atomic states, with a flat internal structure,
- Normal states, which can contain additional entry or exit actions and can be sub-structured in further statechart diagrams.



**Fig. 3.** UML<sup>H</sup> statechart diagram.

The transitions between the states are specified with an optional label, an event, an optional guard and the action part. The following code shows the corresponding code of the statechart section<sup>2</sup> of the model A1:

```

model A1
...
statechart
  state A1SC extends State
    annotation(extent=[-88,86; 32,27]);
    state State1
      extends State;
      exit action x:=0; end exit;
    end State1;
  State1 state1 annotation(extent=[-66,62; -41,48]);
  State A3 annotation(extent=...);
  State I5(isInitial=true)...;
  State F7(isFinal=true)...;
  transition I5->state1 end transition
    annotation(points=[-76,73;-64,71; -64,62]);

  transition l1:state1->A3 event on action x:= 2.0;
  end transition annotation(points=...);

  transition l2:A3->state1 event off guard y < 5
    action x:=3.0;
  end transition ...;

  transition state1->F7 end transition annotation...;
end A1SC;
end A1;
  
```

<sup>2</sup> The new introduced statechart section is part of the Modelica language extension for model structural dynamics [6].

### 3 Example for UML<sup>H</sup>-modeling

The modelling and simulation of a simplified Pool-Billiard game shall demonstrate the advantages of the graphical modelling with UML<sup>H</sup>.

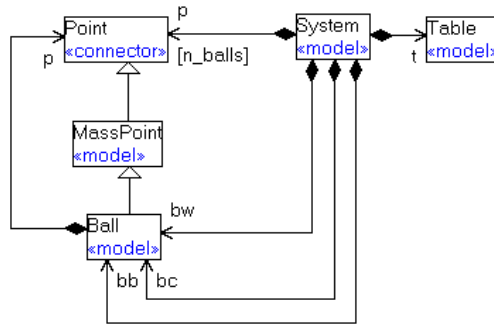
#### 3.1 Model of a Pool-Billiard game

The system model of the Pool-Billiard game includes sub models for the balls and the table. The configuration of the system model is illustrated in Fig. 4. Following simplifications are assumed in the model:

- The Pool-Billiard game knows only a black, a white and a coloured ball.
- The table has only one hole instead of 6 holes.
- The collision-model is strong simplified.
- The balls are moving between the collisions and reflections only on straight directions in the dimension x and y.
- The reflections on the borders take place ideal without any friction losses.
- The rolling balls are slowed down with a linear friction coefficient  $f_r$ :

$$m \cdot \frac{dv_x}{dt} = -v_x \cdot f_r \text{ and } \frac{dx}{dt} = v_x \quad (1)$$

$$m \cdot \frac{dv_y}{dt} = -v_y \cdot f_r \text{ and } \frac{dy}{dt} = v_y \quad (2)$$



**Fig. 4.** UML<sup>H</sup> class diagram of the Pool-Billard model

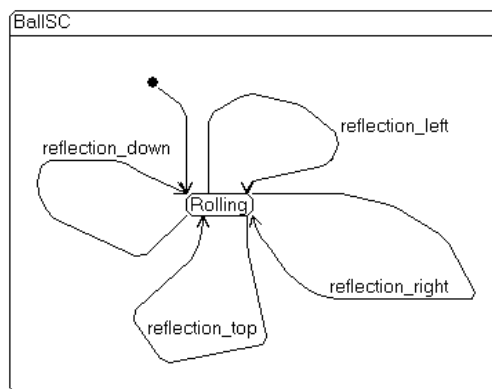
Fig. 5 shows the statechart diagram for the ball model. After the model enter the state *Rolling*, the ball knows four reflection events, for the four different borders of

the billiard table. Depending from the border event, the new initial conditions (velocity and position) after the reflections are set and the ball enters again the state *Rolling*:

```

model Ball
  extends MassPoint(m=0.2);
  parameter SIunits.Length width;
  parameter SIunits.Length length;
  parameter SIunits.Length d = 0.0572 "diameter";
  parameter Real f_r = 0.1 "friction coefficient";
  SIunits.Velocity v_x, v_y;
  event Boolean reflection_left(start = false);
  ...
  equation
    reflection_left = if x < d/2.0;
    m * der(v_x) = - v_x * f_r;
    der(x) = v_x;
    ...
  statechart
    state BallSC extends State;
      State Rolling;
      State startState(isInitial=true);
      ...
      transition startState -> Rolling
      end transition;
      ...
      transition Rolling->Rolling event reflection_left
        action v_x := -v_x; x := d/2.0;
      end transition;
      ...
    end BallSC;
end Ball;

```



**Fig. 5.** UML<sup>H</sup> statechart diagram of the ball model

On the system level two different states (*Playing* and *GameOver*) and two types of events - the collision of two balls and the disappearance of a ball in the hole (compare with Fig. 6 and the program code) exist. If the white ball enters the hole, the game

will be continued with the white ball from the starting point (transition from *Playing* to *Playing*). If the black ball disappears in the hole, the statechart is triggered to the state *GameOver*. If the coloured ball disappeared, the game is reduced for one ball - *remove(bc)* - and the numerical calculation will be continued with a smaller equation system<sup>3</sup>:

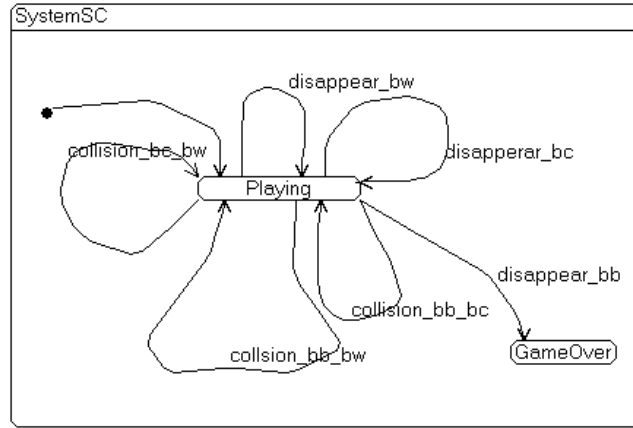


Fig. 6. UML<sup>H</sup>-statechart-diagram for the model

```

model System
    parameter SIunits.Length d_balls = 0.0572;
    parameter SIunits.Length d_holes = 0.15;
    dynamic Ball bw, bb, bc; //structural dynamic submodels
    Table t(width = 1.27, length = 2.54);
    event Boolean disappear_bw(start = false);
    event Boolean disappear_bb(start = false);
    event Boolean disappear_bc(start = false);
    event Boolean collision_bw_bb(start = false);
    ...
    event Boolean push(start = false);

    equation
    push = if fabs(bw.v_x)<0.005 and fabs(bw.v_y) < 0.005;
    disappear_bw = if((p[1].x-0)^2+(p[1].y-0)^2)^0.5
                    < d_holes;
    collision_bw_bb = if((p[2].x-p[1].x)^2
                        +(p[2].y-p[1].y)^2)^0.5 < d_balls;
    ...

    statechart
    state SystemSC extends State;
    State Playing, startState(isInitial=true), GameOver;
    ...
    transition startState -> Playing action
        bw := new Ball(d = d_balls,...); add(bw);
    
```

<sup>3</sup> This model reduction mechanism takes place by using the model structural dynamics from MOSILAB [1].



```

    bb := new Ball(...); add(bb);
    bc := new Ball(...); add(bc);
end transition;

transition Playing->Playing event disappear_bw action
...
remove(bw);
bw := new Ball(x(start=1.27/2.9), y(start=0.6));
end transition;

transition Playing->Playing event disappear_bc action
...
remove(bb);
end transition;

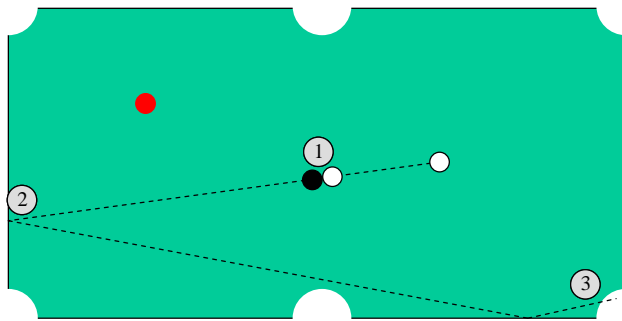
transition Playing -> GameOver event disappear_bb
end transition;

transition Playing->Playing event collision_bw_bb
action
v_x := bw.v_x; v_y := bw.v_y;
bw.v_x := bb.v_x; bw.v_y := bb.v_y;
bb.v_x := v_x; bb.v_y := v_y;
end transition;
end SystemSC;
end System;

```

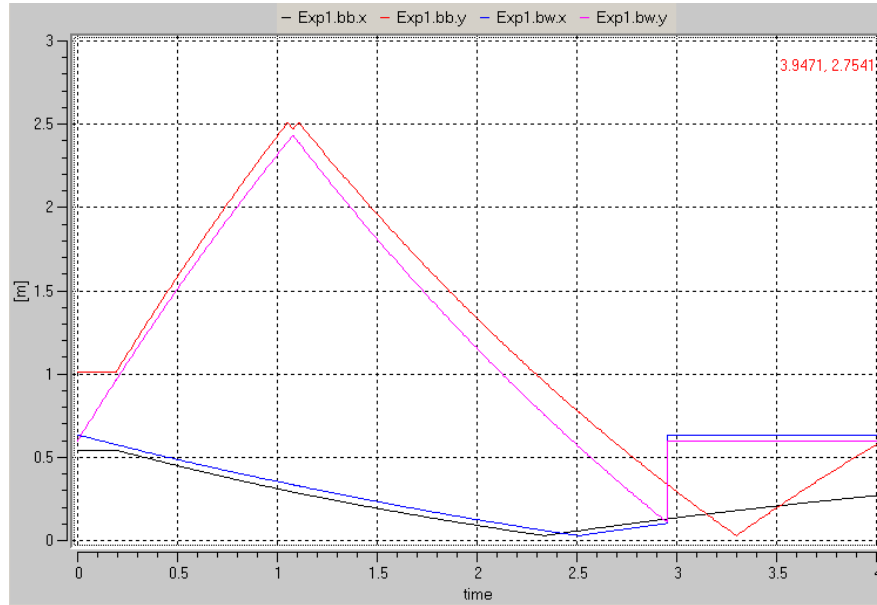
### 3.1 Simulation experiment

The following simulation experiment illustrates the previous explained behaviour of the Pool-Billiard game. The parameter of the model are set in a manner, that all different types of events (1: collision of two balls, 2: reflection on a border, 3: disappearing in the hole) are present during the simulation experiment (see Fig. 7).

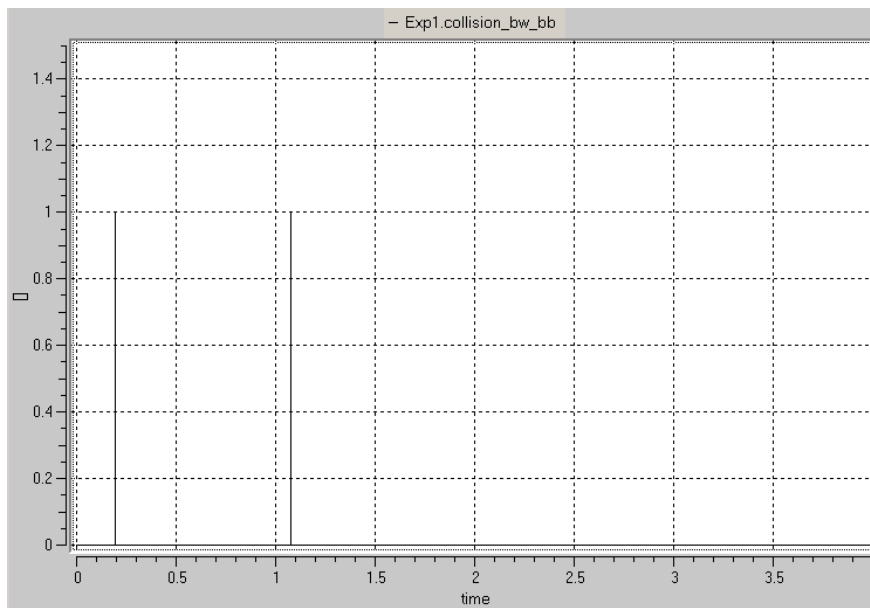


**Fig. 7. Event types in the Pool-Billiard game**

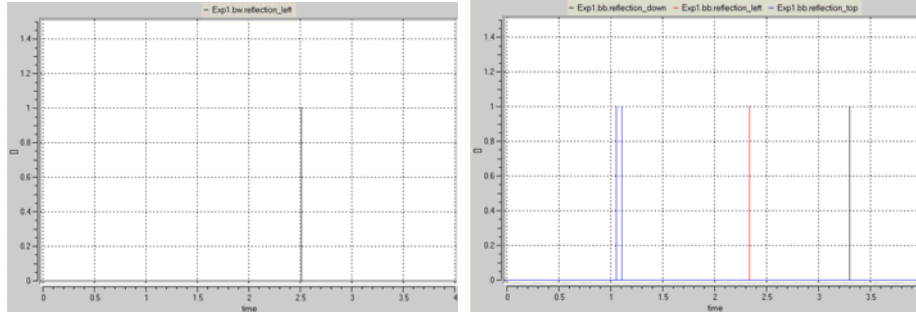
Figure 8 show the positions and the Figures 9 and 10 the reflection and collision events of the white and the black ball during a simulation period of 4 seconds.



**Fig. 8. x- and y-positions of the white and the black ball**



**Fig. 9. Collision events of the white and the black ball**



**Fig. 10. Reflection events of the white ball (left) and the black ball (right)**

After 0.2 seconds, the white ball collides with the black ball. After 1 second, the black ball is reflected twice in a short time period on the top side on the billiard-table and both balls collide again between its reflections. After 2.3 and 2.5 seconds the balls reflect on the left border. At 2.95 seconds the white ball drops into the hole. At the end, the white ball is set again on its starting position.

## 4 Conclusions

The example of the modelling and simulation of a Pool-Billiard game has shown the advantages of the graphical modelling with UML<sup>H</sup> for Modelica models. With UML<sup>H</sup>, the design of a complex system model in Modelica begins with the drawing of its model structure. The class diagrams und the collaboration diagrams describe the object-oriented model construction and the statechart diagrams are used for the formulation of the event-driven model behaviour. If the Modelica tool supports code generation like MOSILAB, the Modelica code can be obtain automatically from the UML<sup>H</sup> model. This pure code has to be filled up by the user with model equations (physical behaviour) of the modelled system.

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# Towards Unified System Modeling with the ModelicaML UML Profile

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**Abstract.** In order to support the development of complex products, modeling tools and processes need to support co-design of software and hardware in an integrated way. Modelica is the major object-oriented mathematical modeling language for component-oriented modeling of complex physical systems and UML is the dominant graphical modeling notation for software. In this paper we propose ModelicaML UML profile as an integration of Modelica and UML. The profile combines the major UML diagrams with Modelica graphic connection diagrams and is based on the System Modeling Language (SysML) profile.

## 1 Introduction

The development in system modeling has come to the point where complete modeling of systems is possible, e.g. the complete propulsion system, fuel system, hydraulic actuation system, etc., including embedded software can be modeled and simulated concurrently. This does not mean that all components are dealt with down to the very smallest details of their behavior. It does, however, mean that all functionality is modeled, at least qualitatively.

In this paper, a UML profile for Modelica, named ModelicaML, is proposed. The ModelicaML UML profile is based on the OMG SysML™ (Systems Modeling Language) profile and reuses its artifacts required for system specification. SysML diagrams are also extended to support all Modelica constructs. We argue that with ModelicaML system engineers are able to specify entire systems, starting from requirements, continuing with behavior and finally perform system simulations.

## 2 SysML and Modelica

The Unified Modeling Language (UML) has been created to assist software development processes by providing means to capture software system structure and behavior. This evolved into the main standard for Model Driven Development [5].

The System Modeling Language (SysML) [4] is a graphical modeling language for systems engineering applications. SysML was developed by systems engineering ex-

perts, and was adopted by OMG in 2006. SysML is built on top of UML and tailored to the needs of system engineers by supporting specification, analysis, design, verification and validation of broad range of systems and system-of-systems.

The main goal behind SysML is to unify and replace different document-centric approaches in the system engineering field with a single systems modeling language. A single model-centric approach improves communication, assists to manage complex system design and allows its early validation and verification.

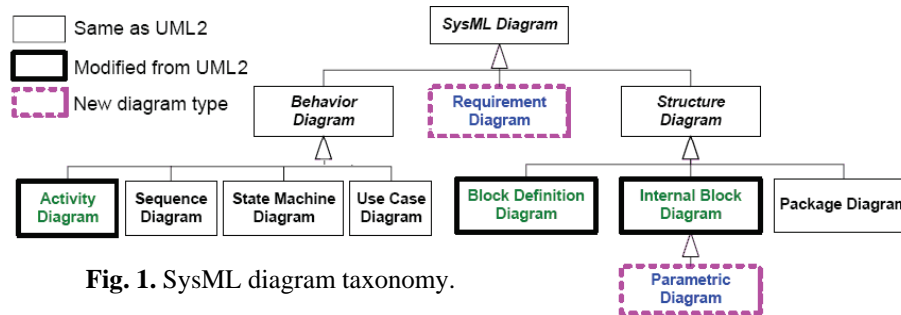


Fig. 1. SysML diagram taxonomy.

The taxonomy of SysML diagrams is presented in Fig. 1. For a full description of SysML see (SysML, 2006) [4]. The major SysML extensions compared to UML are:

- *Requirements diagrams* support requirements presentation in tabular or in graphical notation, allows composition of requirements and supports traceability, verification and “fulfillment of requirements”.
- *Block diagrams* extend the Composite Structure diagram of UML2.0. The purpose of this diagram is to capture system components, their parts and connections between parts. Connections are handled by means of connecting ports which may contain data, material or energy flows.
- *Parametric diagrams* help perform engineering analysis such as performance analysis. Parametric diagrams contain constraint elements, which define mathematical equations, linked to properties of model elements.
- *Activity diagrams* show system behavior as data and control flows. Activity diagrams are similar to Extended Functional Flow Block Diagrams, which are already widely used by system engineers. Activity decomposition is supported. by SysML.
- *Allocations* are used to define mappings between model elements: For example, certain Activities may be allocated to Blocks (to be performed by the block).

SysML block definitions (Fig. 2) can include properties to specify block parts, values and references to other blocks. A separate compartment is dedicated for each of these features. To describe the behavior of a block the “Operations” compartment is reused from UML and it lists operations that describe certain behavior. SysML defines a special form of an optional compartment for constraint definitions owned by a block. A “Namespace” compartment may appear if nested block definitions exist for a block. A “Structure” compartment may appear to show internal parts and connections between parts within a block definition.

SysML defines two types of ports: standard ports and flow ports. Standard ports, which are reused from UML, are service-oriented ports required or provided by a block. Flow ports specify interaction points through which items may flow between

blocks, and between blocks and environment. A flow port definition may include single item specification or complex flow specification through the FlowSpecification interface; flow ports define what “can” flow between the block and its environment. Flow direction can be specified for a flow port in SysML. SysML also defines a notion of Item flows that specify “what” does flow in a particular usage context.

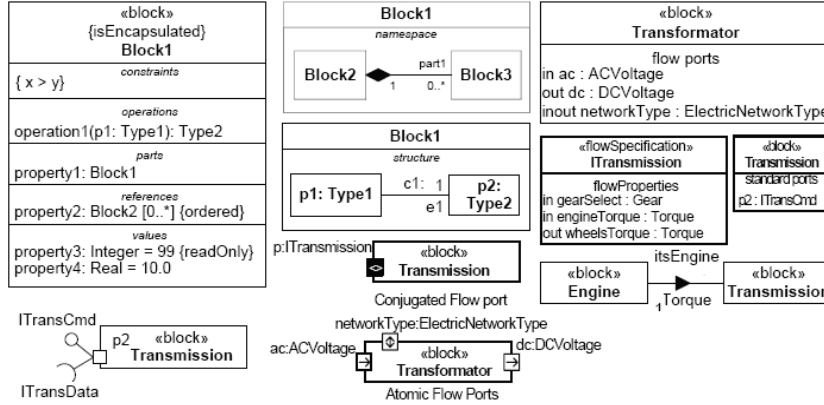


Fig. 2. SysML block definitions.

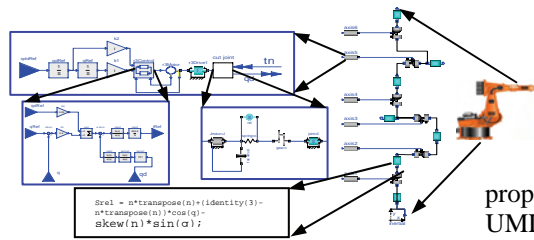
## 2.1 Modelica

Modelica [2] [3] is a modern language for equation-based object-oriented mathematical modeling primarily of physical systems. Several tools, ranging from open-source as OpenModelica [1], to commercial like Dymola [11] or MathModelica [10] support the Modelica specification.

The language allows defining models in a declarative manner, modularly and hierarchically and combining various formalisms expressible in the more general Modelica formalism. The multidomain capability of Modelica allows combining electrical, mechanical, hydraulic, thermodynamic, etc., model components within the same application model. In short, Modelica has improvements in several important areas:

- *Object-oriented mathematical modeling.* This technique makes it possible to create model components, which are employed to support hierarchical structuring, reuse, and evolution of large and complex models covering multiple technology domains.
- *Physical modeling of multiple application domains.* Model components can correspond to physical objects in the real world, in contrast to established techniques that require conversion to “signal” blocks with fixed input/output causality. In Modelica the structure of the model naturally correspond to the structure of the physical system in contrast to block-oriented modeling tools.
- *Acausal modeling.* Modeling is based on *equations* instead of assignment statements as in traditional input/output block abstractions. Direct use of equations significantly increases re-usability of model components, since components adapt to the data flow context in which they are used.

Hierarchical system architectures can easily be described with Modelica thanks to its powerful component model. The *Components* are connected via the *connection mechanism* realized by the Modelica system, which can be visualized in connection diagrams. The *component framework* realizes components and connections, and ensures that communication works over the connections. For systems composed of *acausal* components with behavior specified by equations, the direction of data flow, i.e., the *causality* is initially unspecified for connections between those components and the causality is automatically deduced by the compiler when needed. Components have well-defined *interfaces* consisting of ports, also known as *connectors*, to the external world. A component may internally consist of other connected components, i.e., *hierarchical modeling* as in Fig. 3.



**Fig. 3.** Hierarchical model of an industrial robot.

## 2.2 SysML vs. Modelica

The System Modeling Language (SysML) has recently been proposed and defined as an extension of UML targeting at systems engineers. As previously stated, the goal of SysML is to unify different

approaches and languages used by system engineers into a single standard. SysML models may span different domains, for example, electrical, mechanical and software. Even if SysML provides means to describe system behavior like Activity and State Chart Diagrams, the precise behavior can not be described and simulated. In that respect, SysML is rather incomplete compared to Modelica.

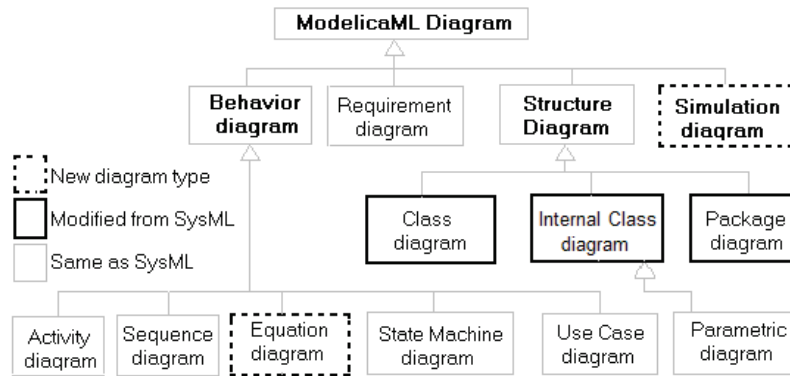
Modelica also, was created to unify and extend object-oriented mathematical modeling languages. It has powerful means for describing precise component behavior and functionality in a declarative way. Modelica models can be graphically composed using Modelica connection diagrams which depict the structure of designed system. However, complex system design is more than just a component assembly. In order to build a complex system, system engineers have to gather requirements, specify system components, define system structure, define design alternatives, describe overall system behavior and perform its validation and verification.

## 3 ModelicaML: a UML profile for Modelica

ModelicaML reuses several diagrams types from SysML without any extension, extends some of them, and also provides several new ones. The ModelicaML diagram overview is shown in Fig. 4. Diagrams are grouped into four categories: Structure, Behavior, Simulation and Requirement. In the following we present the most important ModelicaML profile diagrams. The full description of the ModelicaML profile is presented in [8]. The most important properties of the ModelicaML profile are outlined in the following:



- The ModelicaML profile supports modeling with all Modelica constructs and properties i.e. restricted classes, equations, generics, discrete variables, etc.
- Using ModelicaML diagrams it is possible to describe multiple aspects of a system being designed and thus support system development process phases such as requirements analysis, design, implementation, verification, validation and integration.
- ModelicaML is partly based on SysML, but reuses and extends its elements.
- The profile supports mathematical modeling with equations since equations specify behavior of a (Modelica) system. Algorithm sections are also supported.
- Simulation diagrams are introduced to model and document simulation parameters and results in a consistent and usable way.
- The ModelicaML meta-model is consistent with SysML in order to provide SysML-to-ModelicaML conversion.



**Fig. 4.** ModelicaML diagrams overview.

Three SysML diagram types have been partly reused and changed for the ModelicaML profile. The rest of the diagram types we used in ModelicaML unchanged:

- The SysML Block Definition Diagram has been updated and renamed to *Modelica Class Diagram*.
- The SysML Internal Block Diagram has been updated and renamed to *Modelica Internal Class Diagram* (some of the SysML constructs are disabled).
- The *Package Diagram* has been changed in order to fully support the Modelica language (i.e. Modelica package constants, Generic Packages, etc).
- Other SysML diagram types such as Use Case Diagram, Activity Diagrams and Allocations, and State Machine Diagrams are included in ModelicaML without modifications. ModelicaML reuses Sequence Diagrams from SysML and changes the semantics of message passing. Modelica doesn't support method declaration within a single class but supports declaration of functions as a restricted class type.

Thus, the following diagram types are available in the ModelicaML profile:

- The *Modelica Class Diagram* usually describes class definitions and their relationships such as inheritance and containment.

- The *Modelica Internal Class Diagram* describes the internal class structure and interconnections between parts.
- The *Package Diagram* groups logically connected user defined elements into packages. In ModelicaML the primarily purpose of this diagram is to support the specifics of the Modelica packages.
- Activity, Sequence, State Machine, Use Case, Parametric and Requirements diagrams have been reused without modification from SysML.
- Two new diagrams, *Simulation Diagram* and *Equation Diagram*, not present in SysML, have been included in the ModelicaML profile.

### 3.1 Package Diagram

A UML Package is a general purpose model element for grouping other elements within a separate namespace. With a help of packages, designers are able group elements to correspond to different structures/views of a system. ModelicaML extends UML packages in order to support Modelica packaging features, in particular: package inheritance, generic packages, constant declaration within a package, package “instantiation” and renaming import (see [2] for Modelica packages details).

A diagram which contains package elements and their relationships is called a Package Diagram. Modelica packages have a hierarchical structure containing package elements as nodes. In Modelica, packages are used to structure model elements into libraries. A snapshot of the Modelica Standard Library hierarchy is shown in Fig. 5 using UML notation. Package nodes in the hierarchy are connected via the package containment link as in the example in Fig. 6.

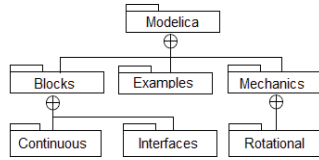


Fig. 5. Package hierarchy modeling.

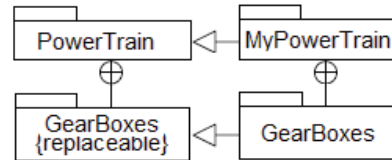


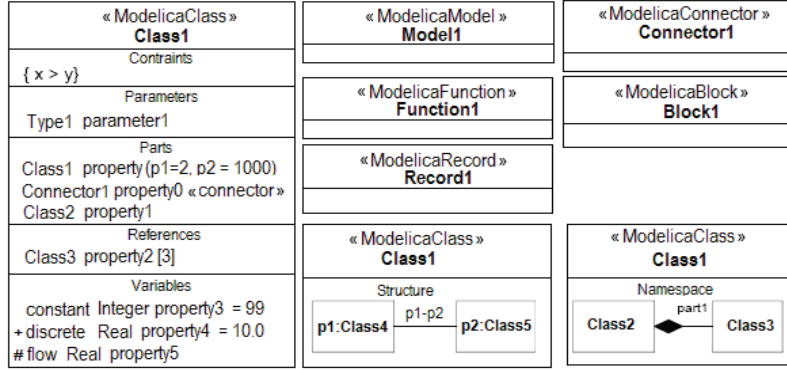
Fig. 6. Package hierarchy modeling

### 3.2 Modelica Class Diagrams

Modelica uses restricted classes such as class, model, block, connector, function and record to describe a system. Modelica classes have essentially the same semantics as SysML blocks specified in [4] and provide a general-purpose capability to model systems as hierarchies of modular components. ModelicaML extends SysML blocks by defining features which are relevant or unique to Modelica.

The purpose of the Modelica Class Diagram is to show features of Modelica classes and relationships between classes. Additional kind of dependencies and associations between model elements may also be shown in a Modelica Class Diagram. For example, behavior description constructs – equations, may be associated with particular Modelica Classes. The detailed description of structural features of Modeli-

caML is provided below. ModelicaML structural extensions are defined based on the SysML block definition outlined in section 2.



**Fig. 7.** ModelicaML class definitions.

### 3.2.1 Modelica Class Definition

The graphical notation of ModelicaML class definitions is shown in Fig. 7. Each class definition is adorned with a stereotype name that indicates the class type it represents. The ModelicaML Class Definition has several compartments to group its features: parameters, parts, variables. We designed the parameters compartment separately from variables because the parameters need to be assigned values in order to simulate a model (see the Simulation Diagram later on). Some compartments are visible by default; some are optional and may be shown on ModelicaML Class Diagram with the help of a tool. Property signatures follow the Modelica textual syntax and not the SysML original syntax, reused from UML. A ModelicaML/SysML tool may allow users to choose between UML or Modelica style textual signature presentation. Using Modelica syntax on a diagram has the advantage of being more compatible with Modelica and being more straightforward for Modelica users. The Modelica syntax is quite simple to learn even for users not acquainted with Modelica.

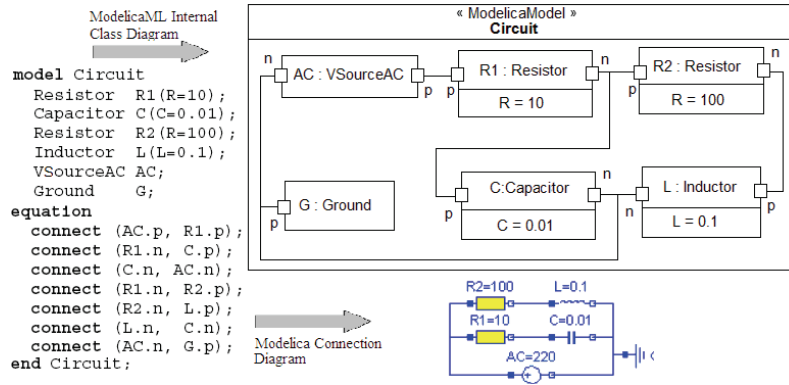
ModelicaML provides extensions to SysML in order to support the full set of Modelica constructs and features. For example, ModelicaML defines unique class definition types ModelicaClass, ModelicaModel, ModelicaBlock, ModelicaConnector, ModelicaFunction and ModelicaRecord that correspond to class, model, block, connector, function and record restricted Modelica classes. We included the Modelica specific restricted classes because a modeling tool needs to impose their semantic restrictions (for example a record cannot have equations, etc).

### 3.2.2 Modelica Internal Class Diagram

The Modelica Internal Class Diagram is based on the SysML Internal Block Diagram but the connections are based on ModelicaConnector. The Modelica Class Diagram defines Modelica classes and relationships between classes, like generalizations, association and dependencies, whereas a Modelica Internal Class Diagram shows the internal structure of a class in terms of parts and connections. The Modelica Internal Class Diagram is similar to Modelica connection diagram, which presents parts in a

graphical (icon) form. An example Modelica model presented as a Modelica Internal Class diagram is shown in Fig. 8.

Usually Modelica models are presented graphically via Modelica connection diagrams (Fig. 8, bottom). Such diagrams are created by the modeler using a graphic connection editor by connecting together components from available libraries. Since both diagram types are used to compose models and serve the same purpose, we briefly compare the Modelica connection diagram to the Modelica Internal Class Diagram. The main advantage of the Modelica connection diagram over the Internal Class Diagram is that it has better visual comprehension as components are shown via domain-specific icons known to application modelers. Another advantage is that Modelica library developers are able to predefine connector locations on an icon, which are related to the semantics of the component. In the case of a ModelicaML Internal Class Diagram a SysML/ModelicaML tool should somehow point out at which side of a rectangular presentation of a part to place a port (connector).



**Fig. 8.** ModelicaML Internal Class vs. Modelica Connection Diagram.

One of the advantages of the Internal Class Diagram is that it directly supports nested structures. However, nested structures are also available behind the icons in a Modelica connection diagram, thus using the drawing area more effectively.

The main advantage of the Internal Class Diagram is that it highlights top-level Modelica model parameters and variables specification in separate compartments.

Other SysML elements, such as Activities and Requirements which do not exist in Modelica but are very important for additional model specification can be combined with both Internal Class Diagram and Modelica connection diagrams.

### 3.4 Parametric Diagrams vs. Equation Diagrams

SysML defines Constraint blocks which specify mathematical expressions, like equations, to constrain physical properties of a system. Constraint blocks are defined in the Block Definition diagram and can be packaged into domain-specific libraries for later reuse. There is a special diagram type called Parametric Diagram which relates block parameters with certain constraints blocks. The Parametric Diagram is included in ModelicaML without any modifications to keep the compatibility with SysML.

The Modelica class behavior is usually described by equations, which also constrain Modelica class parameters, and have a domain-specific usage. SysML constraint blocks are less powerful means of domain model description than Modelica equations. Modelica equations include some type of equations, which cannot be modeled using Constraint blocks, i.e.: `if`, `for`, `when` equations. Also, modeling complexity is an issue, as for example in Fig. 9 there are only four equations, and the diagram is already quite complex. However, grouping constraint blocks into libraries can be useful for system engineers who use Modelica and SysML. SysML Parametric diagram may be used during the initial design phase, when equations related to a class are being identified using Parametric Diagrams and finally associated (via an Equation Diagram) with a Modelica class or set of classes.

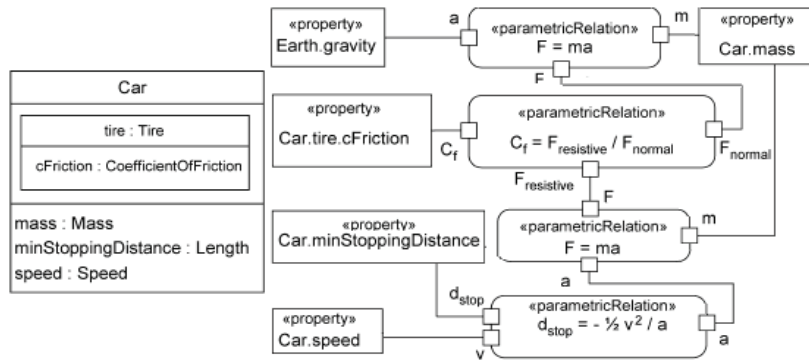


Fig. 9. Parametric Diagram Example

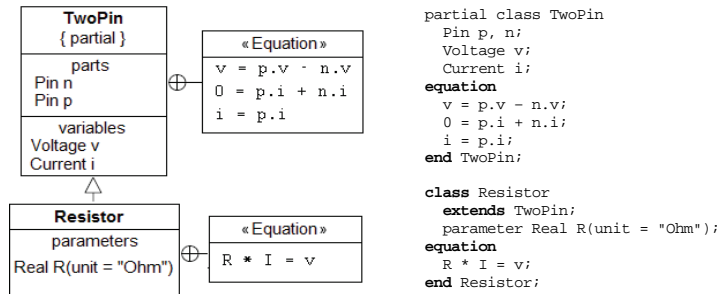


Fig. 10. Equation modeling example with a Modelica Class Diagram.

In Fig. 10, Fig. 11 we present examples of behavior specification via Equation Diagrams in ModelicaML. Equations do not prescribe a certain data flow direction which means that the order in which equations appear in a model do not influence their meaning and semantics. The only requirement for a system of equations is to be solvable. For further details about Modelica equations, see [2]. Besides simple equality equations, Modelica allows other kind of equations be presented within a model. For each of such kind of equations (i.e. when/if/initial equations) ModelicaML defines a graphical construct. It's up to designer to decide whether to use simple equations block representation or specific construct for equation modeling. Algorithm sections are modeled similar to equations, as text.

With a help of Equation Diagram top-down modeling approach is applied to behavior modeling. First, the primarily equations may be captured, then conditional constructs applied, equations text description substituted with mathematical expressions or even equations refactored by moving to other classes. In the similar way as Modelica classes are grouped by physical domain libraries, common equations can be packaged into domain-specific libraries and be reused during a design process. Moreover, equation constructs shown on Equation Diagram can be linked to Activity elements or with Requirement elements to show that a specific requirement has been fulfilled.

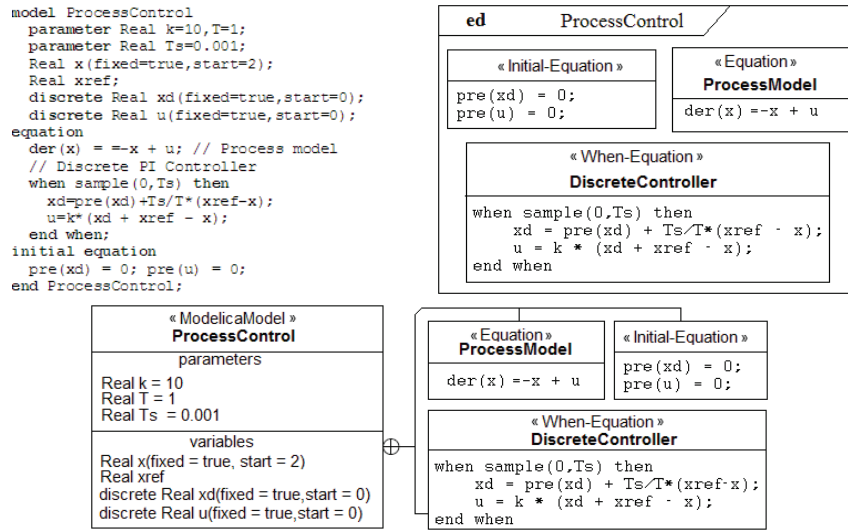


Fig. 11. ModelicaML nested/extern Equation diagrams

### 3.5 Simulation Diagram

ModelicaML introduces a new diagram type, called Simulation Diagram (Fig. 12), used for simulation modeling. Simulation is usually performed by a simulation tool which allows parameter setting, variable selection for output and plotting. The Simulation Diagram may be used to store any simulation experiment, thus helping to keep the history of simulations and its results. When integrated with a modeling and simulation environment, a simulation diagram may be automatically generated.

The Simulation Diagram provides facilities for simulation planning, structured presentation of parameter passing and simulation results. Simulations can be run directly from the Simulation Diagram. Association of simulation results with requirements from a domain expert and additional documentation (e.g. by: Note, Problem Rationale text boxes of SysML) are also supported by the Simulation Diagram. The Simulation Diagram introduces new diagram elements: "Parameter" element and two stereotyped dependency associations, "simParameter" and "simResults". Parameter values are associated with a class via simParameter for a simulation. Simulation results are associated with a model via simResults which specify which variable is to be plotted and for what time interval.

For simulation purposes, the Simulation Diagram can be integrated with any Modelica modeling and simulation environment. We are currently in the process of designing a ModelicaML development environment which integrates with the OpenModelica modeling and simulation environment.

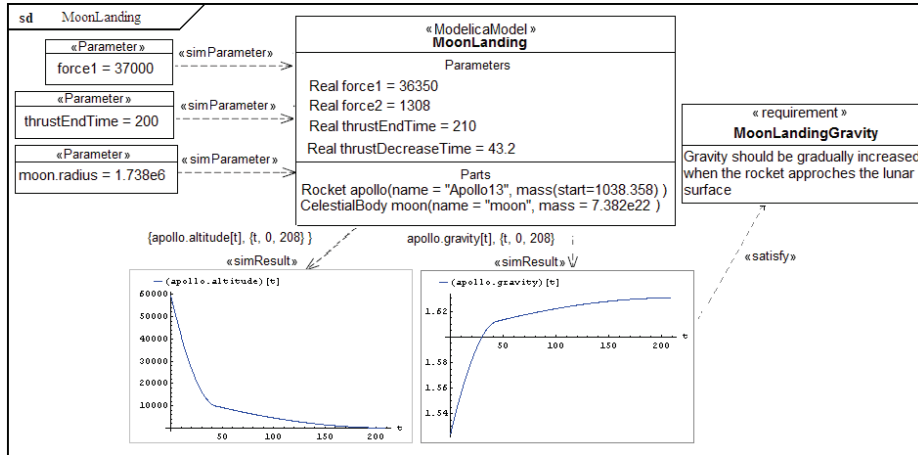


Fig. 12. Simulation Diagram example.

## 4 Conclusion and Future Work

In this paper we propose the ModelicaML profile that integrates Modelica and UML. UML Statecharts and Modelica have been previously integrated, see e.g. [9][15]. SysML is rather new but it was already adopted for system on chip design [13] evaluated for code generation [14] or extended with bond graphs support [12].

The support for Modelica in ModelicaML allows precisely defining, specifying and simulating physical systems. Modelica provides the means for defining behavior for SysML block diagrams while the additional modeling capabilities of SysML provides additional modeling and specification power to Modelica (e.g. requirements and inheritance diagrams, etc).

As a future project we plan to implement an Eclipse-based [6] graphical editor for ModelicaML as a part of our Modelica Development Tooling (MDT) [7].

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# Developing Dependable Automotive Embedded Systems using the EAST-ADL; representing continuous time systems in SysML

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**Abstract.** The architectural description language for automotive embedded systems EAST-ADL is presented in this paper. The aim of the EAST-ADL language is to provide a comprehensive systems modeling approach as a means to keep the engineering information within one structure. This facilitates systems integration and enables consistent systems analysis. The EAST-ADL encompasses structural information at different abstraction levels, requirements and variability modeling. The EAST-ADL is implemented as a UML2 profile and is harmonized with AUTOSAR and a subset of SysML. Currently, different ways to model behavior natively in the language are investigated. An approach for using SysML parametric diagrams to describe equations in composed physical systems is proposed. An example system is modeled and discussed. It is highlighted that parametric diagrams lacks support for separation between effort and flow variables, and why this separation would be desired in order to model composed physical systems. An alternative approach by use of SysML activity diagrams is also discussed.

**Keywords:** EAST-ADL, automotive embedded systems, UML, SysML, parametric diagrams, physical modeling, continuous systems, Modelica

## 1 Introduction and goals

New functionality in automotive systems is increasingly realized by software and electronics. A system level function, such as an adaptive cruise controller will then be

partitioned into functions that are realized by software and electronics (the vehicle embedded system), and other functions realized in mechanical subsystems. The complexity of embedded systems calls for a more rigorous approach to system development compared to current state of practice. A critical issue is the management of the engineering information that defines the embedded system. This issue should be understood in the light of current development practice which is characterized by the involvement of a very large number of specialists/groups/companies: all participants are working on the same system but using different tools, models, information formats, and subsets of the complete information. In current practices, integration of artifacts from different parties takes place at a very late stage of the development process where electronic control units are integrated into the overall embedded system of a vehicle. There is a need to shift this hardware level integration to model level integration [2]. Model based development has the potential to improve the cost efficiency of the products including their quality.

In this paper, we will discuss one aspect of model based development, the modeling of the environment to the developed system. Environment models serve several purposes in an architecture description language (ADL). An environment model defines implicitly the context and relevant use of the systems and functions in the embedded systems architecture. Validation activities such as simulation, interface consistency, formal verification all benefit from an environment model. The environment of an automotive embedded system may include all kinds of systems and behaviors that are part of the embedded system itself. The focus here is on physical continuous time systems. In particular we investigate the representation of continuous systems in SysML parametric diagrams, using Modelica [6] compatible constructs.

The presented approach is a part of an effort to refine an architecture description language for automotive embedded systems. An initial version of this language, EAST-ADL, was developed in the EAST-EEA project [19]. Further work on the the language is pursued in the ATESSST project [17]. For other aspects of the EAST-ADL, see [3].

## 2 Overview of the EAST-ADL

The EAST-ADL is intended to support the development of automotive embedded software by capturing all the related engineering information. The scope is the embedded system (hardware and software) of a vehicle and its environment. The EAST-ADL system model is organized in parts representing different levels of abstraction and thus reflects different views and details of the architecture. The levels implicitly reflect different stages of an engineering process, but the detailed process definition is company specific.

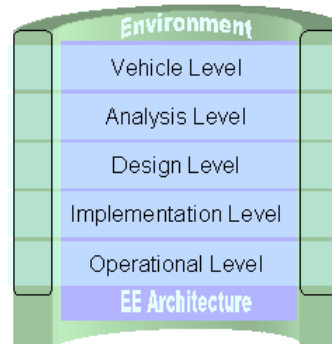
The EAST-ADL language constructs support:

- vehicle feature modeling including concepts to support product families
- concepts for defining variability in all parts of a model
- vehicle environment modeling to define context and perform validation

- structural and behavioral modeling of software and hardware entities in the context of distributed systems.
- requirements modeling and tracing with all modeling entities
- other information part of the system description, such as a definition of component timing and failure modes, necessary for design space exploration and system verification purposes

The language is structured in five abstraction levels (see Fig. 1), each with corresponding environment system representation (in parenthesis):

- *Operational Level* supporting final binary software deployment (operational architecture)
- *Implementation Level* describing reusable code (platform independent) and AUTOSAR compliant software and system configuration for hardware deployment (implementation architecture)
- *Design Level* for detailed functional definition of software including elementary decomposition (design architecture)
- *Analysis Level* for abstract functional definition of features in system context (analysis architecture)
- *Vehicle Level* for elaboration of electronic features (vehicle feature model)



**Fig. 1.** EAST-ADL language abstractions. Note that the environment model spans all abstraction levels, and that requirements and variability constructs apply to modeling elements regardless of abstraction level.

## 2.1 EAST-ADL definition, implementation and relation to other languages

In defining the EAST-ADL language, a two step procedure is adopted. The final language is implemented as a UML2 profile. A domain model is first defined, capturing only the domain specific needs of the language. The domain model thus represents the meta-model, the language definition. Basic concepts of UML are used for this purpose, such as classes, compositions and associations. Based on the Domain model, a UML2 profile implementation, the “UML viewpoint” is defined with

stereotypes, tags and constraints. This implementation is delivered as an XMI file ready for use in UML2 tools. As a proof-of-concept, an Eclipse based prototype tool with supporting analysis features, Papyrus [20], has been implemented. The EAST-ADL language also incorporates relevant aspects from SysML [14] and MARTE [9]. SysML is a modeling language that supports the specification, analysis, design, verification and validation of systems which may include hardware, software, information, processes, personnel, and facilities. SysML is defined in terms of a UML2 profile. MARTE is an ongoing effort to define a UML profile for Modeling and Analysis of Real-Time and Embedded systems, initiated to overcome the UML limitations of modeling such systems. The EAST-ADL is also being harmonized with the new automotive domain standardization AUTOSAR [18]. AUTOSAR focuses mainly on the implementation level of abstraction, whereas the EAST-ADL supports the overall comprehensive systems modeling.

Inspiration in the development of the EAST-ADL is also gathered from the SAE Architecture and Analysis Description Language (AADL) [16] and safety standards such as the ISO 26262 Functional Safety (committed draft planned for beginning 2008).

Considering the multitude of languages that in different ways address embedded systems, a relevant question is how the EAST-ADL relates to other modeling language efforts. This was partly elaborated in the previous text, but the main reasons for introducing “yet another language” are summarized here for clarity:

- EAST-ADL vs. UML: UML is a general modeling language for software engineering, which contains no specifics for automotive embedded systems. The EAST-ADL provides a tailoring of UML2 through a profile dedicated for such systems.
- EAST-ADL vs. SysML: SysML is a UML2 profile for systems engineering. EAST-ADL incorporates several SysML concepts and specializes them as needed for automotive embedded systems.
- EAST-ADL vs. AUTOSAR: AUTOSAR focuses on software and hardware implementation. The EAST-ADL complements AUTOSAR with e.g. functional specifications and requirements and reuses AUTOSAR concepts for the implementation level abstractions.
- Why not proven proprietary tools and languages such as MATLAB/Simulink [13], ASCET [5] or Modelica? The very fragmentation into multiple domain/discipline tools that target different aspects of the system is a key driver for developing the EAST-ADL. The EAST-ADL language provides an information structure for the engineering data required as a basis for automotive embedded systems development. In the ATESSST project, interfaces in terms of model transformations and tool interfaces for the prototype tool Papyrus are developed to domain tools/languages.
- Why not information management tools such as product data management tools (PDM)? Such tools lack an information model for automotive embedded systems and the connections to external domain tools. The EAST-ADL domain model could be used as a basis for information management in existing PDM-like tools. Moreover, the use of UML2 allows native behavior to be defined. The fact that UML2 is a standard allows the EAST-ADL to be used with several UML tools.

## 2.2 Behavior modeling approach in EAST-ADL

With respect to behavior the goal of the EAST-ADL is to provide native behavior descriptions, for primitive components, as well as for compositions. Native behavior is useful for example to describe the desired overall behavior of the vehicle systems as well as the environment. The objective includes a native behavioral notation that allows simulation and verification within the defined system model, but also concerns the integration of external tools, as part of today's industrial practice for designing behavior algorithms of vehicle applications. Integration here refers to the ability to import/export models to and from external tools based on model transformations.

## 3 Physical systems modeling in SysML

The overall goal is to find a representation of continuous systems in SysML to be used in the EAST-ADL. Parametric diagrams and Modelica models have many things in common (equation based, acausal, modular etc.), so the hypothesis was to see how they relate to each other. The approach was to make a proof-of-concept description of a Modelica model using native SysML constructs.

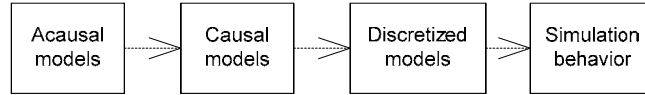
### 3.1 Modeling physical systems

To get a complete description of the system, not only the embedded system needs to be modeled, but also the environment it interacts with. In control theory this is referred to as the plant model. One possibility in the EAST-ADL is to rely on legacy tools, most notably Simulink and ASCET, for defining plant behavior. The functions that compose the environment model define the structure, and a link to a behavioral definition in the external tool is provided for each of these. The complete behavior of the environment model is the result of the composition of the parts. Alternatively, and this is elaborated below, an equation-based behavioral definition is used for environment model behavior. This behavior is defined as a part of the EAST-ADL, and would be understood by EAST-ADL compliant tools.

A typical plant model is described by differential equations, but can also include hybrid systems. Example of the latter kind is a gearbox, or the state-transition between different properties of a road, e.g. from ice to water. The interface between the plant and the embedded system is in form of inputs (actuators act as inputs to the plant) and outputs (sensors act as outputs from the plant to the control system). In the implementation, this will be realized as an interface between a discrete embedded system and a continuous "real world". During modeling and design, this interface could be continuous to continuous at the Analysis Level (from Fig. 1), or discrete to discrete when modeling and simulating a plant model.

Just like the embedded systems, plant models can be described at different levels of abstraction, see Fig. 2. These abstraction levels are adapted from [12], and have been proven useful when discussing how models could be described in a uniform way, e.g.

are the models on the same abstraction level, and can they be integrated on that level? There are also levels orthogonal to this, e.g. more or less advanced models, more or less validated models etc. These abstraction levels are not directly correlated to the EAST-ADL abstraction levels of the embedded system.



**Fig. 2.** Different levels of abstractions of environment models

**Acausal models.** At the highest level of abstraction for environment models, there are acausal models, where models are described by one or several differential or algebraic equations, possibly combined with state machines to model hybrid systems. The acausal Modelica components are on this level of abstraction, where equations and their relation to the outside world are modeled. Acausal models are generally more flexible and reusable than models at a lower abstraction level [6].

**Causal models.** At the causal level of abstraction, it is defined what is input, and what is output inside the system, and between components in a composed system. Typical causal models include bond graph representations [11], or block diagrams, like continuous Simulink models.

**Time-discretized models.** To solve a differential equation numerically it is typically discretized in time. A discretized model is an algorithmic representation in the sense that it generates a defined output for a certain input and internal state. A model can be discretized in different ways, e.g. using forward/backward Euler.

**Simulation behavior.** To perform the calculations of the discretized models, a solver and a scheduler is needed as part of the simulation engine. The simulation engine can decide the time-step, execution order, triggering, communication, etc. of the model. Typical numerical tools to solve differential equations are Simulink and ASCET [5]. Here, the simulation behavior of the continuous time model can be described using the same Model Of Computation (MOC) as for the model components of the embedded system.

Proceeding to lower abstraction levels means that the model gets more sophisticated, in the sense that the model can produce simulation results. On the other hand, the models get more specialized and differ more from the original system. Moreover, numerical errors could be introduced at all transformations. Tools and methods could use many of these abstraction levels, and hide some from the user. For example, a continuous Simulink model is modeled as a causal model. The choice of solver provides a way of (indirectly) discretizing the model since it implies a discrete-time implementation through the simulation behavior built into the tool. The Dymola [4] tool makes a point of taking the equations from the highest abstraction level to the

lowest, while ASCET uses models close to the lowest level, and letting the user have control over the real-time hardware-in-the-loop simulation.

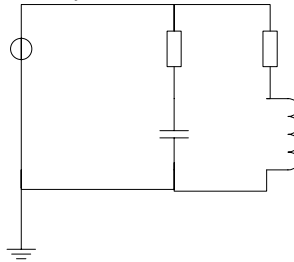
### 3.2 SysML parametric diagrams

SysML consists of four behavioral and five structural diagrams. Seven of these diagrams are partially reused from UML, while two are new: the requirement diagrams and the parametric diagrams [14]. The parametric diagrams' conceptual foundation is the composable objects representation (COBs), developed at Georgia Institute of Technology [10], the academic partner of the SysML team [14]. COBs provide five basic views of a system, Shape Schematic, Relations, Constraint Schematic, Lexical COB structure and Subsystem, of which Relations, Constraint Schematic and Subsystems have equivalent representation in parametric diagrams.

Parametric diagrams describe constraints between variables, like equations, and how they are related to each other. The SysML specification gives an example of Newtons equation, which can be modeled in continuous time [14]. The constraints are acausal, and by combining many modular subsystems, acausal relationships for a large system can be achieved. In addition, state machines can tell which equations to be used in the parametric diagrams, to be able to describe hybrid systems.

## 4 Investigation of an example system

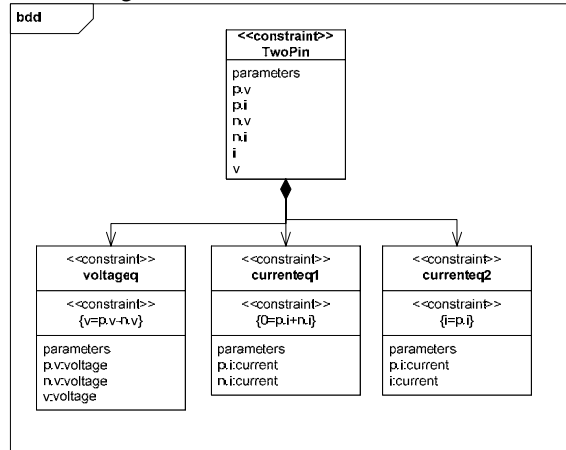
As an example system an electrical circuit from [6] was chosen. It was chosen since the Modelica representation is well-described in this reference, and since it has sufficient complexity: different dynamical features and parallel branches. The model also highlights that different causality is needed for the resistors in the two branches.



**Fig. 3.** The example electrical circuit. An alternating voltage source is applied to a circuit containing two resistors, one capacitance and one inductor.

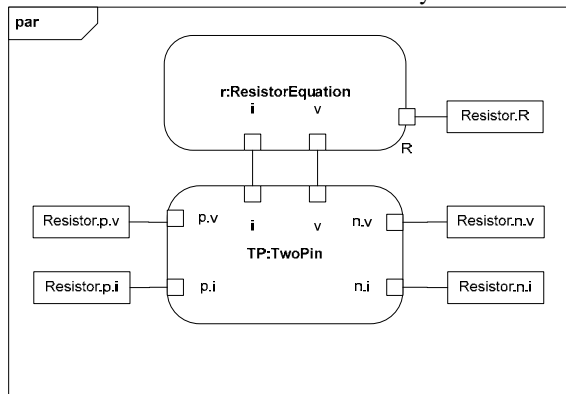
#### 4.1 Definition of a component with SysML

An electrical component is specified using the two-pin class, where general equations common for many electrical components are stored. These equations are also linked together in a parametric diagram.



**Fig. 4.** Constraint definition of the electrical TwoPin Modelica class within a SysML block definition diagram (bdd). There is a separate parametric diagram to show the relation between the equations.

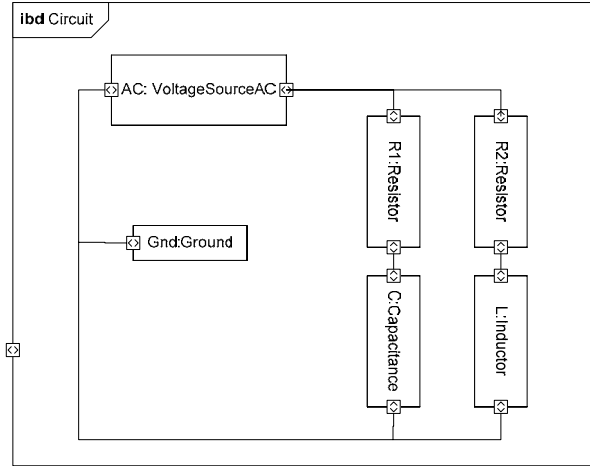
This diagram is also linked in the parametric diagram of the resistor, where variables link to the FlowPorts of the resistor. The component is defined as a SysML block, where bidirectional FlowPorts are used to represent connections. The capacitor, inductance, ACSource etc. can be defined in a similar way.



**Fig. 5.** SysML parametric (par) diagram of the resistor. The `Resistor.x.y` variables are linked to the FlowPorts of the resistor, and the `Resistor.R` is a property of the Resistor.

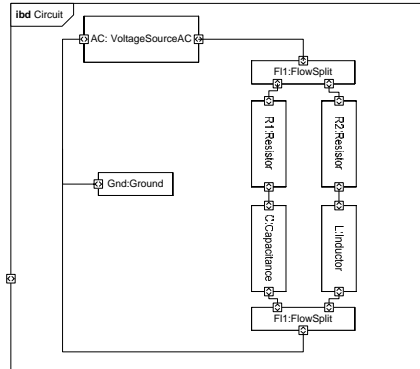


## 4.2 Composing a system



**Fig. 6.** Internal block diagram (ibd) of the circuit. SysML ItemFlow is specified between components, and have the same function as connectors in Modelica

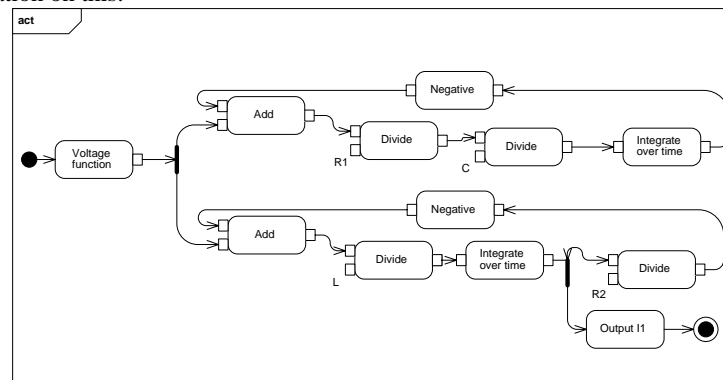
The example system is composed using instances of the components defined in the previous section. The parameters can then be assigned values. In a first attempt, ItemFlow is specified between the components to be connected, used the same way as connectors would be used in Modelica. ItemFlow is a SysML stereotype describing the flow of items across a connector or an association. This is an intuitive way to connect the components, since it is related to the physical layout of the circuit. The problem is that when calculating the current from these parametrics, a wrong result will occur: The current will be the same everywhere in the system, which is not possible due to Kirchhoff's current law. In Modelica this is handled by defining the current as a "sum to zero"-variable, and the voltage as an "equality"-variable. In a connection having many branches, the equality variable will have the same value for all branches, while the "sum to zero" variables will be summed to zero. Here, a workaround has to be made. A possible solution is to include a parallel flow split component, which is shown in Fig. 7. One of the two-pin current equations must also be changed from  $0 = p.i + n.i$  to  $p.i = n.i$ .



**Fig. 7.** An acausal representation of the system, where the equations are fully described by the underlying parametric diagrams

#### 4.3 Block diagram model expressed as an activity diagram

The system could also be described at the causal level of abstraction. The circuit could be transferred to a bond graph representation. There are systematic approaches for electrical circuits, see for example [8]. Bond graphs could be described using SysML activity diagrams [15]. A bond graph model can also be converted to a block diagram [11], which could be expressed as an activity diagram, as described in [1]. This representation is shown in Fig. 8. To execute this diagram, initial values are needed on both “Add”-actions. When simulating a model in Simulink, the execution order is however typically not from input to output, it is decided by which blocks have direct-feed through and those that have internal state, see [13] for more information on this.



**Fig. 8.** An activity diagram describing the circuit. This is similar to the Simulink representation, although the sequencing of the actions differ

## 5 Discussion and Conclusions

An attempt to make modular simulation models of physical systems using SysML has been tested and evaluated. The concepts includes internal block diagrams with attached parametric diagrams, bi-directional FlowPorts, and ItemFlow. The approach is maybe not the intended way to use these diagrams. The intention in SysML is to connect parametric diagrams to each other to generate a composed parametric diagram for the system. Then, the governing equations in this parametric diagram are separated from the structure in the internal block diagrams.

Although parametric diagrams are acausal, they do not contain any separation between effort and flow variables, which is fundamental when modeling physical systems [11]. Both Modelica (using `flow`) [6] and VHDL-AMS (using `across/through`) [7] contain such constructs. The solution of introducing the flow-split block is not an appealing solution; the point of using acausal diagrams is then somehow lost. Another solution would be to introduce stereotypes for flow variables, to show that they are to be summed to zero. A parametric diagram could be manually generated, or in principle automatically generated, based on the properties of the internal block diagram.

The behavior of the composed system could of course also be modeled using a non-modular parametric diagram. An optimized system model of the example system contains seven equations [6], which could easily be modeled using a single parametric diagram.

Using activity diagrams it is possible to model a block model version of the system. The activity diagram is a behavioral diagram, as opposed to the parametric diagrams (which are structural). This is a way to capture the structure of a Simulink model which is the causal representation of the system. This is an interesting path and will be further investigated in the ATESSST project.

The compatibility between Modelica and UML representations needs further investigation. The advantage of having a Modelica compatible description is that it could be translated to Modelica models, and then simulated in existing Modelica tools.

**Acknowledgements.** This work has been carried out within the ATESSST project, 2004-026976 in the EC 6<sup>th</sup> Framework Programme.

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# Hybrid Dynamics in Modelica: Should all Events be Considered Synchronous

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**Abstract.** The Modelica specification is ambiguous as to whether all the events are synchronous or not. Different interpretations are possible leading to considerable differences in the ways models should be constructed and compilers developed. In this paper we examine this issue and show that there exists an interpretation that is more appropriate than others leading to more efficient compilers. It turns out that this interpretation is not the one currently adopted by Dymola but it is closely related to the Scicos formalism.

**Keywords:** Modelica, Synchronous language, Scicos, modeling and simulation.

## 1 Introduction

Modelica ([www.modelica.org](http://www.modelica.org)) is a language for modeling physical systems. It has been originally developed for modeling systems obtained from the interconnection of components from different disciplines such as electrical circuits, hydraulic and thermodynamics systems, etc. These components are represented symbolically in the language providing the compiler the ability to perform symbolic manipulations on the resulting system of differential equations. This allows the usage of acausal components (equation based) without loss of performance.

But Modelica is not limited to continuous-time models [1]; it can be used to construct hybrid systems, i.e., systems in which continuous-time and discrete-time components interact. Modelica specification [2] tries to define the way these interactions should be interpreted and does so by inspiring from the formalism of synchronous languages. Synchronous languages however deal with events, i.e., discrete-time dynamics. So in the context of Modelica, the concept of synchronism had to be extended to encompass continuous-time dynamics as well. It is exactly this extension which is the subject of this paper.

Scicos ([www.scicos.org](http://www.scicos.org)) is a modeling and simulation environment for hybrid systems. It is free software, included in the scientific software package Scilab ([www.scilab.org](http://www.scilab.org)). Scicos formalism is based on the extension of synchronous languages, in particular Signal [3], to the hybrid environment. The class of models that Scicos is designed for is almost the same as that of Modelica. So it is not a surprise that Modelica and Scicos

have many similar features and confront similar problems. Modelica has many advantages for modeling continuous-time dynamics, especially thanks to its ability to represent models in symbolic form, whereas the Scicos formalism has been specifically designed to allow high performance code generation of discrete-time dynamics.

In this paper, we examine the specification of hybrid dynamics in Modelica and propose an interpretation that is fully compatible with the Scicos formalism. This interpretation, which is not contradictory with the official specification, allows us to obtain an efficient compiler/code generator for Modelica inspired by the Scicos compiler.

Here we start with a flat model (obtained from the application of a front-end compiler), and consider only the problems concerning the design of the phase one of a back-end compiler. This phase breaks down the code into independent asynchronous parts each of which can be compiled separately in phase two. Phase two will be presented in a subsequent paper.

## 2 Conditioning and Sub-sampling in Modelica

If a model contains no conditioning and all of its parts function at the same rate, then back-end compilation would be a simple task. But in most real life applications, models contain different dynamics resulting from the inter-connection of heterogeneous systems. A model of such a system would often include conditioning and sub-sampling. We use the term conditioning for a change in the model conditioned on the value of a variable (for example *if  $a > 0$  then*) and the term sub-sampling for the construction of a new, not necessarily regular, clock from a faster clock.

The *when-elsewhen* and *if-then-else* clauses are the basic language constructs in Modelica for performing conditioning and sub-sampling. The description of the ways these constructs function is ambiguous in the Modelica specification. Comparing with the Scicos formalism, we can consider that Modelica's *if-then-else* clause does conditioning and *when* does sub-sampling. But the situation is somewhat more complex because *when* plays two different roles. And, we need to distinguish these two different types of *when* clauses. But before, we need to examine the notion of synchronism in Modelica.

### 2.1. Synchronous versus Simultaneous

In our interpretation of the Modelica specification, two events are considered synchronous only if they can be traced back to a single event source. For example in the following model:

```
when sample(0,1) then
  d:=pre(d)+1;
end when;
when d>3 then
  a:=pre(a)+1;
end when;
```

the event  $d > 3$  is synchronous with the event  $sample(0,1)$ . The former is the source of the latter. But in

```

der(x)=x ;
when sample(0,1) then
  d=pre(d)+1;
end when;
when x>3 then
  a=pre(a)+1;
end when;

```

the two events are not synchronous. There is no unique source of activation at the origin of these events. So these events are considered asynchronous even if the two events are activated simultaneously; even if we can prove mathematically that they always occur simultaneously.

Our basic assumption is that events detected by the zero-crossing mechanism of the numerical solver (or an equivalent mechanism used to improve performance) are always asynchronous. So even if they are detected simultaneously by the solver, by default they are treated sequentially in an arbitrary order. In particular, in the model:

```

when sample(0,1) then
  b=a;
end when;
when sample(0,1) then
  a=b+1;
end when;

```

the variables *a* and *b* can be evaluated in any order.

Dymola on the other hand assumes that all events are synchronous. In particular it assumes that all the equations in both *when* clauses may have to be satisfied simultaneously. That is why Dymola finds an algebraic loop in this example.

To see the way Dymola proceeds, consider the following example:

```

equation
der(x)=1;
der(y)=1;
when (x>2) then
  z=pre(z)+3;
  v=u+1;
end when;
when (y>2) then
  u=z+1;
end when;

```

The simulation shows that the equations (assignments) are ordered as follows:

```
z=pre(z)+3; u=z+1; v=u+1;
```

this means that the content of a *when* clause is split into separate conditional clauses. In stark contrast, in our interpretation of the Modelica specification, the code within an asynchronous *when* clause is treated synchronously and never broken up. Both interpretations are valid and consistent; however our interpretation has many advantages as we will try to show here.

At first glance, the non determinism that may be encountered in our approach when two zero-crossing events occur simultaneously may seem unacceptable. However, treating two simultaneous zero-crossings as synchronous is not a solution because it is not robust. Indeed, when dealing with nonlinear and complex models, there is no guarantee that the numerical solver would detect two zero-crossings simultaneously even if

theoretically they are simultaneous. In general one is detected slightly before or after the other. And in any case, in most cases treating such an accidental synchronism is not of any use for the construction of the model. Even if the model depends for some reason on the simultaneous detection of two events by the solver, a mechanism should be provided by the language to specify explicitly what should be done in that case. One way would be to introduce a *switchwhen* clause [4], which can be used to explicitly specify what equations are activated in every possible case. The possible cases when we have, for example, two zero-crossings are: the first surface has crossed but not the second, the second has crossed but not the first and finally both surfaces have crossed zero together.

Dymola's interpretation imposes constraints, which in most cases are useless. Moreover, when all zero-crossing events are considered synchronous, the complexity of static scheduling increases with the number of zero-crossings. The solution based on using the *switchwhen* clause allows the user to specify explicitly what possible synchronisms must be considered. It turns out that in most cases, no synchronism is to be considered.

## 2.2. Primary and Secondary *when* Clauses

So far we have seen two types of *when* clauses, or more specifically *when* clauses based on two types of events: events depending on variables evolving continuously in time such as  $time > 3$  or  $x < 2$  where  $x$  is a continuous variable; and events depending on discrete variables. *when* clauses conditioned on events of the former type are called primary, the latter ones are called secondary.

An event associated with a secondary *when* clause is necessarily synchronous with events associated to one or more primary *when* clauses. These primary clauses are those in which the discrete variables involved in the definition of the event are defined.

But not all *when* clauses can easily be classified as primary or secondary. Let us consider a simple example:

```
when sample(0,1) then
  d=pre(d)+j;
  c=b;
end when;
when time>d then
  b=a;
end when;
```

The question is whether or not the above two *when* clauses are primary or not. Clearly the first one is, but the second hides in reality two distinct *when* clauses that is because the event  $time > d$  can be activated in two different ways:

- time increases and crosses  $d$  continuously (zero-crossing event so asynchronous),
- at a sample time  $d$  jumps, activating the  $time > d$  condition; this event is clearly synchronized with *sample(0,1)*.

We call such *when* clauses mixed. We handle this situation by implementing the simulation in such a way that  $time > d$  is activated only when time crosses continuously  $d$  and placing a duplicate of the content of this *when* where  $d$  is defined within a condition that guarantees that the content is activated only if  $time > d$  is activated due to a jump:

```
when sample(0,1) then
```



```

d=pre(d)+j;
c=b;
if ((time>d) and not(time>pre(d))) then
  b=a ;
end if ;
end when;
when time>d then
  b=a;
end when;

```

The second solution amounts to considering that a clause such as *when c>0* where *c* is a continuous variable is activated only if *c* crosses zero continuously (the way that is detected by zero-crossing mechanisms built into numerical solvers such as LSODAR or DASKR). This seems to be an appropriate way to handle mixed *when* clauses, however to stay compatible with the Modelica specification, at a pre-compilation phase, the content of these clauses must be duplicated as explained above.

Note that the code we obtain after the pre-compilation phase is not correct according to the Modelica specification (because *b* is defined twice). This however is not a problem because this code is only used within the compiler. But in any case, we consider this restriction too restrictive and we think it should be relaxed. This will be discussed later.

There still remains a situation that needs clarification. Consider the following example:

```

discrete Real a(start=0);
Real x(start=0);
equation
der(x)=0;
when x>3 then
  a=pre(a)+1;
end when;
when time>2 then
  reinit(x,x+4);
end when;

```

Here *x* is a continuous variable, but it is also discrete because at time 2 it jumps from 0 to 4 (activation of *reinit*). This jump activates the content of the first *when*. The *reinit* primitive in this case must be considered as a definition of “discrete” *x*, so following the rule discussed previously, the content of the clause *when x>3* must be copied inside the other *when*:

```

discrete Real a(start=0);
Real x(start=0);
equation
der(x)=0;
when x>3 then
  a=pre(a)+1;
end when;
when time>2 then
  reinit(x,x+4);
  if edge(x>3) then
    a=pre(a)+1;
  end if ;
end when;;

```

This transformation is just a special case of the situation we have considered previously. To see this more clearly, note that

```

when time>2 then
  reinit(x,x+4);
end when;

```

should really be expressed as follows:

```

when time>2 then
  x=pre(x)+4;
end when;

```

### 2.3. Restrictions on the Use of *when* and *if*

Modelica imposes hard constraints on the usage of *when* and *if-then-else* clauses.

In the case of *when*, a variable is not allowed to be defined in two *when* clauses. For example the following code is not allowed in an *equation* section:

```

when sample(0,1) then
  b=pre(b)+1 ;
end when ;
when time>3.5 then
  b=0 ;
end when ;

```

According to the specification, this can lead to a contradiction if the two *when* clauses are activated at the same time. This statement would make sense if the two *when* clauses were synchronous but not in this case. Lifting this restriction, in the case of primary *when* clauses, is without danger and facilitates the task of modeling in many situations. However, it creates an important difference as far some interpretation of the primitive *pre* is concerned. With the current restriction, we are sure that in the following code:

```

when sample(0,1) then
  b=pre(b)+1 ;
end when;

```

*pre(b)* is the previous value of *b* defined by *b=pre(b)+1* the last time this *when* clause was activated, i.e. one unit of time before. So without even having to examine the rest of the code, we can be sure that *b* indicates the time. This will no longer be true if the constraint is lifted; consider:

```

when sample(0,1) then
  b=pre(b)+1 ;
end when;
when sample(.5,1) then
  b=pre(b)+1 ;
end when;

```

In this case the value of *b* used to update it in each clause is computed by the instruction in the other clause. But this is not a problem as long as the rules are clear.

We thus propose the following modifications: this restriction be lifted for primary *when* clauses and this restriction be lifted in all *when* clauses as long as the equations defining common variables are identical (such identical equations can arise in transformations applied by the compiler which includes duplicating parts of the code). For example for all conditions *c1*, *c2* (synchronous or not), accept:

```

equation

```

```

when c1 then
  b=a;
end when;
when c2 then
  b=a;
end when;

```

The second modification may seem strange. Indeed why would a model contain identical statements in synchronous *when* clauses. The reason is that our Modelica compiler performs a series of transformations each one generating a new Modelica code from a Modelica code in which such a situation may come up (this happens in particular when processing the union of events construct, see Section 2.6). By lifting this restriction, we make sure that we obtain a valid Modelica code at every stage. But a specific test must be applied to the original model to issue at least a warning to the user for such cases.

Another important restriction concerns the use of *elsewhen*. The Modelica specification states that all the branches of a *when-elsewhen* clause must define the same set of variables. We don't believe this constraint is justified. This constraint is probably a consequence of a similar condition on the use of *if-then-else* clauses. Indeed Modelica imposes that the number of equations in different branches of such a clause be identical. This may be acceptable as far as continuous-time variables are concerned<sup>1</sup>, but it is not for discrete variables. So we propose to lift this restriction and accept models including for example the following code:

```

equation
when sample(0,1) then
  if u>0 then
    v=1;
  end if;
end when;

```

Normally in Modelica we should have an *else* branch defining *v*. Note that our proposal is not just an editing facility (i.e., a way to avoid writing code which can be added in automatically later); this code is not equivalent to

```

equation
when sample(0,1) then
  if u>0 then
    v=1;
  else
    v=pre(v) ;
  end if;
end when;

```

In the absence of the *else* branch, the variable *v* is sub-sampled. This would not be the case if *v=pre(v)* were used. Even if the simulation result would be the same, the construction by sub-sampling leads to the generation of more efficient code. Lifting this restriction is again important for transformed models. A specific test can be used on the original model to impose the constraint if desired.

---

<sup>1</sup> Removing the restriction in the continuous case makes it possible to model Simulink's enabled Super Blocks in Modelica.

## 2.4. Continuous-Time Dynamics

Our objective is to reduce the Modelica code into a number of asynchronous *when* clauses each of which can be treated separately. The continuous dynamics is no exception. What we call continuous dynamics includes everything within the *equation* section but outside *when* clauses. These equations are always active (Scicos terminology) even when a *when* clause is activated. So these equations are synchronous with all the *when* clauses.

The way this situation is handled in Scicos is to introduce a fictitious clock generating a continuous activation signal. To do the same in Modelica amounts to defining a special *when* clause:

```
when continuous then
```

the content of which would be active all the time except at event instances associated to other *when* clauses. Doing so allows us to consider the “continuous” event as asynchronous with the rest and treat this *when* clause as primary. To preserve the dynamics of the original model, the continuous dynamic equations must also be copied inside all the *when* clauses. For example:

```
equation
y=sin(time) ;
der(x)=y ;
when x<.2 then
  a=y ;
end when ;
```

becomes

```
equation
when continuous then
  y=sin(time) ;
  der(x)=y ;
end when ;
when x<.2 then
  y=sin(time) ;
  der(x)=y ;
  a=y ;
end when ;
```

During the simulation, the content of the *when continuous* clause is used to respond to the queries of the numerical solver, and in particular to generate the value of  $der(x)$  in this case. In other *when* clauses, the equations defining derivative values can be dropped, especially in the explicit case. In the implicit case (DAE case), the computation of the derivatives can be used to help the re-initialization of the solver.

The point to retain from this section is that the clause *when continuous* is primary and that its content can be treated like any other.

## 2.5. Initial Conditions

In Modelica, variables can be initialized in different ways but in a flat model (after the application of the front end), they should all be grouped within a single *when* clause:

```
when initial then
```

```

a=0 ;
d=3 ;
...
end when ;

```

This would be a primary *when* clause and would contain the initialization of all discrete and continuous variables.

A *when terminal* clause can similarly be used to specify whatever needs to be done at the end of the simulation.

## 2.6. Union of Events

The *when* and *elsewhen* clauses can be activated at the union of events. In Modelica the syntax is as follows:

```

when {c1,c2,c3} then
  < eq1 >
  < eq2 >
end when;

```

In this case, *c1*, *c2*, *c3* may be synchronous or not. Note that the content of synchronous *when* clauses should not be executed more than once. For example in:

```

when sample(0,1) then
  d=pre(d)+1;
end when;
when {d>2,2*d>4} then
  a=pre(a)+1 ;
end when;

```

*a* must be incremented only once, passing from zero to one. But in:

```

when sample(0,1) then
  d=pre(d)+1;
end when;
when sample(0,1) then
  e=pre(e)+1;
end when;
when {d>2,e>2} then
  a=pre(a)+1 ;
end when;

```

*a* is incremented twice (its value must jump from zero to two). But Dymola considers the *d>2* and *e>2* synchronous and increments *a* just once in this case. Similarly in:

```

when sample(0,3) then
  d=pre(d)+1;
end when;
when time>=3 then
  e=pre(e)+1;
end when;
when {d>1,e>0} then
  a=pre(a)+1 ;
end when;

```

in Dymola *d>1*, *e>0* are synchronous (*a* is incremented only once at time 3). As we have said previously, we think that this interpretation must be avoided.

The counterpart of the union of events is the sum of activation signals in Scicos. The two formalisms coincide perfectly in this case.

In one of the early phases of the compilation of Modelica code, we propose the following transformation which removes all event unions. For example the first *when* clause presented in this section would be transformed as follows:

```
when c1 then
  < eq1 >
  < eq2 >
end when;
when c2 then
  < eq1 >
  < eq2 >
end when;
when c3 then
  < eq1 >
  < eq2 >
end when;
```

This code is correct if we take into account all the modifications suggested previously whether the  $c_i$ ,  $i=1,2,3$ , are synchronous or not.

### 3 Back-end Compiler

The back-end compiler can be divided into two phases. The objective of the first phase is to transform the model into one in which all the *when* clauses are primary. This will allow us to generate, in phase two, static code for each one independently of the others.

Consider the following example:

```
when time>3 then
  d=pre(d)+1;
end when;
when d>3 then
  a=pre(a)+1;
end when;
when a>3 then
  b=a;
end when;
```

We want to remove the secondary *when* clauses. Clearly in this case we have to remove the last two *when* clauses. We pick one (say *when a>3*) and copy its content everywhere the variables involved in the definition of the corresponding event are computed. In this case the only variable involved is *a*, which is defined in the second *when* clause:

```
when time>3 then
  d=pre(d)+1;
end when;
when d>3 then
  a=pre(a)+1;
  if edge(a>3) then
    b=a;
  end if ;
```

```

        end when;
and then
    when time>3 then
        d=pre(d)+1;
        if edge(d>3) then
            a=pre(a)+1;
            if edge(a>3) then
                b=a;
            end if ;
        end if ;
    end when;

```

This example shows how secondary *when* clauses can be removed to obtain a single primary *when* clause at the end. If the model contains more than one primary *when* clause, the procedure would still be the same as illustrated in the following example:

```

    when time>2 then
        a=1 ;
    end when ;
    when time>3 then
        b=pre(b)+1 ;
    end when ;
    when a>b then
        c=1 ;
    end when ;

```

In this case the first two *when* clauses are primary. We now remove the secondary *when*:

```

    when time>2 then
        a=1 ;
        if edge (a>b) then
            c=1 ;
        end if ;
    end when ;
    when time>3 then
        b=pre(b)+1 ;
        if edge (a>b) then
            c=1 ;
        end if ;
    end when ;

```

In this example, a variable is defined twice in two different primary (so asynchronous) *when* clauses. Clearly, this is not a problem. But the application of the transformation, can also lead to a variable being defined more than once in the same *when* clause. Let us examine the following example:

```

    when time>2 then
        a=pre(a)+1 ;
    end when ;
    when a>d then
        b=pre(b)+1 ;
    end when ;
    when {a>2,b>2} then
        n=pre(n)+1 ;
    end when ;

```

We start by removing the operator “union of events. Then we remove the secondary clauses as previously described. We obtain (in two steps):

```

when time>2 then
  a=pre(a)+1 ;
  if edge(a>d) then
    n=pre(n)+1 ;
  end if ;
  if edge(a>2) then
    b=pre(b)+1 ;
    if edge(b>2) then
      n=pre(n)+1 ;
    end if ;
  end if ;
end when ;

```

This code, once *edge* replaced with its definition, may seem to be ordered properly and usable as a sequential code. But this is not the case since  $n=pre(n)+1$ , in some cases, can be executed twice instead of once. As discussed in the previous section, it is allowed to have a variable defined twice synchronously as long as the equations defining it are identical. This is of course the case here (this is the case in general when it happens because of the application of our transformations). The second phase of the compilation will transform the code into a sequential code correctly.

## 4 Conclusion

We have examined the notion of synchronism in Modelica and have shown that by abandoning the fully synchronous assumption, it is possible to design more efficient compilers without loss of rigor in the language specification. We have done that by proposing a methodology to implement the first phase of a back-end compiler. The second phase, which is closely related to the second phase of the Scicos compiler, will be presented in a future.

## References

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# Extensions to Modelica for efficient code generation and separate compilation

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**Abstract.** In the current Modelica specification, the only admitted external entities are memory-less functions. We propose an extension to allow parts of the model containing internal states, conditioning and discrete dynamics, to be definable as external functions. This opens the door to separate compilation of Modelica code. For this purpose, we introduce the language construct *switchwhen* and the type *Event*. These extensions are directly inspired by the Scicos formalism.

## 1 Introduction

Modelica ([www.modelica.org](http://www.modelica.org)) is a language for modeling physical systems. It has been originally developed for modeling systems obtained from the interconnection of components from different disciplines such as electrical circuits, hydraulic and thermodynamics systems, etc. Modelica can also be used to construct hybrid systems, i.e., systems in which continuous-time and discrete-time components interact. With that respect, Modelica is similar to Scicos ([www.scicos.org](http://www.scicos.org)), a modeling and simulation environment for hybrid systems. Scicos is included in the free open-source scientific software package Scilab ([www.scilab.org](http://www.scilab.org)). Modelica specifications are provided in an official document; the current version is available in [1].

Unlike Scicos, in which model components are defined as grey-box modules (blocks), the Modelica language requires that the complete model be expressed in the Modelica language (except for simple memory-less external functions). It is currently virtually impossible to isolate a sub-model and compile it separately or develop it in a different language. But this is exactly what is needed when simulation environments are used to design, validate and generate code for real-time applications.

This limitation in Modelica may seem surprising, especially when compared to Scicos. Contrary to Modelica, the Scicos compiler requires only some macroscopic information about each block (direct input/output dependency, presence of state, etc.; see [4] for more information on Scicos block structure). The detail of the algorithm used within each block is irrelevant to the compiler; the internal of each block is fully isolated from the outside. A similar mechanism to isolate a sub-model does not exist in Modelica. In this paper, inspired by the Scicos formalism, we propose a solution to this problem. This is particularly important for applications where real-time code generation is sought. In addition, it allows a harmonious integration of the two environments Modelica and Scicos.

## 2 Language Extensions

The *when-elsewhen* and *if-then-else* clauses are the language constructs in Modelica for performing conditioning and sub-sampling. We present extensions to these clauses, which not only facilitate programming in Modelica, but also in conjunction with the introduction of a new type, *Event*, allows for module isolation and separate compilation.

### 2.1. switch and switchwhen

The *switch* construct is a very natural extension of *if-then-else*:

```
switch (n)
  case 1 :
    < eq1 >
    < eq2 >
    ...
  case 2 :
    < eq3 >
    < eq4 >
    ...
  case default:
    < eq5 >
    < eq6 >
    ...
end switch;
```

One and only one case is active depending on the value of the integer *n*. The counterpart in Scicos of this construct is realized with the *ESelect* block.

The *switchwhen* construct generalizes *when-elsewhen*.

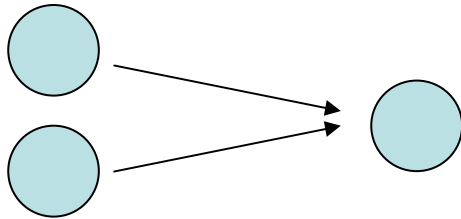
```
switchwhen {c1, c2, c3}
  case '001' :
    < eq1 >
    < eq2 >
    ...
  case '010' :
    < eq3 >
    < eq4 >
    ...
  case '111' :
    < eq5 >
    < eq6 >
    ...
end switchwhen;
```

The content of one and only one case is activated depending on what combination of events *c1*, *c2* and *c3* is generated. For example if events *c1* and *c3* are simultaneously generated but not *c2*, then the case '101' is activated. For this to happen, *c1* and *c2* must be synchronous or be time events which are simultaneously detected (for example by the zero-crossing mechanism of the numerical solver).

In most cases, simultaneous detection of time events (e.g. zero-crossings) needs not be considered as a special case. By default, time events are considered asynchronous, in case of “accidental” simultaneous detection, one event can usually be activated after another (no specified order). For special cases where simultaneous detection does matter, the *switchwhen* construct allows us to take advantage of this additional information. This is useful in some applications as we shall see in the next example:

### Example of Usage of *switchwhen*

Consider the problem of contact between three balls:



Ignoring the possibility of simultaneous contact, the dynamics of this system can be modeled as a 1D problem as follows:

```
equation
der(x1)=v1;der(x2)=v2;der(x3)=v3;
der(v1)=0;der(v2)=0;der(v3)=0;
when x3-x1<=1 then
  reinit(v1,pre(v3));
  reinit(v3,pre(v1));
end when;
when x3-x2<=1 then
  reinit(v2,pre(v3));
  reinit(v3,pre(v2));
end when;
```

Here  $x_i$ ,  $i=1,2,3$ , denote the positions of the balls and we suppose that the balls 1 and 2 situated on left come into contact with the ball number 3 positioned on the right.

The simulation result shows that in the case of simultaneous contact, the model is incorrect. The reason is that after a double contact, after treating one, the jump in the speeds of the balls make it so that the second contact never happens. But this contact is already detected and must be treated. This problem can be avoided by adding a test to make sure that after treating the first contact, the result is taken into account before treating the second:

```
when x3-x1<=1 then
  if v1>v3 then
    reinit(v1,pre(v3));
    reinit(v3,pre(v1));
  end if;
end when;
when x3-x2<=1 then
  if v2>v3 then
    reinit(v2,pre(v3));
    reinit(v3,pre(v2));
  end if;
end when;
```

```
end when;
```

This solution is consistent but it is not very flexible. In particular, it does not allow us to specify explicitly what should happen in the case of a double contact. Using *switchwhen*, the dynamics of simultaneous contact can be explicitly expressed:

```
equation
der(x1)=v1;der(x2)=v2;der(x3)=v3;
der(v1)=0;der(v2)=0;der(v3)=0;
switchwhen {x3-x1<=1,x3-x2<=1} then
case "10":
  reinit(v1,pre(v3));
  reinit(v3,pre(v1));
case "01":
  reinit(v2,pre(v3));
  reinit(v3,pre(v2));
case "11":
  <TO DO IN CASE OF SIMULTANEOUS CONTACT>
end switchwhen;
```

We are not suggesting that *switchwhen* is a universal solution to the problem of contacts in mechanics, which is a difficult problem in general. This example was simply chosen to illustrate the usage of *switchwhen* in case of time events.

The most important use of *switchwhen* is for module isolation and applies to the case of synchronous events as we shall see later.

## 2.2. Type *Event* and Primitive *event*

Currently events in Modelica are coded by Booleans:

```
e=edge(time>2);    e=sample(0,1);
```



But these Booleans are not “normal” Booleans: they are of impulsive type. We can consider then that events are coded by this type of Booleans. However, the edge operator, which is usually used to generate events, does not always generate this type of Boolean. For example if  $k$  is a discrete variable, then

```
when k>0 then
  c=edge(b) ;
```



generates a “standard” Boolean. This shows that the *edge* does not always produce an impulsive Boolean (as the name suggests). In fact, *edge(b)* is not just a function of  $b$ , it depends on the argument of the *when* clause in which it is placed.

We see that there is no real distinction between a “standard” Boolean and one that is used as an event. This can be very confusing as it can be seen in the following example:

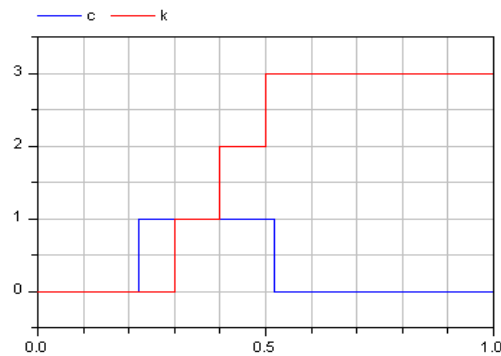
```
discrete Real d,k;
Boolean b,c;
```

```

equation
when sample(0,.1) then
  if c then
    k=pre(k)+1;
  else
    k=pre(k);
  end if;
end when;
when sample(.22,.3) then
  b=d>0;
  c=edge(b);
  d=pre(d)+1;
end when;

```

The simulation result over the period  $[0,1]$  is given below:



It shows that  $k$  is incremented three times during a single  $edge(b)$ ! This is not what we would expect and comes from the confusion between Booleans and events.

To remedy this problem, and for more important reasons that we shall see later, we introduce a new type called *Event*. The type *Event* codes the times of events as floats:

```

Event e1(start=0), e2 ;
equation
when e1 then
  e2=e1+1 ;
.....

```

In this case,  $e2$  is an event, delayed by one with respect to  $e1$ . The Modelica function  $sample(0,1)$  can be emulated easily using this type:

```

Event e(start=0) ;
equation
when pre(e) then
  e=pre(e)+1 ;
end when ;

```

We can then use  $when e$  instead of  $when sample(0,1)$ . Note that to realize  $sample(t0,T)$ , we should proceed as follows to avoid accumulation of numerical errors:

```

Event e(start=t0) ;
discrete Integer k(start=0) ;
equation
when pre(e) then

```

```

        k=pre(k)+1 ;
        e=k*T+t0 ;
    end when ;

```

*Events* can also be generated from zero-crossings. We need a mechanism to generate such *Events*. We have seen that the *edge* operator is not fully appropriate, that is why we propose a new operator: *event*.

```

    Event e1,e2;
    .....
    equation
    der(x)=sin(x);
    e1=event(x>.2) ;
    when e1 then
        d=pre(d)+1 ;
        e2=event(d>4);
    end when ;
    when e2 then
        <xxx>
    end when ;

```

Note that the two *event* operators in the above code both generate *Events*, but the mechanism by which they do it is very different. The first one is used outside a *when* clause, so it is realized in the compiler/simulator by the zero-crossing detection mechanism of the numerical solver. The second on the other hand is inside a *when*, it is synchronous and will be removed in the compilation phase. In this case this is done as follows:

```

    Event e1;
    .....
    equation
    der(x)=sin(x);
    e1=event(x>.2) ;
    when e1 then
        d=pre(d)+1 ;
        if (d>4) and not (pre(d)>4) then
            <xxx>
        end if ;
    end when ;

```

The use of the type *Event* clarifies the situation to a point that we propose that only *Events* be allowed as arguments of *when* clauses<sup>1</sup>. For example the 3 ball problem introduced previously would be expressed as follows:

```

    equation
    der(x1)=v1;der(x2)=v2;der(x3)=v3;
    der(v1)=0;der(v2)=0;der(v3)=0;
    E1= event(x3-x1<=1);
    E2=event(x3-x2<=1);
    switchwhen {E1,E2} then
        case "10":
            ...

```

---

<sup>1</sup> This can be done gradually to reduce the problems of backward compatibility.

### 3 Applications

In this section we examine the applications of using the proposed extensions.

#### 3.1. Compiler/Simulator Simplification

The ability to manipulate event times explicitly simplifies model construction. In particular there is no need to use artificial tests against time. For example, consider the problem of modeling the propagation delay in a digital circuit, which requires a variable dependent event delay. This type of delaying operation can be realized as follows:

```
when time>c_time then
  d_time=c_time+u;
end when;
when time>d_time then
  ...
```

But using *Event*, the code can be made simpler:

```
when c_time then
  d_time=c_time+u;
end when;
when d_time then
  ...
```

But representing events explicitly, also simplifies the job of the compiler. In particular, the compiler no longer needs to “figure out” what “tests” are simple enough to be implemented without using the solver’s zero-crossing mechanism.

Another simplification comes from the canonical representation of the model when *Events* are explicitly identified (declared) and used in conditioning of the *when* statements. Consider the following example:

```
Event e1,e2,e3,...;
.....
equation
.....
e1=event(...
when initial then
  ...
end when;
when e1 then
  k=pre(k)+1;
  e2=event(k>1);
  ...
when e2 then
  e3=time+1;
  ...
end when;
.....
```

In this model, we can readily identify the *Events* and their types. Clearly *e1* and *e3* are asynchronous *Events*; the first one is of type zero-crossing. But *e2* is synchronous and thus can be eliminated. This simplifies the task of the compiler, which in the first phase

of the compilation, removes *e2* yielding a model containing only primary *when* clauses (see [2] for the definition of a primary *when* clause):

```

Event e1,e3,...;
.....
equation
when continuous then
  e1=event(...)
  ...
end when;
when initial then
  ...
end when;
when e1 then
  k=pre(k)+1;
  if (k>1) and not(pre(k)>1) then
    e3=time+1;
  ...
end when;
.....

```

The important thing to note here is that at the end of the first phase of the compilation, we end up with a model that contains only asynchronous *Events*. Note also that asynchronous events, explicitly declared as *Event*, are of two types:

- Zero-crossing: implemented using zero-crossing mechanism of the numerical solver
- Predictable: e.g., *e2=e1+1*;

The type of *Event* is coded in the model by the user, not guessed by the compiler (we may consider allowing the compiler to switch type from zero-crossing to predictable when possible).

The phase two of the compilation performs static scheduling independently for the codes associated with each *Event*, and for sections: “continuous”, “initial” and “terminal” (see [2] for the definition of these sections). The simulator interacts with the code through these *Events*. It uses an “Event Scheduler” on run-time.

### 3.2. Separate Compilation

Currently there is no mechanism in Modelica that allows us to isolate a part of a model and compile it separately. But being able to isolate a module has many applications. For example in control applications, often the user models the plant and the controller to validate the performance by simulation and then wants to generate code for the controller part only.

We think that module isolation can be realized using *input/output Events*. Consider the following example, which is a counter that slows down as time advances:

```

model SlowDownCounter
event_delay BB;
Event E(start=0);
discrete Real U(start=1);
discrete Integer k(start=1);
equation

```



```

when pre(E) then
  k=pre(k)+1;
  (E)=BB(pre(E),U);
end when;
end SlowDownCounter

```

where *event\_delay* is considered as an external function that can be expressed in C, Java, Modelica, etc. For that, we consider an extension of the current notion of function, which currently allows only immediate functions with no states, no sub-sampling, etc. The Modelica program for this function could be the following:

```

function event_delay
input Event e1;
output Event e2;
input Real u;
equation
when e1 then
  e2=e1+u;
end when;
end event_delay;

```

In this case *SlowDownCounter* can be compiled without any knowledge of the content of the *event\_delay* function. This function can be compiled separately too or written in C (the Scicos block routine of the *Event-delay block* does exactly this). The use of “*function*” may not be the best solution. We may consider using *block* instead of *function*, and declare it as *external* in *SlowDownCounter*.

To see why we need *input Event* in this case, note that without it, flattening the model yields a *when* clause within another *when* clause. Nested *when* clauses are not accepted in Modelica and it is not obvious to see how they can be interpreted if they are allowed.

Isolated modules can be a lot more general than in this example; they can have *input/output Events*, internal states (*der* and *pre*), conditioning (*if-then-else* and *switch*) and sub-sampling under the isolation condition: **all Events within the module must either come from input or be of asynchronous type (for example of type zero-crossing).**

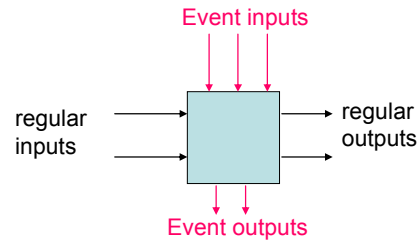
The isolation condition guarantees that the calling environment knows when to call the external module. Specifically it avoids nested *when* clauses which are meaningless.

Note that some information concerning module must be provided to the calling environment so that it can be compiled separately. These information are needed for proper scheduling (finding the right order of execution) but also for proper interfacing with the numerical solver:

- what inputs affect the outputs directly (direct feedthrough),
- is block always active (contains continuous variables),
- if the module contains *der()*, the continuous state and its derivative must be part of the input and the output.

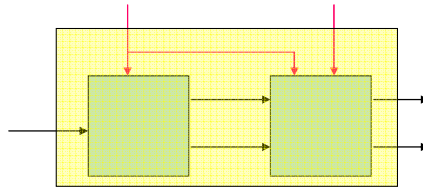
These conditions are exactly the block properties provided to the compiler in Scicos. They are enough for separate compilation and code generation. In Scicos, the internal function of the block is not known to the compiler; its code is in general provided as a *dll*. What the simulator does is to call the “black box” routines associated with the blocks in the right order. This is exactly what the Modelica simulator would do with external modules.

An isolated Modelica module can be represented as follows:



As stated previously, the routines of the block can be written in C (such as a Scicos block, or a Simulink block) or in Modelica. Note that in such a block, output events are never synchronous with input events. Input events however can be synchronized, that is why the *Switchwhen* clause is sometimes needed inside the block.

Under certain conditions, a sub-model obtained from connected blocks can be converted to a single block:



This construction is very similar to that of a Super Block in Scicos compiled into a block using the code generation mechanism.

### 3.3. Scicos Interface

As we have seen, the Scicos formalism and the Modelica language become very similar when the type *Event* is introduced in Modelica. It is then natural to consider developing an interface between the two so that:

- Scicos blocks can be used in a Modelica model,
- Modelica isolated modules can be used as Scicos blocks (this has been the subject of the Simpa Project [5]).

The routines associated to blocks in Scicos have a specific prototype:

```
void my_block(scicos_block *block, int flag)
```

where *scicos\_block* is a structure containing block data and *flag* indicates the task that the function must realize. The following table indicates the tasks that the function may have to realize:

Flag	Job
0	Compute state derivative
1	Compute outputs
2	Update states
3	Output event dates
4	Initialization
5	Ending
9	Compute zero crossings and modes

The simulation engine interacts with the block by calling the functions associated with the blocks, with different *flags*, to advance time. This is exactly the way we implement the Modelica simulator. This implementation separates completely the simulation engine from the model. It also allows the use of both Modelica and Scicos components in the same model.

## Conclusion

We have introduced extensions to the Modelica language that would allow for model isolation and separate compilation. Besides obvious advantages, this allows Modelica programs to be interfaced with other simulation environments such as Scicos and Simulink.

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# Enhancing Modelica towards variable structure systems

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**Abstract.** This paper explains the motivation behind variable structure systems and analyses the current Modelica language with respect to those concerns. The major flaws and shortcomings are discussed to raise the awareness for the most relevant problem sets. Furthermore we sketch our current research activity in broad terms and explain our approach that consists of a new modeling language. Finally, a small example is presented.

**Keywords:** object-oriented modeling, variable structure systems.

## 1 Motivation

Many contemporary models contain structural changes at simulation run time. These systems are typically denoted by the collective term: variable structure systems. The motivations that lead to the generation of such systems are manifold:

- The structural change is caused by ideal switching processes. Classic examples are ideal switching processes in electric circuits, rigid mechanical elements that can break apart, e.g. a breaking pendulum or the reconfiguration of robot models [4].
- The model features a variable number of variables: This issue typically concerns social or traffic simulations that feature a variable number of agents or entities, respectively.
- The variability in structure is to be used for reasons of efficiency: A bent beam should be modeled in more detail at the point of the buckling and more sparsely in the remaining regions.
- The variability in structure results from user interaction: When the user is allowed to create or connect certain components at run time, this usually reflects a structural change.

The term variable structure system turns out to be a rather general term that applies to a number of different modeling paradigms, such as adaptive meshes in finite elements, discrete communication models of flexible computer networks, etc. We focus on the paradigm that is represented by Modelica: declarative models that are based on DAEs with hybrid extensions. Within such a paradigm, a structural change

is typically reflected by a change in the set of variables, and by a change in the set of relations (i.e., equations) between these time-dependent variables. These changes may lead to severe changes in the model structure. This concerns the causalization of the equation system, as well as the perturbation index of the DAE system.

A general modeling language supporting variable structure systems offers a number of important benefits. Such a potential language incorporates a general modeling methodology that enables the convenient capture of knowledge concerning variable structure systems, and provides means for organizing and sharing that knowledge both by industry and science. A corresponding simulator is a valuable tool for engineering and science education.

In concrete terms, our research is intended to aid the further extension of the Modelica framework. This benefits primarily the prevalent application areas of mechanics and electronics.

- Ideal switching processes in electronic circuits (resulting from ideal switches, diodes, and thyristors) can be more generally modeled. Occurring structural singularities can be handled at run time.
- The modeling of ideal transitions in mechanical models, like breaking processes or the transition from friction to stiction, become a more amenable task.

Additional applications may occur in domains that are currently foreign to Modelica. This might concern for instance:

- Hybrid economic or social simulations that contain a variable number of entities or agents, respectively.
- Traffic simulations.

Finally, more elaborate modeling techniques become feasible. For instance multi-level models can be developed, whereby the appropriate level of detail is chosen at simulation run time in response to computational demands and/or level of interest.

## 2 Analysis of Modelica

Unfortunately, the modeling of variable structure systems within the current Modelica framework is very limited. This is partly due to a number of technical restrictions that mostly originate from the static treatment of the DAEs. Specific techniques, like inline-integrations [2] can help in certain situations, but they do not provide a general solution. Although the technical restrictions represent a major limiting factor, other issues need to be concerned as well. An important problem is the lack of expressiveness in the Modelica-language.

To get a better understanding, we analyze the Modelica language with respect to the modeling of structural changes and list the most problematic points in the following subsections.

## 2.1 Lack of Conditional Declarations

Modelica is a declarative language that is based upon the declaration of equations, basic variables and sub-models. Modelica offers conditional blocks (i. e.: if, when) that enable the convenient formulation of changes in the system-equations. However, the declaration of variables or sub-models is kept away from these conditional blocks and is restricted to the unconditional header-section. Hence there is no mechanism for instance creation or removal at run-time.<sup>1</sup>

## 2.2 No Dynamic Linking

The linking of an identifier to its instance is always static in Modelica. To conveniently handle objects that are created at run time, a dynamic linking of identifiers to their instances becomes desirable. Consequently, the linking must be assigned by the use of appropriate operators. Sub-models have now to be treatable as an entity.

## 2.3 Nontransparent type-system

Such assignments that operate on complete model-instances also increase the emphasis on the type analysis like type-compatibility. Modelica is based on a structural type system [1] that represents a powerful and yet simple approach. Sadly, the actual type is not made evident in the language for a human reader since type-members and non-type members mix in the header-section. Also the header section itself might be partitioned in different parts. Hence it becomes hard to identify the type of sub-models just by reading its declaration. This becomes a crucial issue when objects need to be treated dynamically.

## 2.4 Accessing the Environment

Each model in Modelica is defined as a closed entity that cannot access by itself any outside variables. Whereas such a restriction is meaningful in most of the cases, it is inappropriate for certain tasks. One of these tasks is for instance the automatic connection of mass-holding objects to a gravity field. Modelica offers the concept of outer-models for this purpose. Unfortunately this approach is quite limited and represents not a feasible approach for more complex data-structures. At most, outer-models could be used to create pools for mutual gravitational attraction [8] or potential collisions [3]. But to enable such pools, the single-pool members had to be manually assigned to an appropriate integer-ID. This is not a convenient solution.

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<sup>1</sup> In fact, there exists a small mechanism for conditional declaration in Modelica that is supported by Dymola, but the conditions are based upon parameters and the way it is done restricts the access on such a conditional object to connect-statements.

The dynamic creation of sub-models increases the importance of a feasible solution for this task. When objects are created dynamically, they also need to be connected to other objects in their environment. Connections to other sub-models need to be established automatically at simulation time.

## 2.5 Insufficient Handling of Discrete-Events

Processes for the creation, removal and handling of dynamic instances represent discrete processes. Hence a powerful support for discrete-event handling is necessary. Modelica offers hybrid extension for such modeling tasks that are inspired by the synchronous data-flow principle [5]. However, for larger systems the current implementation may lead to an computational overkill and hence more elaborated concepts are needed.

The creation and connection have to be managed by discrete events. During such a construction process, singular equation systems may temporarily occur. However, they are not meant to be evaluated. Thus, a synchronous evaluation of the complete system represents an infeasible approach for such tasks, since it can lead to the inappropriate evaluation of intermittent singular systems.

In addition, the discrete event handling is insufficiently specified in the Modelica-language definition. There is a clear lack of specification for describing what is supposed to happen exactly if one event is subsequently causing (or canceling) other events during the same point of simulation time.<sup>2</sup>

## 2.6 Tedious complexity

In the attempt to enhance the Modelica-language with regard to certain application-specific tasks, the original language has lost some of its original beauty and clarity. An increasing amount of specific elements have been added to the language that come with rather small advantages. Several of these small add-ons are potential sources for problems when structural variability is concerned. Thus, a clean-up of the language is an inevitable prerequisite for any further development in this field. Furthermore the language is subverted in daily practice by foreign elements, i.e., so-called annotations.

## 2.7 Summary

To express structural changes, a corresponding modeling language has to meet certain requirements. The language must support discrete events and hence support hybrid modeling, since structural changes clearly represent a discrete event. Furthermore, it

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<sup>2</sup> This concerns for example the MultiBondLib [8] and its impulse-models. The correctness of these models cannot be proven on the basis of the language-specification. Indeed, the correct simulation of these models is bound to the specific implementation in Dymola.



must be allowed to state relations between variables or sub-models in a conditional form, so that the structure can change depending on time and state. In addition, variables and sub-models should be dynamically declarable, so that the corresponding instances can be created, handled, and deleted at run time. Modelica meets these requirements only partly and provides only very limited means for the description of such models.

## **2.8 MOSILAB**

MOSILAB[7] offers a first approach to handling variable structure systems in a more general sense. It combines an extensive subset of Modelica with a description language for statecharts to handle the transition between different modeling modes. MOSILAB features the dynamic creation of sub-model instances, although it does so in a limited way. For us, the use of statecharts represents a practical but limited solution. However, statecharts do not integrate too well into the object-oriented and declarative framework of Modelica. Hence the complexity of the language had to be increased significantly and the beauty and clarity of the original Modelica language suffered in the process of extending the language.

## **3 Sol - a Derivative Language of Modelica**

In attempting an enhancement of Modelica's capabilities with respect to variable structure systems, one arrives at the conclusion that a straight-forward extension of the language will not lead to a persistent solution. The introduction of additional dynamics inevitably violates some of the fundamental assumptions of the original language design and of its corresponding translation and simulation mechanisms.

Hence we have taken the decision to design a new language, optimized to suit the new set of demands. This language is called Sol. In the design process, we intend to maintain as much of the essence of Modelica as possible. To this end, we review the major strengths of Modelica:

- Modelica owns natural readable, intuitive syntax. Models can be understood even by outsiders, and beginners are enabled to quickly acquaint themselves with the language.
- The declarative, equation-based modeling approach enables the modeler to concentrate on what should be modeled, rather than forcing him or her to consider, how precisely the model is to be simulated.
- Modelica offers convenient object-oriented means for the organization of knowledge and type-generation. This makes large projects feasible and eases the knowledge transfer.
- The structural type-system of Modelica separates type-generation and implementation. Thus, even separate implementations can be compatible and exchangeable. The generic connection mechanism enables an intuitive and convenient modeling.

### 3.1 Sol – a New Language for Variable Structure Systems

All those considerations of the previous sections have been taken into account for the design process of Sol. The decision to design a new language enables us to take a more radical, conceptually stronger approach. Hence, Sol attempts to be a language of low complexity that still enables a high degree of expressiveness.

Like Modelica, Sol provides means for declaring synchronous, non-causal relations between variables (i.e., equations). As an extension to Modelica, we furthermore offer a convenient way for declaring asynchronous, causal transmissions from one variable (or sub-model) to another. All of these declarations can be grouped in an almost arbitrary fashion. These groups of declarations may be activated or deactivated in accordance with conditions, events or predetermined sequences.

Unlike in Modelica, also the declaration of variables and sub-models can occur at the beginning of each group or subgroup. Since these groups can be stated in a conditional form, variables and sub-models may be dynamically created and deleted at run time. Hence instance creation and deletion does not need to be stated in the (typical) imperative form. It results from the activation and deactivation of declarative groups. The dynamically created objects can be handled in an unambiguous way by the declaration of asynchronous transmissions. Identifiers can also link dynamically to an instance.

Hence systems that are expressed in Sol are described in a constructive way, where the path of construction and the corresponding interrelations might change in dependence on the current system's state or on current evaluations. Conditional declarations enable a high degree of variability in structure. The constructive approach avoids memory leaks and the description of error-prone update-processes.

The new language will be well-structured, easily readable, and intuitive to understand. The language will provide various object-oriented tools that enable the efficient handling of complex systems. The syntax and grammar of Sol is significantly stricter than the grammar of Modelica. Alternative writings have been discarded and the different sections of a model must obey a given order. This strictness unifies the writing and intends to guide towards a clear and understandable modeling style.

### 3.2 Example

Without going into the details concerning Sol's grammar and semantics, we provide a small, introductory example to show its potential usage. Due to Sol's similarity to Modelica and its intuitive syntax, the example should be understandable in its main functionality. In addition to classic equations Sol features copy-transmission ( $\llcorner$ ) and move-transmissions ( $\llcorner-$ ). We model a simple machine, consisting of an engine that drives a fly-wheel. Two models are provided for the engine: The first model "Engine1" applies a constant torque on the flange. In the second model "Engine2", the torque is dependent on the positional state similar to a piston-engine. The

machine-model connects the engine and the fly-wheel. It contains a structural change that is reflected by a substitution of the engine-models. Initially, the fly-wheel is at rest, and the more complex engine model is used. When the speed exceeds a certain threshold, it seems appropriate to average the torque. Thus, the simpler engine-model is used instead.

```

package Rotational

  connector Flange
  interface:
    static potential Real phi;
    static flow Real t;
  end flange;

  partial model Engine
  interface:
    parameter Real meanTorque << 1;
    static Flange f;
  end Engine;

  model Engine1 extends Engine;
  implementation:
    f.t = meanTorque;
  end Engine1;

  model Engine2 extends Engine;
  implementation:
    static Real transmission;
    transmission = 1+sin(f.phi);
    f.t = meanTorque*transmission;
  end Engine2;

  model FlyWheel
  interface:
    parameter Real inertia << 1;
    static Flange f;
    static Real w;
  implementation:
    static Real z;
    w = der(f.phi);
    z = der(w);
    f.t = inertia*z;
    when initial then w=0; f.phi=0; end;
  end FlyWheel;

```

```

model Machine
implementation:
    static FlyWheel Wheel1{inertia << 10};
    static Boolean fast;
    if fast then
        static Engine1 E{meanTorque << 100};
        connection(E.f,Wheel1.f);
    else then
        static Engine2 E{meanTorque << 100};
        connection(E.f,Wheel1.f);
    end;

    when initial then fast << false; end;
    when Wheel1.w > 50 then fast << true; end;
end Machine;

end Rotational;

```

The structural change is contained in the model “Machine”. It declares a Boolean state-variable “fast” that determines which model to use. Please note, that the conditional if-clauses also contain declarations of sub-models. This enables a convenient, easily readable formulation of the structural change based on the current system state. There is also no need for an explicit model of the transition or manual disconnections.

The example code below presents an alternative solution for the machine-model. The identifier E is declared to be “dynamic”. This means: It can be dynamically linked to any model-instance that is type-compatible with “Engine”. The corresponding instances are simply declared anonymously in the conditional when-clauses. The type of a model is solely defined by its interface-section.

```

model Machine
implementation:
    static FlyWheel Wheel1{inertia << 10};
    dynamic Engine E;
    connection(E.f,Wheel1.f);
    when initial then
        E <- Engine2{meanTorque << 100};
    end;
    when Wheel1.w > 50 then
        E <- Engine1{meanTorque << 100};
    end;
end Machine;

```

This simple example contains only a very simple structural change that is basically reflected by the replacement of a single equation. Hence this could have also been modeled in Modelica, but not at this level of abstraction. The complete replacement of a model, as it is done here, can as well be used for more elaborate multi-level models.

## 4 Implementation and Ongoing Development

A first version of the language definition of Sol has been written down in the form of an internal report. It forms the fundamentals for a corresponding implementation that is currently under development. This implementation will be represented by an interpreter that parses the model-file, instantiates a selected model and starts simulation. In addition to its main task, the interpreter will provide various tools for the analysis of the object-hierarchy, type-structure, etc.

Whereas the pair of a compiler and a simulator is the preferred choice for high-end simulation tasks, an interpreter is an appropriate tool for research work on language design. The development process becomes much easier, faster and more flexible. Hence the development of the interpreter can proceed in parallel with a further refinement of the language. Also, new debugging techniques will be needed that can be better provided by an interpreter, since all necessary meta-information is available. Of course, any interpreter (even if it is well written) suffers from a certain computational overhead that may prevent its usage for highly demanding simulation applications. Hence an important aspect will be to sketch the development of a corresponding compiler.

### 4.1 Future goals

Sol is a language primarily conceived for research purposes. We want to explore the full power of a declarative modeling approach and how it can handle potential, future problem fields. Some of our goals and motivations are similar to [6], although we are coming from a different direction. The implementation of Sol will be a small and open project that should enable other researchers to test and validate their ideas with a moderate effort. The longer term goal of our research is to significantly extend Modelica's expressiveness and range of application. Furthermore, the Sol-project gives us a development-platform for technical solutions that concerns the handling of structurally changing equation systems. This includes solutions for dynamic recausalization or the dynamic handling of structural singularities.

It is not our target to immediately change the Modelica standard or to establish an alternative modeling language. Our scientific work is intended to merely offer suggestions and guidance for future development. This will primarily benefit the future development of Modelica, but our results may also prove useful to other modeling communities and researchers.

## 5. Conclusion

The development of a new modeling language should be a well considered step, since it incorporates a lot of effort. This does not only concern the developers of the language and the corresponding software, it includes as well the potential modelers

and users that are expected to get themselves acquainted with the new methodology. However, the continuous progress of modeling technology generates a new set of demands. This makes such a step finally inevitable.

In this workshop-paper, we offered a first glance of Sol, our new modeling language. Sol has been designed to enable the modeling of variable-structure systems using an equation-based framework. While its development is currently still at the beginning, we expect to make significant progress in the near future. In the longer term, we hope that our research will benefit Modelica's future development.

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# Functional Hybrid Modeling from an Object-Oriented Perspective

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*Declaration:* This paper is closely based on [19] that was published in the Proceedings of Practical Aspects of Declarative Languages (PADL) 2003. The paper has been updated and adapted for the Equation-Based Object-Oriented Languages and Tools (EOOLT) 2007 Workshop.

**Abstract.** The modeling and simulation of physical systems is of key importance in many areas of science and engineering, and thus can benefit from high-quality software tools. In previous research we have demonstrated how *functional programming* can form the basis of an expressive language for *causal* hybrid modeling and simulation. There is a growing realization, however, that a move toward *non-causal* modeling is necessary for coping with the ever increasing size and complexity of modeling problems. Our goal is to combine the strengths of functional programming and non-causal modeling to create a powerful, strongly typed *fully declarative modeling language* that provides modeling and simulation capabilities beyond the current state of the art: in particular, support for highly structurally dynamic systems. Additionally, we think our approach could serve as a semantical framework for studying modeling and simulation languages supporting structural dynamism, and maybe even as a core language in systems where the surface syntax is more conventional. Although our work is still in its very early stages, we believe that this paper clearly articulates the need for improved modeling languages and shows how functional programming techniques can play a pivotal role in meeting this need.

## 1 Introduction

*Modeling and simulation* is playing an increasingly important role in the design, analysis, and implementation of real-world systems. In particular, whereas modeling fragments of systems in isolation was deemed sufficient in the past, considering the interaction of these fragments *as a whole* is now necessary. The resulting models are large and complex, and span multiple physical domains.

Furthermore, these models are almost invariably *hybrid*: they exhibit both continuous-time and discrete-time behaviors. In fact, the very structure of the modeled system changes over time. Such models are known as *structurally dynamic*. In general, the total number of structural configurations, or *modes*, can be enormous, or even unbounded. We refer to systems whose number of modes cannot be practically predetermined as *highly structurally dynamic*. While supporting structural dynamism is hard, supporting highly structurally dynamic systems is even harder as this necessitates comprehensive and flexible solutions of a number of important subproblems: see Sect. 5.

There are two broad language categories of modeling and simulation languages. *Causal* (or *block-oriented*) languages are most popular; languages such as Simulink and Ptolemy II [13] represent this style of modeling. In causal modeling, the equations that represent the physics of the system must be written so that the direction of signal flow, the *causality*, is explicit. The second, but less populated, class of language is *non-causal*, where the model focuses on the interconnection of the components of the system being modeled, from which causality is then inferred. Such languages often support an *object-oriented* approach to modeling. Examples include Dymola [5] and Modelica [15].

The main drawback of causal languages is the need to explicitly specify the causality. This hampers modularity and reuse [2]. Non-causal languages address this problem by allowing the user to describe a model in a way which does not commit to any specific causality. The appropriate causality constraints are then inferred using both symbolic and numerical methods depending on how the model is being used. Unfortunately, current non-causal modeling languages tend to sacrifice generality when it comes to hybrid modeling: in particular, we are not aware of any *declarative* non-causal modeling language that supports highly structurally dynamic models, even if recent efforts like MOSILAB [20] and Sol [28] are important steps in that direction.

In previous research at Yale, we have developed a framework called *Functional Reactive Programming* (FRP) [26], which is suited for causal hybrid modeling. This framework is embodied in a language called *Yampa*.<sup>4</sup> as an extension of Haskell. Yampa permits highly structurally dynamic hybrid systems to be described clearly and concisely [18].<sup>5</sup> In addition, because the full power of a functional language is available, it exhibits a high degree of modularity, allowing reuse of components and design patterns. It also employs Haskell's polymorphic type system to ensure that signals are connected consistently, even as the system topology changes. The semantic foundations of Yampa are well defined and understood, making models expressed using Yampa suited for formal manipulation and reasoning. Yampa and its predecessors have been used in robotics simulation and control as well as a number of related domains [23, 24]. It has even been used for video games [4, 3]. We are currently investigating biological cell population modeling, where Yampa's support for highly structurally dynamic

<sup>4</sup> See <http://haskell.org/yampa>.

<sup>5</sup> However, at present, Yampa lacks integration with sophisticated numerical solvers, and its applicability for serious simulation work is thus limited



systems provides an interesting declarative approach to handling cell division in contrast to the imperative approach of agent-based simulators [12].

Non-causal modeling and FRP complement each other almost perfectly. We therefore aim to integrate the core ideas of FRP with non-causal modeling to create *Hydra*, a powerful, fully declarative modeling language combining the strengths of each. If we treat causality and dynamism as two dimensions in the modeling language design space, we see that Hydra occupies a unique point:

	Mostly static structure	Highly dynamic structure
Causal	Simulink	Yampa
Non-causal	Modelica	Hydra

MOSILAB and Sol are somewhere between Modelica and Hydra.

We refer to the combined paradigm of functional programming and non-causal, hybrid modeling as *Functional Hybrid Modeling*, or FHM. Conceptually, FHM can be seen as a generalization of FRP, since FRP's *functions* on signals are a special case of FHM's *relations* on signals. In its full generality, FHM, like FRP, also allows the description of structurally dynamic models.

The main contribution of this paper is that it outlines how notions appropriate for non-causal, hybrid simulation in the form of *first-class relations on signals* and *switch constructs* can be integrated into a functional language, yielding a non-causal modeling language supporting structural dynamism. It also identifies key research issues, and suggests how recent developments in the field of programming languages could be employed to address those issues.

## 2 Yampa

To help readers who are not familiar with Functional Reactive Programming put the ideas of this paper into context, we provide a brief review of the key ideas of Yampa in the following. For further details, see earlier papers on Yampa [9, 18]

### 2.1 Fundamental Concepts

Yampa is based on two central concepts: *signals* and *signal functions*. A signal is a function from time to a value:

$$\text{Signal } \alpha \approx \text{Time} \rightarrow \alpha$$

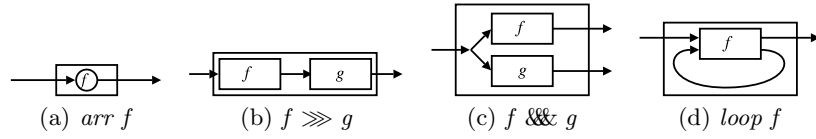
*Time* is continuous, and is represented as a non-negative real number. The type parameter  $\alpha$  specifies the type of values carried by the signal. For example, the type of a varying electrical voltage might be *Signal Voltage*.

A *signal function* is a function from *Signal* to *Signal*:

$$\text{SF } \alpha \beta \approx \text{Signal } \alpha \rightarrow \text{Signal } \beta$$

When a value of type  $SF\ \alpha\ \beta$  is applied to an input signal of type  $Signal\ \alpha$ , it produces an output signal of type  $Signal\ \beta$ . Signal functions are *first class entities* in Yampa. Signals, however, are not: they only exist indirectly through the notion of signal function. In general, the output of a signal function at time  $t$  is uniquely determined by the input signal on the interval  $[0, t]$ . If a signal function is such that the output at time  $t$  only depends on the input at the very same time instant  $t$ , it is called *stateless*. Otherwise it is *stateful*.

## 2.2 Composing Signal Functions



**Fig. 1:** Basic signal function combinators.

Programming in Yampa consists of defining signal functions compositionally using Yampa’s library of primitive signal functions and a set of combinators. Yampa’s signal functions are an instance of the arrow framework proposed by Hughes [10]. Three combinators from that framework are  $arr$ , which lifts an ordinary function to a stateless signal function, and the two signal function composition combinators  $\lll$  and  $\&\&\&$ :

$$\begin{aligned} arr &:: (a \rightarrow b) \rightarrow SF\ a\ b \\ (\lll) &:: SF\ b\ c \rightarrow SF\ a\ b \rightarrow SF\ a\ c \\ (\&\&\&) &:: SF\ a\ b \rightarrow SF\ a\ c \rightarrow SF\ a\ (b, c) \end{aligned}$$

We can think of signals and signal functions using a simple flow chart analogy. Boxes represent signal functions, with one signal flowing in to the box’s input port and another signal flowing out of the box’s output port. Figure 1 illustrates some of the central arrow combinators using this analogy. The similarity to a block-oriented modeling language like Simulink is hopefully clear. The main difference is that the notion of composing blocks into larger blocks has been formalized through a handful of composition combinators, which is helpful from a semantical perspective, in contrast to the more unstructured approach of connecting outputs to inputs in an arbitrary fashion.

## 2.3 Arrow Syntax

While the arrow framework provides a useful semantical structure, it is not convenient for expressing large networks. It is much easier to simply connect whatever needs to be connected Simulink style, e.g. by naming nodes and then

explicitly stating the connection topology. Fortunately, it is easy to provide a layer of syntax that allows this, and then translate this into a network description in terms of the core arrow combinators. Paterson’s arrow notation [22] does exactly that. An expression denoting a signal function has the form:

```
proc pat → do
  pat1 ← sfeexp1 ↗ exp1
  pat2 ← sfeexp2 ↗ exp2
  ...
  patn ← sfeexpn ↗ expn
  return A ↗ exp
```

The keyword **proc** is analogous to the  $\lambda$  in  $\lambda$ -expressions, *pat* and *pat*<sub>*i*</sub> are patterns binding signal variables pointwise by matching on instantaneous signal values, *exp* and *exp*<sub>*i*</sub> are expressions defining instantaneous signal values, and *sfe**exp*<sub>*i*</sub> are expressions denoting signal functions. The idea is that the signal being defined pointwise by each *exp*<sub>*i*</sub> is fed into the corresponding signal function *sfe**exp*<sub>*i*</sub>, whose output is bound pointwise in *pat*<sub>*i*</sub>. The overall input to the signal function denoted by the **proc**-expression is bound by *pat*, and its output signal is defined by the expression *exp*. The signal variables bound in the patterns may occur in the signal value expressions, but *not* in the signal function expressions (*sfe**exp*<sub>*i*</sub>). An optional keyword **rec**, applied to a group of definitions, permits signal variables to occur in expressions that textually precede the definition of the variable, allowing recursive definitions (feedback loops).

For a concrete example, consider the following:

```
sf = proc (a, b) → do
  (c1, c2) ← sf1 &&& sf2 ↗ a
  d ← sf3 <<< sf4 ↗ (c1, b)
  rec
    e ← sf5 ↗ (c2, d, e)
  return A ↗ (d, e)
```

Note the use of the tuple pattern for splitting *sf*’s input into two “named signals”, *a* and *b*. Also note the use of tuple expressions and patterns for pairing and splitting signals in the body of the definition; for example, for splitting the output from *sf1* &&& *sf2*. Also note how the arrow notation may be freely mixed with the use of basic arrow combinators.

## 2.4 Events

While some aspects of a program are naturally modeled as continuous signals, other aspects are more naturally modeled as *discrete events*. To this end, Yampa introduces the *Event* type, isomorphic to Haskell’s *Maybe* type:

```
data Event a = NoEvent | Event a
```

The instantaneous value of signal of type *Event T* for some type *T* is either *NoEvent* or *Event x* for some value *x* of type *T*, thus mimicking a discrete-time signal that is only defined at discrete points in time.

## 2.5 Switching

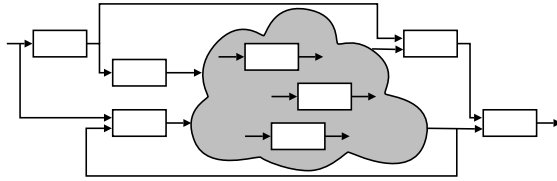
The structure of a Yampa system may evolve over time. These structural changes are known as *mode switches*. This is accomplished through a family of *switching* primitives that use events to trigger changes in the connectivity of a system. The simplest such primitive is *switch*:

$$\text{switch} :: SF\ a\ (b, \text{Event}\ c) \rightarrow (c \rightarrow SF\ a\ b) \rightarrow SF\ a\ b$$

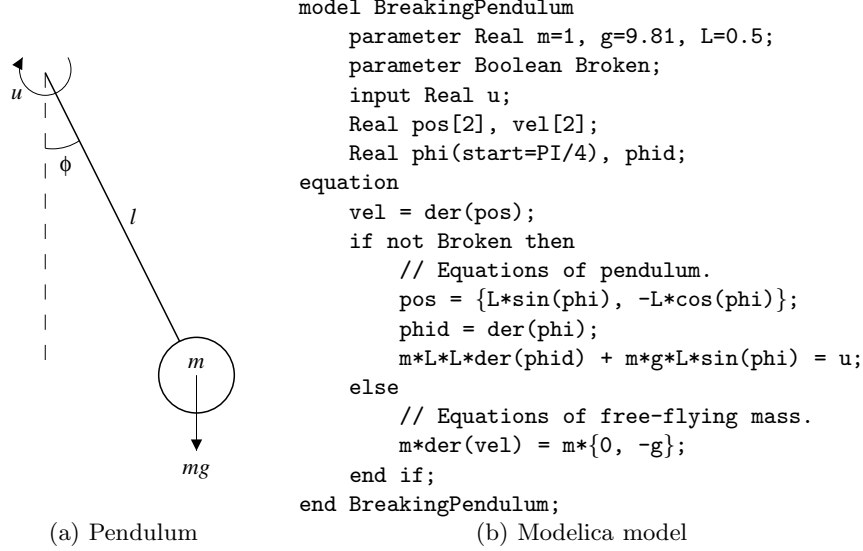
The *switch* combinator switches from one subordinate signal function into another when a switching event occurs. Its first argument is the signal function that initially is active. It outputs a pair of signals. The first defines the overall output while the initial signal function is active. The second signal carries the event that will cause the switch to take place. Once the switching event occurs, *switch* applies its second argument to the value of the event and switches into the resulting signal function.

Thus, note that the second argument of *switch* is a *function* of type  $c \rightarrow SF\ a\ b$ , that, when given the value of type *c* carried by the event, *dynamically computes* a new signal function to switch into. Using a Simulink analogy, *switch* in principle rips out a block, and then dynamically instantiates a parameterized block as a replacement. The design of *switch* thus exploits the fact that signal functions (“blocks”) are first class entities in Yampa.

Yampa also includes *parallel* switching constructs that maintain *dynamic collections* of signal functions connected in parallel [18]. Signal functions can be added to or removed from such a collection at runtime in response to events, while *preserving* any internal state of all other signal functions in the collection; see Fig. 2. The first class status of signal functions in combination with switching over dynamic collections of signal functions makes Yampa an unusually flexible language for describing hybrid systems. For example, this makes it possible to handle systems where the number of modeled entities varies over time, like cell population models as mentioned earlier (Sect. 1).



**Fig. 2:** System of interconnected signal functions with varying structure



**Fig. 3:** A pendulum, subject to externally applied torque and gravity.

### 3 Non-Causal and Hybrid Modeling

While the simulation of pure continuous systems is relatively well understood, hybrid systems pose a number of unique challenges [16, 1]. Problems include handling a large number of modes, event detection, and consistent initialization of state variables. The integration of hybrid modeling with non-causal modeling raises further problems. Indeed, current non-causal modeling languages are quite limited in their ability to express hybrid systems. Many of the limitations are related to the symbolic and numerical methods that must be used in the non-causal approach. But another important reason is that most such systems insist on performing all symbolic manipulations *before* simulation begins [16]. Avoiding these limitations is an important part of our approach, see Sec. 5.

Since Modelica is representative of state-of-the-art, non-causal, hybrid modeling languages, we illustrate the limitations of present languages with an example from the Modelica documentation [14, pp. 31–33]. The system is a pendulum in the form of a mass  $m$  at the end of a rigid, mass-less rod, subject to gravity  $mg$  and an externally applied torque  $u$  at the point of suspension; see Fig. 3(a). Additionally, the rod could break at some point, causing the mass to fall freely.

Figure 3(b) shows a Modelica model of this system that, on the surface, looks like it achieves the desired result. Note that it has two modes, described by conditional equations. In the non-broken mode, the position `pos` and velocity `vel` of the mass are calculated from the state variables `phi` and `phid`. In the broken mode, `pos` and `vel` become the new state variables. This implies that state information has to be transferred between the non-broken and broken mode. Furthermore, the causality of the system is different in the two modes. When

non-broken, the equation relating `vel` and `pos` is used to compute `vel` from `pos`. When broken, the situation is reversed.

These facts make simulation hard. Modelica attempts to simplify matters by avoiding too radical structural changes. To that end, Modelica either requires the condition for selecting between two sets of equation to be a *parameter*, and thus unchanging during simulation, or else that the number of equations in each set are the same. In this case, as the number of equations is not the same, `Broken` has to be declared a parameter. Therefore the model above does not really solve the hybrid simulation problem at all! In order to actually model a pendulum that dynamically breaks at some point in time, the model must be expressed in some other way. The Modelica documentation suggests a causal, block-oriented formulation with explicit state transfer. Unsurprisingly, the result is considerably more verbose, nullifying the advantage of working in a non-causal language.

Moreover, even if `Broken` were allowed to be a dynamic variable, a fundamental problem would remain: once the pendulum has broken, it cannot become whole again. Modelica provides no way to declaratively express the *irreversibility* of this structural change. The best that can be done is to capture this fact indirectly through a state machine model that control the value of `Broken`.

## 4 Integrating Functional Programming and Non-Causal Modeling

In the preceding sections we discussed the advantages of non-causal modeling and the importance of hybrid modeling. We also pointed out serious shortcomings in current modeling languages with respect to these features. In this section, we describe a new way to combine non-causal and hybrid modeling techniques that addresses these issues. Inspired by FRP and Yampa, the two key ideas are to give first-class status to relations on signals and to provide constructs for discrete switching between relations. The result is Hydra, a functional hybrid modeling language capable of representing structurally dynamic systems.

While we, based on our experience of Yampa, believe that a language like Hydra would be a very expressive and powerful modeling and simulation language in its own right, we would like to emphasize that we also think our approach could serve as a valuable semantical framework for general study of modeling and simulation languages that supports structural dynamism, and maybe even as a core language in systems where the surface syntax is more conventional. Thus, what is important in the following is not the syntax (which is tentative and likely lacking in many ways), but the underlying principles.

### 4.1 First-Class Signal Relations

A natural mathematical description of a continuous signal function is that of an ODE in explicit form. A function is just a special case of the more general concept of a *relation*. While functions usually are given a causal interpretation, relations are inherently non-causal. Differential Algebraic Equations (DAEs), which are

at the heart of non-causal modeling, express dependences among signals without imposing a causality on the signals in the relation. Thus it is natural to view the meaning of a DAE as a non-causal *signal relation*, just as the meaning of an ODE in explicit form can be seen as a causal signal function. Since signal functions and signal relations are closely connected, this view offers a clean way of integrating non-causal modeling into an Yampa-like setting.

In the following, first-class signal relations are made concrete by proposing a (tentative) system for integrating them into a polymorphically typed functional language. Signal functions are also useful, but since they are just relations with explicit causality, we need not consider them in detail in the following.

Similarly to the signal function type  $SF$  of Yampa (Sect. 2.1), we introduce the type  $SR\ \alpha$  for a relation on a signal of type  $Signal\ \alpha$ . Specific relations use a more refined type; e.g., for the derivative relation *der* we have the typing:

$$der :: SR\ (Real, Real)$$

Since a signal carrying pairs is isomorphic to a pair of signals, we can understand *der* as a binary relation on two real-valued signals.

Next we need a notation for defining relations. Inspired by the arrow notation (Sect. 2.3), we introduce the following to denote a signal relation:

**sigrel** *pattern where equations*

The pattern introduces *signal variables* that at each point in time are bound to the *instantaneous* value of the corresponding signal. Given  $p :: t$ , we have:

**sigrel**  $p$  **where** ...  $:: SR\ t$

Consequently, the equations express relationships between instantaneous signal values. This resembles the standard notation for differential equations in mathematics. For example, consider  $x' = f(y)$ , which means that the instantaneous value of the derivative of (the signal)  $x$  at every time instant is equal to the value obtained by applying the function  $f$  to the instantaneous value of  $y$ .

We introduce two styles of equations:

$$\begin{aligned} e_1 &= e_2 \\ sr &\diamond e_3 \end{aligned}$$

where  $e_i$  are expressions (possibly introducing new signal variables), and  $sr$  is an expression denoting a signal relation. We require equations to be well-typed. Given  $e_i :: t_i$ , this is the case iff  $t_1 = t_2$  and  $sr :: SR\ t_3$ .

The first kind of equation requires the values of the two expressions to be equal at all points in time. For example:

$$f(x) = g(y)$$

where  $f$  and  $g$  are functions.

The second kind allows an arbitrary relation to be used to enforce a relationship between signals. The symbol  $\diamond$  can be thought of as *relation application*;

the result is a constraint which must hold at all times. The first kind of equation is just a special case of the second in that it can be seen as the application of the identity relation.

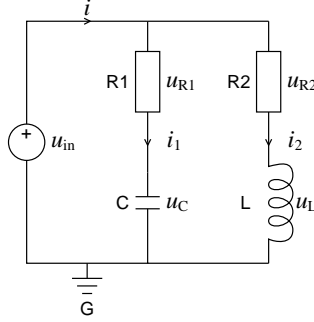
For another example, consider a differential equation like  $x' = f(x, y)$ . Using our notation, this equation could be written:

$$der \diamond (x, f(x, y))$$

where  $der$  is the relation relating a signal to its derivative. For convenience, a notation closer to the mathematical tradition should be supported as well:

$$\mathbf{der}(x) = f(x, y)$$

The meaning is exactly as in the first version.



**Fig. 4:** A simple electrical circuit.

We illustrate our language by modeling the electrical circuit in Fig. 4.1 (adapted from [14]). The type  $Pin$  is a record type describing an electrical connection. It has fields  $v$  for voltage and  $i$  for current.<sup>6</sup>

$$\begin{aligned} twoPin &:: SR (Pin, Pin, Voltage) \\ twoPin &= \mathbf{sigrel} (p, n, u) \mathbf{where} \\ &\quad u = p.v - n.v \\ &\quad p.i + n.i = 0 \end{aligned}$$

$$\begin{aligned} resistor &:: Resistance \rightarrow SR (Pin, Pin) \\ resistor(r) &= \mathbf{sigrel} (p, n) \mathbf{where} \\ &\quad twoPin \diamond (p, n, u) \\ &\quad r \cdot p.i = u \end{aligned}$$

<sup>6</sup> The name  $Pin$  is perhaps a bit misleading since it just represents a pair of physical quantities, *not* a physical “pin component”; i.e.,  $Pin$  is the type of *signal variables* rather than *signal relations*.



```

inductor :: Inductance → SR (Pin, Pin)
inductor(l) = sigrel (p, n) where
    twoPin ◇ (p, n, u)
    l · der(p.i) = u

capacitor :: Capacitance → SR (Pin, Pin)
capacitor(c) = sigrel (p, n) where
    twoPin ◇ (p, n, u)
    c · der(u) = p.i

```

As in Modelica, the resistor, inductor and capacitor models are defined as extensions of the *twoPin* model. However, we accomplish this directly with functional abstraction rather than the Modelica class concept. Note how parameterized models are defined through functions *returning* relations. Since the parameters are normal function arguments, *not* signal variables, their values remain unchanged throughout the lifetime of the returned relations.<sup>7</sup>

To assemble these components into the full model, we adopt a Modelica-like **connect**-notation as a convenient abbreviation for connection equations. This is syntactic sugar which is expanded to proper connection equations, i.e. equality constraints or sum-to-zero equations depending on what kind of physical quantity is being connected. We assume that a voltage source model *vSourceAC* and a ground model *ground* are available in addition to the component models defined above. Moreover, we are only interested in the total current through the circuit, and, as there are no inputs, the model thus becomes a *unary* relation:

```

simpleCircuit :: SR Current
simpleCircuit = sigrel i where
    resistor(1000) ◇ (r1p, r1n)
    resistor(2200) ◇ (r2p, r2n)
    capacitor(0.00047) ◇ (cp, cn)
    inductor(0.01) ◇ (lp, ln)
    vSourceAC(12) ◇ (acp, acn)
    ground ◇ gp
    connect acp, r1p, r2p
    connect r1n, cp
    connect r2n, lp
    connect acn, cn, ln, gp
    i = r1p.i + r2p.i

```

## 4.2 Modeling Systems with Dynamic Structure

In order to describe structurally dynamic systems we need to represent an evolving structure. To this end, we introduce two Yampa-inspired switching constructs: the *recurring switch* and the *progressing switch*. The recurring switch

---

<sup>7</sup> Compare to Modelica's **parameter**-variables mentioned in Sect. 3.

allows repeated switching between equation groups. In contrast, the progressing switch expresses that one group of equations *first* is in force, and then, *once* the switching condition has been fulfilled, another group, thus irreversibly progressing to a new structural configuration. For either sort of switching, difficult issues such as state transfer and proper initialization have to be considered.

We revisit the breaking pendulum example from Sect. 3 to illustrate these switching constructs. To deal with initialization and state transfer, we introduce special initialization equations that are only active at the time of switching, that is, during *events*, and we allow such equations to refer to the values of signal variables just prior to the event through a special **pre**-construct devised for that purpose. The initialization equations describe the initial conditions of the DAE after a switch. Mathematically, these equations must yield an initial value for every state variable in the new continuous equations. It is important that each branch of a switch can be associated with its own initialization equations, since each such branch may introduce its proper set of state variables. Initialization equations typically state continuity assumptions, like *pos* and *vel* below.

First, consider a direct transliteration of the equation part of the Modelica model using a recurring switch. The necessary initialization equations have also been added:

```

vel = der(pos)
switch broken
  when False then
    init phi = pi/4
    init phid = 0
    pos = {l · sin(phi), -l · cos(phi)}
    phid = der(phi)
    m · l · l · der(phid) + m · g · l · sin(phi) = u
  when True then
    init vel = pre(vel)
    init pos = pre(pos)
    m · der(vel) = m · {0, -g}

```

A recurring switch has one or more **when**-branches. The idea is that the equations in a **when**-branch are in force whenever the pattern after **when** (which may bind variables) matches the value of the expression after **switch**. Thus, whenever that value changes, we have an event and a switch occurs (this is similar to **case** in a functional language).

To express the fact that the pendulum cannot become whole once it has broken, we refine the model by changing to a progressing switch:

```

vel = der(pos)
switch broken
  first
  ...
  once True then
  ...

```

A progressing switch has one **first**-branch and one or more **once**-branches. Initially, the equations in the **first**-branch are in force, but as soon as the value of the expression after **switch** matches one of the **once**-patterns, a switch occurs to the equations in the corresponding branch, after which no further switching occurs (for that particular instance of the switch).

By combining recursively-defined relations and progressing switches, it is possible to express very general sequences of structural changes over time, from simple mode transitions to making and breaking of connections between objects. A simple example of a recursively defined relation parameterized on a discrete state variable  $n$  is shown below. Initially, the relation behaves according to the equations in the **first**-branch, which may depend on  $n$ . Whenever the switching condition is fulfilled, the relation switches to a new instance of itself with the parameter  $n$  increased by one. In functional parlance, this is a form of tail call.

```

sysWithCntr :: Int → SR (Real, Real)
sysWithCntr(n) = sigrel (x, y) where
    switch ...
    first
    ...
    once ... then
        sysWithCntr(n + 1) ◇ (x, y)

```

As explained in Sect. 2.5, Yampa supports even more radical structural changes, including dynamic addition and deletion of objects. Our goal is to carry over as much as possible of that functionality to Hydra.

## 5 Implementation Issues

There are a number of challenges that must be addressed in an implementation of a language like Hydra. The primary issues are ensuring model correctness, simulation in the presence of dynamic mode changes, and mode initialization.

It is critical that dynamic changes in the model should not weaken the static checking of the model, i.e. we want to ensure *compositional correctness*. A Haskell-like polymorphic type system, as in Yampa, ensures that the system integrity is preserved. In addition we would like to find at least necessary conditions for statically ensuring that causality analysis can always be carried out, that the equations at least could have a solution, and so on, regardless of how relations are composed dynamically. An example of a necessary but not sufficient condition is that the number of equations and number of variables agree, and that each variable can be paired with one equation. Since it will be necessary to keep track of the balance between equations and variables across relation boundaries, it is natural to integrate this aspect into the type system. Similar considerations apply to the number of initialization equations and continuous state variables. Recent work on dependent types is relevant here [27]. We also aim at extending the type system to handle physical dimensions [11].

In a highly structurally dynamic language, it is impossible to identify all possible operating modes and then factor them out as separate systems. Modes thus have to be generated *dynamically* during simulation as follows. Whenever a switch occurs, a new, global, “flattened” DAE has to be generated. The DAE is obtained by first carrying out the necessary discrete processing, which amounts to standard functional evaluation, including evaluation of the *relational expressions* in the equations that are to be active after the switch. The evaluation of relational expression is what creates *new instances* of relations, and carrying out the instantiation dynamically when switching occurs is what enables modeling of highly structurally dynamic systems. Once the new flattened DAE has been generated, it is subjected to causality analysis and other symbolic manipulations in preparation for simulation using suitable numerical methods [21, 6, 7]. The result is causal simulation code.

The hybrid bond graph simulator HYBRSIM has demonstrated the feasibility of this dynamic approach, and that it indeed allows some difficult cases to be handled [17]. However, HYBRSIM is an *interpreted* system. Simulation is thus slowed down both by occasional symbolic processing and by the interpretive overhead. To avoid interpretive overhead, we intend to leverage recent work on run-time code generation, such as ‘C [8] or Cyclone [25]. We will need to adapt the sophisticated mathematical techniques used in existing non-causal modeling languages [21, 6, 7] to this setting. In part, it may be possible to do this systematically by *staging* the existing algorithms in a language like Cyclone.

The initial conditions of the (new) differential equations must be determined on transitions from one mode to another. However, arriving at consistent initial conditions is, in general, hard. Some state variables in the continuous part of the system may exhibit discontinuities at the time of switching while others will not: simply preserving the old value is not always the right solution. Structural changes could change the set of state variables, and the relationship between the new and old states may be difficult to determine. One approach is to require the modeler to provide a function that maps the old state to the new one for each possible mode transition [1]. However, the declarative formulation of non-causal models means that the simulator sometimes has a choice regarding which continuous variables should be treated as state variables. Requiring the user to provide a state mapping function is therefore not always reasonable.

A key to the success of HYBRSIM is that bond graphs are based on physical notions such as energy and energy exchange, which are subject to continuity and conservation principles. We intend to generalize this idea by exploring the use of *declarations* for stating such principles, along the lines illustrated in Sec. 4.2. It may also be possible to infer continuity and conservation constraints automatically based on physical dimension types.

## 6 Related Work

There has been substantial interest in supporting structural dynamism within the non-causal modeling community recently. The most advanced effort at present

is probably MOSILAB [20]. Similarly to what is proposed here, MOSILAB supports dynamic addition and deletion of behavioral objects. The switching is controlled through a form statecharts. A modern, sophisticated DAE solver, with support for computing consistent initial conditions, is used.

However, the statechart approach implies an explicit enumeration of the modes, and even if the number of modes could be large due to combinatorial effects, this rules out a Yampa-style, truly dynamic number of simulation objects, which is the ultimate goal of Hydra.

Another aspect of MOSILAB is the use of Python for various meta-modeling tasks, such as writing “experiment scripts”. We think that Hydra in itself, thanks to being a general-purpose functional language with first-class signal relations and functions, should be expressive enough to mostly provide equivalent meta-modeling capabilities, all in a uniform, declarative setting, without resorting to external imperative languages.

Sol [28] is another effort to create a non-causal modeling and simulation language supporting structural dynamism. It expressively avoids the statechart approach to retain more of the declarative clarity of languages like Modelica. It is also claimed that the Sol approach to dynamism scales better. A key aspect of the Sol approach is the capability to dynamically determine model instances. This idea seems to be somewhat similar to the notion of first-class signal functions and relations in Hydra. Like MOSILAB, Sol seems to stop short of the Hydra goal of supporting systems with a dynamic number of objects.

## 7 Conclusions

Hybrid modeling is a domain in which the techniques of declarative programming languages have the potential to greatly advance the state of the art. The modeling community has traditionally been concerned more with the mathematics of modeling than language issues. As a result, present modeling languages do not scale in a number of ways, particularly in hybrid systems that undergo significant structural changes. Hydra uses functional programming techniques to describe dynamically changing systems in a way that preserves the non-causal structure of the system specification and allows arbitrary switching among modes, yielding expressive power beyond current non-causal modeling languages.

Although we have not completed an implementation of Hydra, this paper demonstrates our basic design approach and maps out the design landscape. We expect that further research into the links between declarative languages and hybrid modeling will produce significant advances in this field.

**Acknowledgments** The authors would like to thank the anonymous EOOLT reviewers for many useful suggestions for adapting the paper to this venue.

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# Important Characteristics of VHDL-AMS and Modelica with Respect to Model Exchange

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**Abstract.** Modeling and simulation have been established as fundamental facilities in the development of analog and analog-digital systems. Essential advances have been achieved by the usage of behavioral modeling languages. These languages can be considered as a link between the technical problem and the mathematical model that can be evaluated by computational methods. The paper outlines the various possibilities that are offered by the language VHDL-AMS – standardized by the IEEE to describe analog and mixed-signal systems – and the language Modelica. The underlying modeling approaches are compared. Last but not least, the potential to transform models written in one language into models of the other language is discussed.

**Keywords:** VHDL-AMS, Modelica, model exchange

## 1 Introduction

A multitude of behavioral modeling languages has been developed during recent years [1, 2, 3, 4]. They provide among other things a link between the mathematical description of an object and a representation that can be used by a simulation engine. Currently it can not be expected that all languages are supported by all simulation tools. On the other hand, large efforts have been spent on developing model libraries. Possibilities and limitations of the transformation of models written in VHDL-AMS [1] and Modelica [2] are discussed in the paper.

VHDL-AMS is a behavioral modeling language that is an extension of VHDL for the analysis of analog and mixed-signal systems. The language is standardized by IEEE. VHDL was originally developed to describe and model digital electronic circuits. The analog and mixed signal extensions were partly based on the experiences with the program Spice [5]. This is a program to simulate Kirchhoffian networks. Spice-like simulation tools have been available for more than thirty years. The first implementations of tool-specific behavioral languages to describe the analog behavior include for instance MAST [6] which is one of the ancestors of VHDL-AMS.

Modelica is a modeling language that allows the specification of complex systems. Especially it is widely used to describe non-electrical systems. The description of analog systems is supported as well as discrete, hybrid, and concurrency modeling.

A very active community has developed many models and libraries during recent years.

From a general point of view, both languages have a lot of similarities. Modeling is primarily based on equations. A system description can be established by connecting subsystems in a hierarchical way. A system of equations that considers the restrictions of the unknowns with respect to the terminal behavior of the subsystems, the topology of the final system, and some augmentation sets (for example to describe initial conditions) are generated. Both languages support the modeling of time-continuous conservative systems (based on a network approach), non-conservative systems (described by signal-flow blocks), and time-discrete systems. Modeling of multidomain systems that consist of electrical and non-electrical components is possible.

The time-continuous part of a well-established analysis problem is described by a differential algebraic system of equations [7] of form

$$F(x(t), x'(t), t) = 0 \quad (1)$$

$$\text{with } F : R^n \times R^n \times R^+ \rightarrow R^n \text{ and } x : R^+ \rightarrow R^n$$

The solution method is not defined by the language descriptions.

The VHDL-AMS language reference manual [1] defines characteristic equations that must be fulfilled by the solution. The structural set of these equations considers the Kirchhoff laws and represents the equality of quantities at the connection points of non-conservative ports. The explicit set is given by the simultaneous statements of the models. In the case of a conservative network, these equations describe the branch constitutive relations. Additional restrictions that must be considered in the initialization phase and at discontinuities are established by the augmentation set. It has to be considered during modeling that these equations must be fulfilled by a solution of the simulation problem [8]. A solvability check is done at the design unit level. Roughly spoken, the number of essential unknowns that are contributed by a model to the characteristic equations of the entire system must match the number of explicitly added model equations. In the case of VHDL-AMS simulators, the unknowns that fulfill the characteristic expressions are determined using a modified nodal analysis approach [9]. In this way, the dimension  $n$  of the system of form (1) that is solved by the simulator is reduced compared to the number of characteristic equations. The time-discrete part is solved by event-driven simulation approaches. Besides time domain simulation, small signal frequency and noise domain simulation are supported by VHDL-AMS. The language reference manual describes how to establish the characteristic expressions in these cases. They base on a linearization of (1) around the operating point.

A similar mechanism is used in Modelica. For each connection point of instances of an entire system connection equations contribute to the system (1). These equations correspond to the structural set in VHDL-AMS. The non-flow variables are set equal and the flow variables are summed to zero. The equations of each instance are included to build up (1). They represent the explicit set in VHDL-AMS. Additional requirements can be expressed to initialize unknowns in the initialization phase or to reinitialize values after discontinuities. This is similar to the augmentation set in VHDL-AMS. In contrast to VHDL-AMS, violations of these augmented equations

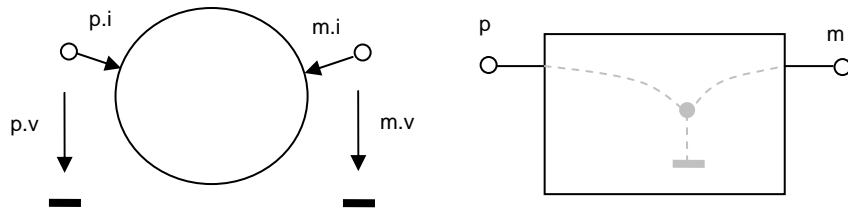
are handled “liberal” [10] (section 8.4.1.5). In the context of Modelica, reduction algorithms are applied to simplify the system (1). The number of unknowns and equations of the reduced system must be in accordance. A check on the unit level as in VHDL-AMS is not required. Similar to the mixed-simulation cycle in VHDL-AMS, Modelica defines how to handle hybrid differential algebraic representations that consist of differential, algebraic, and discrete equations. Table 1 compares some aspects of both languages in a general way. More details can be found in [1, 2, 10, 11].

**Table 1.** Comparison of some aspects of VHDL-AMS and Modelica

Aspect	VHDL-AMS	Modelica
Definition	IEEE Std. 1076.1 (2007) [1]	Modelica Specification 2.2 [2] Modelica Association
Time-continuous	Conservative (Kirchhoff networks) non-conservative	physical modeling block-oriented modeling
Time-discrete	event-driven	event-driven, but no event queue supported
Interaction	mixed-signal simulation cycle	solution of hybrid DAE's
Model interface	<b>entity</b>	<b>model, block</b>
Model parameter	<b>generic</b> parameter	<b>parameter</b>
Connection point	<b>port</b> (terminal,quantity,signal)	specified by connector classes
Model behavior (general)	<b>architecture</b> (one ore more corresponding to one entity)	declarations and <b>equation</b> part of <b>model, algorithm</b>
Analog behavior	equation oriented; simultaneous statements (for instance <i>expr1 = = expr2;</i> )	equation oriented; equations (for instance <i>expr1 = expr2;</i> )
Event-driven behavior	assignment of values; concurrent statements (for instance <b>process</b> )	assignment of values; conditional equations ( <b>when</b> -equations)
Analog waveform	<b>quantity</b>	dynamic variable
Connection point characterization	<b>nature</b> for terminals ( <b>through, across, reference</b> )	<b>connector</b> ( <b>flow, non-flow</b> )
Digital waveform	<b>signal</b>	<b>discrete</b>
Simulation	Time domain, small-signal frequency and noise analysis	Time domain analysis
Simulation phase information	DOMAIN signal	<b>initial()</b>
Initial conditions	<b>break</b> statement	<b>initial equation</b> <b>fixed start</b> values
Values after discontinuities	<b>break</b> statement	<b>reinit()</b>
D/A conversion	'RAMP, 'SLEW	<b>smooth()</b>
Vector operations	Overloading of operators	Built-in functions
Inheritance	Not supported	Widely used
Netlists	instance oriented ( <b>port map</b> )	pin oriented ( <b>connect</b> )

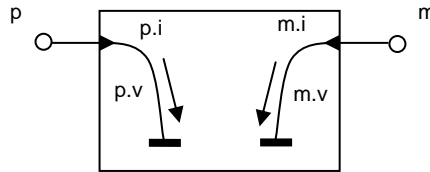
The classes of problems that can be described with both languages seem to be very similar. Thus, it is obvious to check the possibilities to use models described in both languages together or to transform from one language into the other. In the following, aspects of the transformation of Modelica models into VHDL-AMS models are especially considered.

## 2 Modeling Approaches in VHDL-AMS and Modelica



**Fig. 1.** Terminal behavior of conservative (electrical) systems in Modelica and VHDL-AMS (examples)


A physical Modelica model describes the relation between pin flow and pin potential variables (see for example in Fig. 1 p.i and p.v resp.) by a set of equations. Additional variables can be introduced. Usually the number of equations equals the sum of the number of pins and additional variables. This is not strictly required but usually fulfilled. It is only required that the final system (1) is well-defined. In VHDL-AMS, an internal graph structure is declared. The constitutive relations between branch across (non-flow) and through (flow) quantities are described by simultaneous statements. Additional free quantities can be declared to establish the constitutive relations. Without considering all details, it is in principle strictly required that the number of branches and free quantities equals the number of simultaneous statements.



**Fig. 2.** Internal branch structure of a Modelica model (example)

A branch structure can be assigned to a Modelica model in a direct way - see Fig. 2. The associated model descriptions of a resistor using Modelica and VHDL-AMS are shown in Table 2.

**Table 2.** Example of a Modelica model and the associated VHDL-AMS model

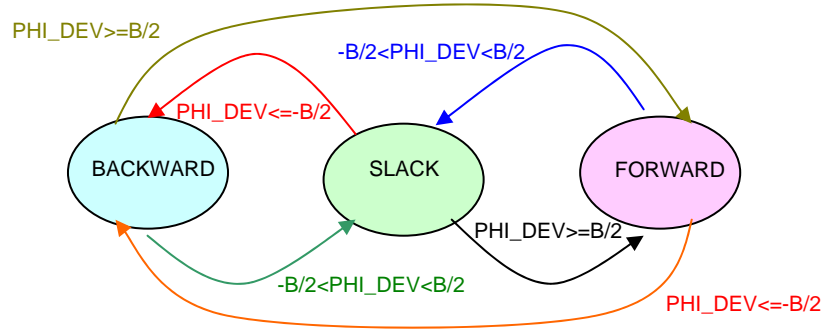
Modelica	VHDL-AMS
 <pre> <b>partial model</b> OnePort   "Component with two pins"   ...Voltage v "p.v - n.v";   ...Current i "from p to n";   ...PositivePin p;   ...NegativePin n; <b>equation</b>   v = p.v - n.v;   0 = p.i + n.i;   i = p.i; <b>end</b> OnePort;  <b>model</b> Resistor "Ideal resistor"   <b>extends</b> ...OnePort;   <b>parameter</b> ...Resistance R=1;   <b>equation</b>   R*i = v; <b>end</b> Resistor; </pre>	<pre> <b>library</b> IEEE; <b>use</b> IEEE.ELECTRICAL_SYSTEMS.all;  <b>entity</b> RESISTOR <b>is</b>   <b>generic</b>(R : RESISTANCE := 1.0);   <b>port</b>      (<b>terminal</b> P: ELECTRICAL;              <b>terminal</b> N: ELECTRICAL); <b>end entity</b> RESISTOR;  <b>architecture</b> MODELICA <b>of</b> RESISTOR <b>is</b>   <b>quantity</b> P_V <b>across</b> P_I <b>through</b> P;   <b>quantity</b> N_V <b>across</b> N_I <b>through</b> N;   <b>quantity</b> V : REAL;   <b>quantity</b> I : REAL; <b>begin</b>   V == P_V - N_V;   0.0 == P_I + N_I;   I == P_I;   R*I == V; <b>end architecture</b> MODELICA;  <b>architecture</b> IDEAL <b>of</b> RESISTOR <b>is</b>   <b>quantity</b> VNOR <b>across</b> INOR     <b>through</b> P <b>to</b> N; <b>begin</b>   VNOR == R*INOR; <b>end architecture</b> A0; </pre>

The architecture MODELICA of the VHDL-AMS model is a direct transformation of the corresponding Modelica model on the left side of Table 2. This description is correct from a formal point of view. The better description with a reduced set of internal quantities and branches is given by the architecture A0. This simple example shows the limits of a formal straight forward transformation of a Modelica model into VHDL-AMS. In section 4, we will discuss a way to handle this problem. By the way, constructing an appropriate internal branch structure is not only helpful in the model transformation process. It produces also a better understanding of existing models (in any case from an electrical engineer's point of view) - see for example Fig. 3.



**Fig. 3.** Internal branch structure of the sliding mass model of the Translational Modelica Mechanics Standard Library (see [10])

Facilities to handle finite state machines are available in both, Modelica and VHDL-AMS. The associated statements allow establishing event-driven analog models. The equations used in a time-continuous analog model depend on the value of a digital STATE. The STATE is updated by evaluating the transition conditions.

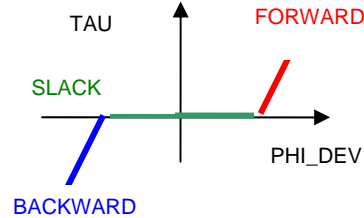


**Fig. 4.** Finite State Machine for event-driven analog modeling (example)

Handling finite state machines and time-discrete simulation problems is well supported in VHDL-AMS. It must be qualified how the translation of the constructs for hybrid simulation in Modelica (in VHDL-AMS named as mixed-signal simulation) into VHDL-AMS and vice versa can be done. A simple example of an event-driven analog model is shown in Fig. 5. The corresponding Modelica description can be found in [10] (chapter 13).

```
library IEEE;
use IEEE.MECHANICAL_SYSTEMS.all;

entity SIMPLE_ELASTOBACKSLASH is
  generic (
    B      : REAL ;
    C      : REAL := 1.0E5;
    PHI_REL0 : REAL := 0.0 );
  port (
    terminal FLANGE_A : ROTATIONAL;
    terminal FLANGE_B : ROTATIONAL);
end entity SIMPLE_ELASTOBACKSLASH;
```



```
architecture BASIC of SIMPLE_ELASTOBACKSLASH is
  -- STATE declaration for finite state automaton

  type STATE_TYPE is (SLACK, FORWARD, BACKWARD);
  signal STATE : STATE_TYPE;

  quantity PHI_REL across TAU through FLANGE_B to FLANGE_A;
  quantity PHI_DEV : REAL;

begin
  -- angle deviation from zero position
  PHI_DEV == PHI_REL - PHI_REL0;

  -- finite state automaton
```

```

STATE <= BACKWARD when not PHI_DEV'ABOVE(-B2) else
        FORWARD when PHI_DEV'ABOVE(B2)         else
        SLACK;

-- constitutive relations

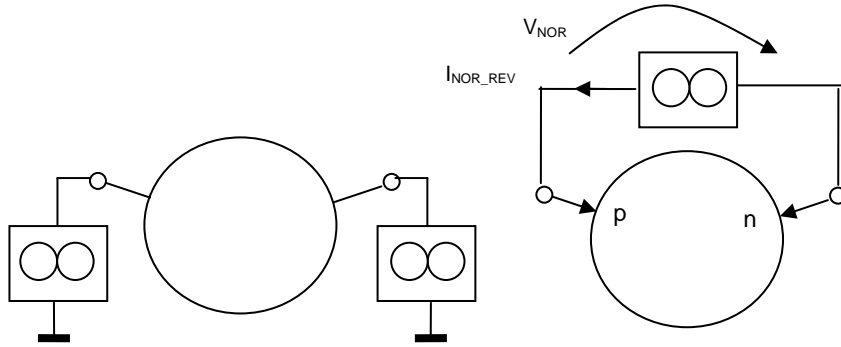
if      STATE = FORWARD use
    TAU == C*(PHI_DEV - B/2.0);
elsif  STATE = BACKWARD use
    TAU == C*(PHI_DEV + B/2.0);
else   TAU == 0.0;
end use;

break on STATE;
end architecture BASIC;

```

**Fig. 5.** Event-driven analog VHDL-AMS model of a simple elastobackslash

### 3 Potential to Establish Small VHDL-AMS Models



**Fig. 6.** Structures to determine the terminal behavior

In section 2, a direct way how to transform a physical Modelica model into a VHDL-AMS model was demonstrated. We will now draft how to simplify the structure and equations of the target model.

The terminal behavior of a conservative system can be determined by an interconnection with a tree structure of norator circuit elements [12]. A norator is a one-branch network in which branch voltage and current are completely arbitrary [13]. There are no restrictions concerning branch voltage and branch current.

If the sum of the flows into the component that has to be modeled is unequal zero, all pins have to be connected by norators to the corresponding reference node (see left part of Fig. 6). If the sum equals zero, one terminal can be used as local reference terminal and the other terminals are connected to the local reference by norator branches.

The structure of the final model is the same as the structure of the norator tree(s). The analysis of the test structure delivers the model equations that are expressed with the branch quantities of the norator branches.

### Example

Let us have a look at the Modelica model from Table 2. The sum of the currents  $p.i$  and  $n.i$  is 0.0 (see the equations of the partial model OnePort). Thus, we can use a test circuit similar to the right part of Fig. 6. The network equations are given by

$$\begin{pmatrix} 1 & -1 & . & . & -1 & . & . & . \\ . & . & 1 & 1 & . & . & . & . \\ . & . & 1 & . & . & -1 & . & . \\ . & . & . & . & 1 & -R & . & . \\ 1 & -1 & . & . & . & . & -1 & . \\ . & . & 1 & . & . & . & . & -1 \\ . & . & . & 1 & . & . & . & 1 \end{pmatrix} \cdot \begin{pmatrix} p.v \\ n.v \\ p.i \\ n.i \\ v \\ i \\ V_{NOR} \\ I_{NOR\_REV} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (2)$$

The first four equations represent the equation part of the Modelica model from Table 2. The last three equations result from the Kirchhoff laws. There is no additional constitutive relation that restricts branch voltage and current of the norator. Thus, there are fewer equations than unknowns.

These equations (2) can easily be transformed (for instance using a row echelon algorithm) into the following form

$$\begin{pmatrix} 1 & -1 & . & . & . & . & . & -R \\ . & . & 1 & . & . & . & . & -1 \\ . & . & . & 1 & . & . & . & 1 \\ . & . & . & . & 1 & . & . & -R \\ . & . & . & . & . & 1 & . & -1 \\ . & . & . & . & . & . & 1 & -R \\ . & . & . & . & . & . & . & . \end{pmatrix} \cdot \begin{pmatrix} p.v \\ n.v \\ p.i \\ n.i \\ v \\ i \\ V_{NOR} \\ I_{NOR\_REV} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (3)$$

The constitutive relation of the model only depends on the equations that now restrict the branch voltage and current  $V_{NOR}$  and  $I_{NOR\_REV}$  resp. in the norator branch

$$V_{NOR} - R \cdot I_{NOR\_REV} = 0 \quad (4)$$

This equation and the used norator structure are the basis of the architecture A0 of the VHDL-AMS model in Table 2. ■

We will try to outline the general procedure that was applied in the example:



1. Starting point is a physical (conservative) Modelica model.
2. Combine pins in one group if the sum of the associated flows equals zero.
3. Select a local reference pin in each group. Connect all other pins of the group with the associated reference pin by norator branches. Connect the remaining pins by norator branches with the associated global reference node. In the norator branches, voltages and currents are contrarily counted.
4. Establish the network equations of the designed test circuit

$$F(x(t), x(t)', t) = 0 \quad (5)$$

$$\text{with } F : R^n \times R^n \times R^+ \rightarrow R^m \text{ and } x : R^+ \rightarrow R^n$$

5. Try to transform this equations into

$$F_1(x_1(t), x_2(t), x_3(t), x_1'(t), x_2'(t), x_3'(t), t) = 0 \quad (6.1)$$

$$F_2(x_2(t), x_3(t), x_2'(t), x_3'(t), t) = 0 \quad (6.2)$$

$$\text{with } F_1 : R^{n_1} \times R^{n_2} \times R^{n_3} \times R^{n_1} \times R^{n_2} \times R^{n_3} \times R^+ \rightarrow R^{m_1}$$

$$F_2 : R^{n_2} \times R^{n_3} \times R^{n_2} \times R^{n_3} \times R^+ \rightarrow R^{m_2}$$

$$x_1 : R^+ \rightarrow R^{n_1}, x_2 : R^+ \rightarrow R^{n_2}, x_3 : R^+ \rightarrow R^{n_3}$$

All norator branch voltages and currents are summarized in  $x_3$ .

6. If  $m_2 = n_2 + \frac{n_3}{2}$ , a VHDL-AMS model can be established. The internal branch structure of the model is given by the structure of the norator branches and their voltage directions. Additional free quantities that represent the  $x_2$  waveforms must be declared in the architecture. The simultaneous statements of the model are given by  $F_2$ .
7. In a final step the VHDL-AMS model has to be checked against the Modelica model.

The procedure cannot only be applied to a basic model but also to an interconnection of models. It can be simplified if the transformation of the equations (5) into (6.1) and (6.2) can make use of the facilities of a simulator. This requires access to the reduced equations created by the simulator.

The transformation from VHDL-AMS to Modelica can be done in a similar way.

## 4 Conclusions

Some aspects of the transformation of Modelica models into VHDL-AMS descriptions were discussed.

A pure code-based transformation will not deliver the expected quality in a lot of cases. A method was prototyped how the transformation could be supported by analyzing appropriated test circuits. This could be a basis for a model transformation process if the reduced equations are available as a result of the evaluation of a Modelica description. The availability of differential algebraic equations that are

established by the simulation program would offer a lot of opportunities concerning model transformation. This is also valid for a transformation from VHDL-AMS to Modelica.

Many other aspects of model transformation could not be taken into consideration in this paper in detail. A special problem is the handling of initial values for instance. There are a lot of facilities in Modelica (for instance based on the experiences with index problems of mechanical system analysis). VHDL-AMS offers a solution based on the experiences with operating point analysis of electronic circuits.

An exchange of models between the two languages should not bring up problems in principle. However, a full automatic transformation of models seems only possible under special conditions.

## Acknowledgement

The authors would like to thank the referees for helpful comments.

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# Modeling Structural - Dynamics Systems in MODELICA/Dymola, MODELICA/Mosilab and AnyLogic

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**Abstract.** With the progress in modeling dynamic systems new extensions in model coupling are needed. The models in classical engineering are described by differential equations. Depending on the general condition of the system the description of the model and thereby the state space is altered. This change of system behavior can be implemented in different ways. In this work we focus on three state-of-the-art DAE simulation environments, Dymola, Mosilab and AnyLogic, and compare the possibilities of coupling of different state spaces. This can be done either using a parallel model setup, a serial model setup, or a combined model setup. The analogies and discrepancies are figured out on the basis of the classical constrained pendulum as defined in ARGESIM comparison C7.

**Keywords:** Structural dynamics, state charts, Dymola, Mosilab, AnyLogic

## 1 Introduction

In the last decade the increase of computer power and the apace growth of model complexity leads to a new generation of simulation environments. Concurrently ambitions pointed towards establishing standardization. Especially the Modelica organization develops a wide range of syntax description and standard libraries.

This paper will compare the solutions of the constrained pendulum as an easy to model example, implemented in the most common Modelica simulator Dymola, Mosilab, a product from six Fraunhofer Institutes which uses Modelica syntax with extensions for state charts, and the simulator AnyLogic from Xjtek in St.Petersburg. This simulator also has object oriented structure and is fully implemented in JAVA.

We will focus on how the model can be implemented and we will have a look in which time slot the state events are and if there is a significant difference referring to the implementation method.

## 2 Model

The constrained pendulum is a classical nonlinear model in simulation techniques. This model has been presented in the definition of ARGESIM comparison C7 [1]. There is no exact analytical solution to this problem. Therefore, the results have to be obtained by numerical methods. In this section a description of the model will be given.

The motion of the pendulum is given by

$$ml\ddot{\varphi} = -mg \sin(\varphi) - dl\dot{\varphi} \quad (1)$$

Where  $\varphi$  denotes the angle measured in counter clockwise direction from the vertical position. The parameter  $m$  is the mass and  $l$  is the length of the pendulum. The damping is realized with the constant  $d$ .

In the case of a constrained pendulum a pin is fixed at a certain position given by the angle  $\varphi_p$  and the length  $l_p$ . If the pendulum is swinging it may hit the pin. In this case the pendulum swings on with the position of the pin as the point of rotation and the shortened length  $l_s = l - l_p$ .

Two experiments have been defined. The first one is starting in the long pendulum modus and is swinging towards the pin. The second experiment is a model where the starting conditions are set in a way that the pendulum is shortened in the beginning of the simulation run.

## 3 Simulation Environments

In this section the focus is on three simulation environments. Two simulators, namely Dymola and Mosilab, are based on the model description standard Modelica [2]. Modelica is a freely available, object-oriented language for modeling of large, complex, and heterogeneous physical systems.

One of its most important features is non-causal modeling. In this modeling paradigm, users do not specify the relationship between input and output signals directly (in terms of a function), but they rather define variables and the equations that must be satisfied.

It is suited for multi-domain modeling, for example, mechatronic models in robotics, automotive and aerospace applications involving mechanical, electrical, hydraulic and control subsystems, process oriented applications. Modelica is designed that it can be utilized in a similar way as an engineer builds a real system: first trying to find standard components like motors, pumps and valves from manufacturers catalogues with appropriate specifications and interfaces and only if there does not exist a particular subsystem, a component model would be newly constructed based on standardized interfaces.

The actual version of the Modelica Standard Library is 2.2.1, which has been released in April 2006.

### **3.1 Dymola**

Dymola, DYnamic MOdeling LABoratory, is an environment for modeling and simulation of integrated and complex systems. It has unique multi-engineering capabilities which mean that models can consist of components from many engineering domains.

The basic structure of the simulator is divided into two separate parts: the Modeling layer and the Simulation layer. Thereby the modeling layer is separated in three parts. One part, the so called ICON layer, is used to define the shape of the new defined blocks. The DIAGRAM layer is the interface for graphical modeling. The third plane is the MODELICA TEXT part where the Modelica source code can be implemented directly.

Dymola has a strong focus on using symbolic methods for mass-matrix inversion and equation sorting.

Integration algorithms for non-real-time simulation typically handle discontinuities by detecting when certain variables cross a boundary. They then calculate the time of the event by iteration and then change the step size to advance the time exactly to the time of the event (crossing) [3].

The default integration method is the Dassl code as defined by Petzold. The method can also be freely chosen out of 15 standard solvers, including algorithms for stiff systems. There is until now no possibility implemented to make graphical model switching for subsystems with different state space dimension.

### **3.2 Mosilab**

The simulator Mosilab (MOdeling and SIMulation LABoratory) is an environment developed from the Fraunhofer-Institutes FIRST, IIS/EAS, ISE, IBP, IWU and IPK in the research project GENSIM.

It has been developed for time-continuous and time-discrete analysis of heterogeneous technical systems. The main innovation from point of simulation techniques view in this simulator is the illustration of condition-based changes in the model structure (model structure dynamics). With this mechanism it is possible develop and simulate models with different modeling depth.

The model description in general is done in the Modelica standard. Additional features to assure high flexibility during modeling and the concept of structural dynamics is implemented. This is done by extending the Modelica standard with state charts, controlling dynamic models. The extended object-oriented model description language resulting is called MOSILA [1,4] Moreover simulator coupling with standard tools (e.g. MATLAB / Simulink, FEMLAB) is realized.

Code generation is done in a quite similar way as in Dymola/Modelica. This makes sense, because this relatively new simulator will also be able to simulate problems defined in the standard Modelica notation with other tools, which use the same syntax. The main difference is the extension for graphical representation of state charts. This is solved with an interface where the user can define UML state charts.

The analysis part of the model is split into two layers: the simulation and the post processing layer. The defined code is translated into C++. The default integration method is the so called idadassl. Other implemented methods are for example the implicit trapeze method and the explicit or implicit euler.

### **3.3 AnyLogic**

AnyLogic is a multiparadigm simulator supporting Agent Based modeling as well as Discrete Event modeling, which is flowchart-based, and System Dynamics, which is a stock-and-flow kind of description. Due to its very high flexibility AnyLogic is capable of capturing arbitrary complex logic, intelligent behavior, spatial awareness and dynamically changing structures. It is possible to combine different modeling approaches making AnyLogic a hybrid simulator. AnyLogic is highly object oriented and based on the Java programming language. To a certain degree this ensures a compatibility and reusability of the resulting models.

The development of AnyLogic in the last years has been towards business simulation. In version 6 of AnyLogic it is possible to calculate problems from engineering, but there are certain restrictions. For example the integration method cannot be chosen freely and there is no state event finder.

When a model starts, the equations are assembled into the main differential equation system. During the simulation, this DES is solved by one of the numerical methods built in AnyLogic. AnyLogic provides a set of numerical methods for solving ordinal differential equations (ODE), algebraic-differential equations (DAE), or algebraic equations (NAE).

AnyLogic chooses the numerical solver automatically at runtime in accordance to the behavior of the system. When solving ordinal differential equations, it starts integration with forth-order Runge-Kutta method with fixed step. Otherwise, AnyLogic plugs in another solver – Newton method. This method changes the integration step to achieve the given accuracy.

## 4 Solution methods

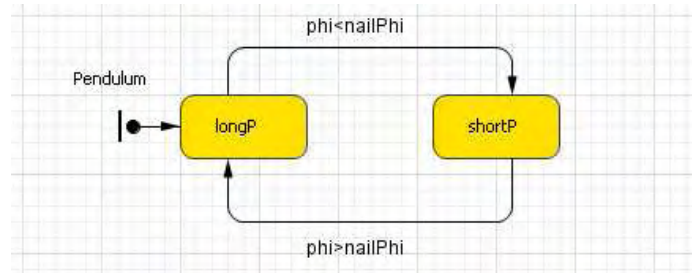
New advantages in computer numerics and the fast increase of computer capacity lead to necessity of new modeling and simulation techniques. In many cases of modern simulation problems state events have to be handled.

There exist more or less different categories of structural dynamic systems which should be focused on and solved.

The first class of hybrid systems are the one, where the state space dimension does not change during the whole simulation time and also the system equations stay the same. Only so called parameter events occur at discrete time points. These are the more or less simplest form of state events. Modern simulators offer different solution methods. A Part of them have a *discrete section* or as implemented in Dymola and Mosilab a so called *algorithm section*. In this part the user can define the parameter value change using the commands *when*, *if*, etc.. In this section the use of acausal modeling has to be switched off. This means that we have to make assignments for the parameter values at time point the event occurs.

Furthermore many software environments support the usage of UML state charts. This is a very intuitive and convenient way to describe a system which contains multiple discrete states. In the combination with dynamical equations this approach enables a simple implementation of structural dynamics. The dynamic equations or parameters are dependent of the discrete state of the model. On the other hand the states can be altered in dependence of the dynamic variables.

In case of the constrained pendulum the states are normally swinging (state 'long') or swinging with shortened length around the pin (state 'short'). The discrete state of the model depends on the angle  $\varphi$  and the pins angle  $\varphi_p$ . The state alters the model parameters or the models set of equations, see figure 1.

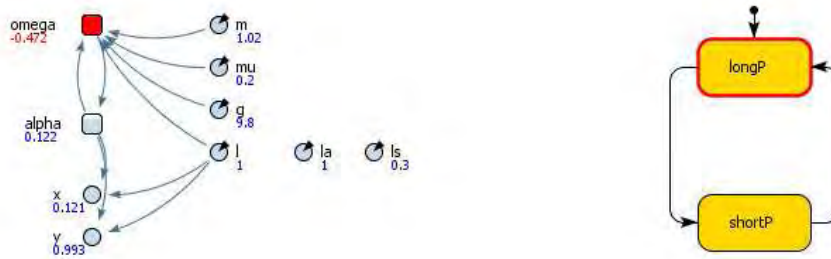


**Figure 1:** UML state diagram controlling the pendulum

### 4.1 Switching states

When the state of a system changes, often the state space of the model stays unchanged, thus the same set of differential equation can be used for different states. In this situation only certain parameters must be changed when a state is entered.

In case of the constrained pendulum the differential equation for movement stays the same for both states 'long' and 'short'. If the state changes the parameter length and angular velocity are updated before the calculation can continue, see figure 2.

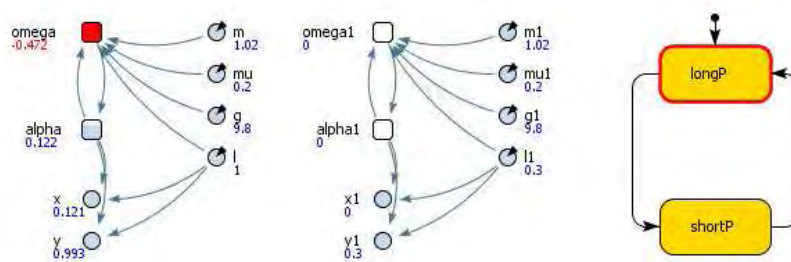


**Figure 2:** The parameters of the model are changed by an UML state diagram.

#### 4.2 Switching models

Often the previous approach is not possible. Sometimes situation occur where the state space of the model changes, thus a simple change of parameters is not possible. Normally the whole set of differential equations, thus the complete model, must be changed. In many simulation environments this approach can lead to complication.

In case of the constrained pendulum two differential equations are set up describing the movement of the pendulum. One describes the normal pendulum the other one the shortened pendulum. Which equation is set to be active is determined by the state diagram. When the states are switched the initial values must be passed on the equation must be activated and the other one must be frozen, see figure 3.



**Figure 3:** The differential equations of the system are switched in dependence of the UML state diagram.



## 5 Dymola

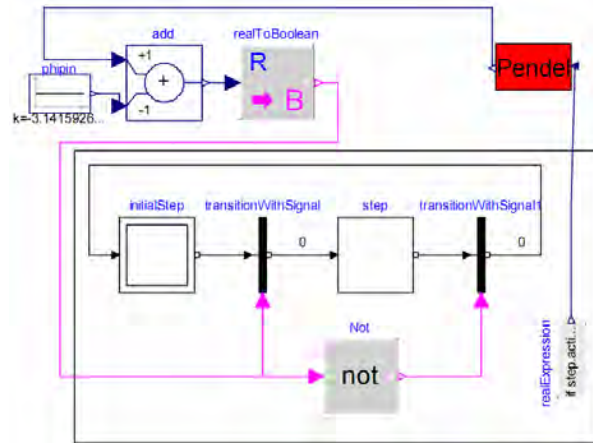
The implementation of the constrained pendulum has been done in two more or less different ways. As Dymola does not support the UML – notation for state charts and there is in the moment no method implemented to switch between two or more independent models during one simulation run, the solution methods described in section 4.1 and 4.2 can not be used.

In our example the state event, which appears every time when the rope of the pendulum hits the pin or looses the connection to it, is modeled in an *algorithm* section. This can be done with the following code digest:

```
algorithm
  if (phi<=phipin) then
    length:=ls;
  end if;
  if (phi>phipin) then
    length:=ll;
  end if;
```

Another method for implementing the constrained pendulum in Dymola is the use of standard blocks in combination with a predefined model which includes the equations or using only the *Modelica.Blocks* components.

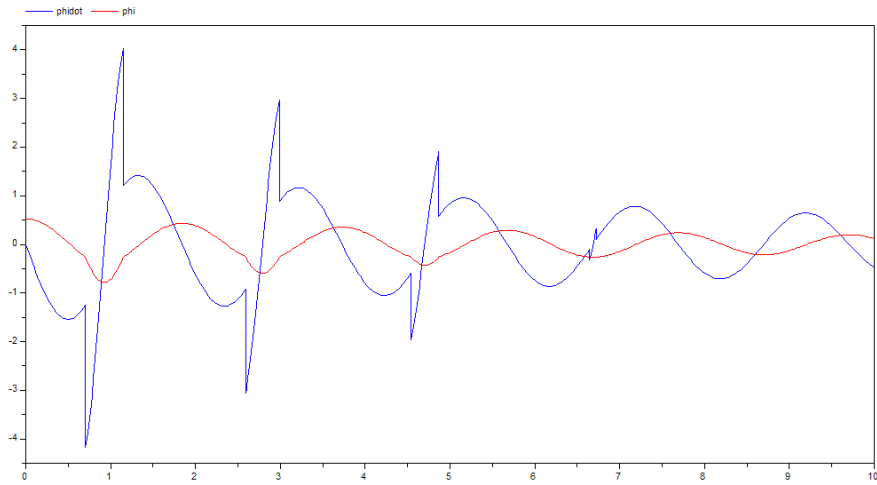
In this example the solution is made by using standard blocks with little extension. Figure 4 shows a screenshot of the Diagram layer of this model.



**Figure 4:** The screenshot of the Diagram layer in Dymola/Modelica

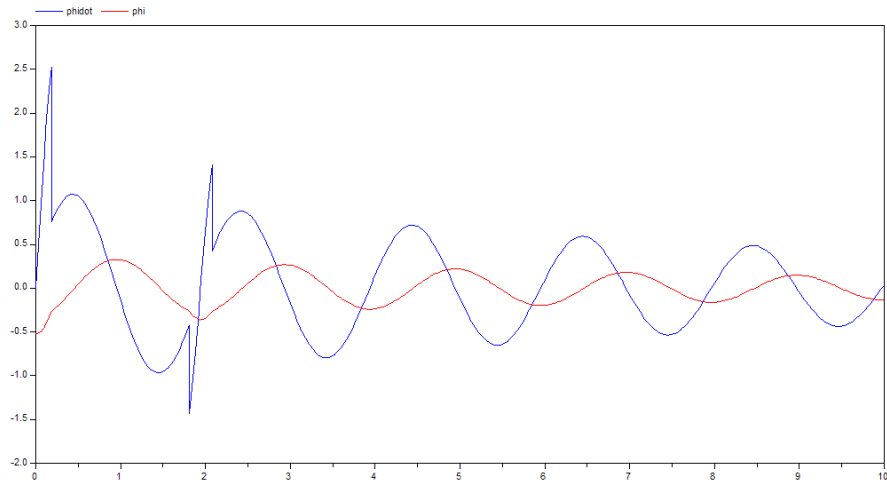
The main difference is that no algorithm section is used in the model. The lower part of the system shown in figure 4 is solved with *Modelica.StateGraph* library.

The simulations are done for both tasks and the solutions are compared. This is done by plotting all the results in one picture. The time of the last event in task a (figure 5) is in both cases the same, namely 6.72198 seconds. There is no easy possibility to plot the difference of special variables from different simulation runs.



**Figure 5:** *angle (red) and angular velocity (blue) as a function of time [s] as described in chapter 2*

The same model has to be checked with other starting values. This is done in next step. The figure 6 shows the plot for starting angle  $\varphi = -\pi/6$  instead of  $\pi/6$ .



**Figure 6:** *angle (red) and angular velocity (blue) as described in chapter 2*

## 6 Mosilab

Similar to the way the solutions in Dymola were calculated, the system can be solved with Mosilab. But as mentioned before, this structure can not handle changes in the state space dimension. The implemented Modelica extension enables the handling of discrete elements as well as structure changes in the general description.

We focus on two different solution methods for the constrained pendulum.

First approach: State charts may be used instead of if- or when- clauses (similar to 4.1 Switching states), with much higher flexibility and readability in case of complex conditions. Boolean variables define the status of the system and are managed by the state chart. The most important part of the source code is as follows:

```
equation
lengthen=(phi>phipin); shorten=(phi<=phipin);
.. here /*pendulum*/ -equations
statechart
state LengthSwitch extends State;
    State Short,Long,Initial(isInitial=true);
transition Initial -> Long end transition;
transitionLong->Shorthevent shorten action
    length := ls;
end transition;
transitionShort->Longeventlengthen action
    length := ll;
end transition;
end LengthSwitch;
```

From the modeling and mathematical point of view, this description is equivalent to the description with if-clauses. The question is, how the Mosilab translator generates the implementation of the equations in both cases. The Mosilab/Modelica simulator performs simulation by handling the state event within the integration over the simulation horizon.

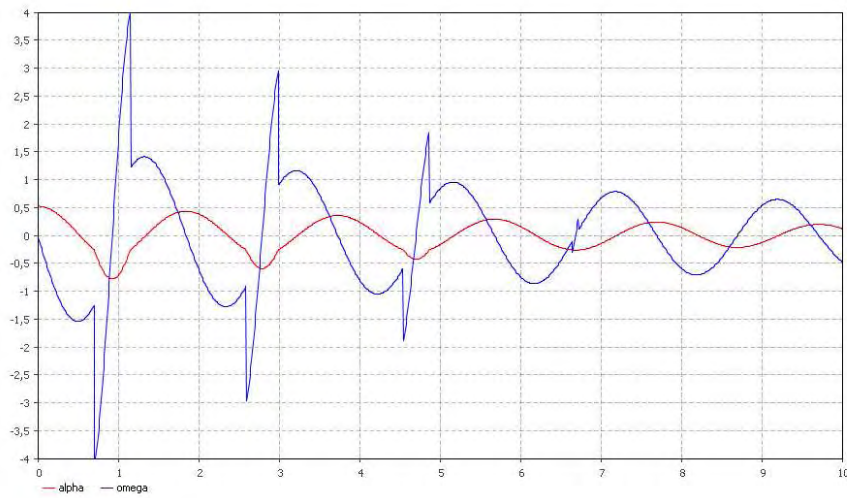
Second approach: These models are the conversion of concepts from chapter 4.2, which is switching models into Mosilab notation. For the constrained pendulum, we decompose the system into two different models, a *short* and a *long* pendulum model, controlled by a state chart. This can again be done with graphical aid in the form of UML-diagrams.

In the development status at the end of 2006, there still occurred several problems with the graphical interface of the state chart layer. The functionality of the system is not restricted. The results are similar to the solutions done with Dymola/Modelica.

## 7 AnyLogic

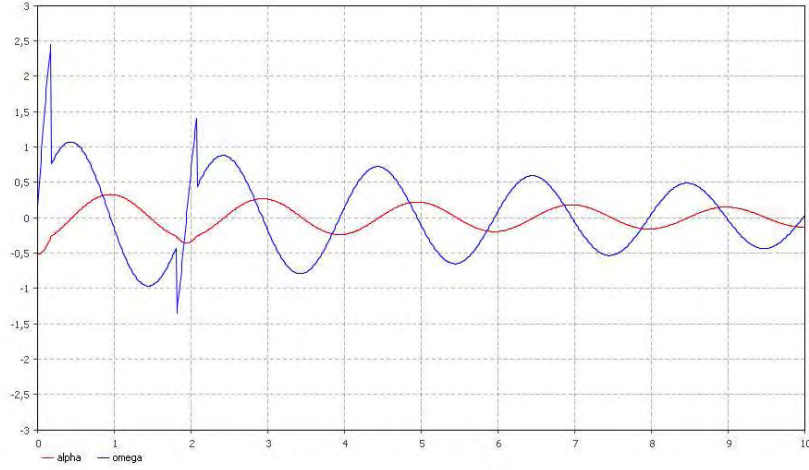
The implementation of the constrained pendulum has been done in two different ways. In the first approach only the parameter states have been switched corresponding to section 4.1, in the second approach the whole differential equation is switched corresponding to section 4.2. Both examples from chapter 2 have been calculated with both approaches. The results in AnyLogic are identical in both methods because the times of the state transitions are the same.

In the first approach the model consists of two ordinary differential equations describing the movement of the pendulum. In these equations four parameters are used length  $l$ , mass  $m$ , damping  $d$ , and gravity  $g$ . Further a state diagram with states 'long' and 'short' and two transitions are used to update the equations. When the state changes length  $l$  and angular velocity  $\omega$  are updated. The results calculated by AnyLogic 6 are plotted in figure 7 and figure 8.



**Figure 7:** Results for example 1: angle (red), angular velocity (blue)

The second approach uses two separate models. The implemented model consists of two times two ordinary differential equations. Both equations have four parameters separately: length  $l$ , mass  $m$ , damping  $d$ , and gravity  $g$ . A state diagram is implemented analog to the first approach. If the state changes the right differential equations are activated and their initial values are set, while the other differential equation is frozen.



**Figure 8:** Results for example 2: angle (red), angular velocity (blue)

## 8 Discussion

For this nonlinear model, there exists no exact solution. For this reason we can only calculate the numerical solutions and compare, for example, the time points where the last state event appears. This is the moment when the rope of the pendulum loses the connection to the pin the last time. In the first model under investigation, this happens after the fourth time shortening the pendulum, which means after eight state events all together. In the second simulation run, this occurs earlier, namely already after two times lengthening the rope, which means after three state events, because of the special initial condition (pendulum is in short modus at starting time).

The solutions are calculated with the default simulation method, if possible. With this approach we try to test the simulation environments from the user's point of view. Many programmers and modelers do not care that much about the implemented integration methods. For this reason the standard method has to produce reliable results in an appropriate calculation time.

The solution in the Mosilab simulator with standard Modelica components cannot be calculated with the standard method (*Dassl* code), because during simulation of this task a numerical error occurs and therefore the calculation is interrupted. The integration method pins at the time point of the first state event. Because of this reason the *Implicit Trapez* method was chosen. The other results are all done with the standard integration method and the given step sizes/number of intervals.

**Table 1.** End time of the last shortening of the pendulum for example 1

Simulator	Simulation	Method
Dymola/Modelica	6.72198	Dassl 500 intervals
Mosilab/Modelica	6.7204	IDA Dassl
Switch models		Min. step 1e-6 Max. step 0.08
Mosilab/Modelica	6.7199	Impl. Trapez
Pure Modelica		Min. step 1e-6 Max. step 1e-4
Mosilab/Modelica	6.7224	IDA Dassl
Parameter switching		Min. step 1e-6 Max. step 0.08
AnyLogic	6.725	No influence Step size 0.001

Table 1 shows that the solutions with Dymola and Mosilab are equivalent, if the solution is rounded towards two digits after the comma. By contrast, the solution in AnyLogic differs. We can try to explain this difference by taking a look on state event finding. This is not implemented in AnyLogic and is missing as an important standard feature of modern simulation environments. The lack of influence on the numerical methods can be explained by the main field of application of AnyLogic. Its main focus is on production and logistics, not on simulation of DAE systems.

In table 1 we see that there is only one row for Dymola/Modelica. This is because of equivalent results in all three implementations. Also AnyLogic delivers the same result for both methods. As we see, in this case Dymola outperforms Mosilab, because the result does not depend on the way of implementation. On the other hand we cannot implement real structural dynamics without blowing up the state space and problems in starting variable definition.

The graphical user interface for UML diagrams is a big advantage of Mosilab and AnyLogic compared to the possibilities of Dymola. But we have to keep in mind, that this feature is not Modelica standard, which complicates model exchange between different simulators based on Modelica.

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# Abstract Syntax Can Make the Definition of Modelica Less Abstract

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**Abstract.** Modelica is an open standardized language used for modeling and simulation of complex physical systems. The language specification defines a formal concrete syntax, but the semantics is informally described using natural language. The latter makes the language hard to interpret, maintain and reason about, which affect both tool development and language evolution. Even if a completely formal semantics of the Modelica language can be seen as a natural goal, it is a well-known fact that defining understandable and concise formal semantics specifications for large and complex languages is a very hard problem. In this paper, we will discuss different aspects of formulating a Modelica specification; both in terms of *what* should be specified and *how* it can be done. Moreover, we will further argue that a "middle-way" strategy can make the specification both clearer and easier to reason about. A proposal is outlined, where the current informally specified semantics is complemented with several grammars, specifying intermediate representations of abstract syntax. We believe that this kind of evolutionary strategy is easier to gain acceptance for, and is more realistic in the short-term, than a revolutionary approach of using a fully formal semantics definition of the language.

## 1 Introduction

Modelica is an open standard language aimed primarily at modeling and simulation of complex physical systems. The first language specification 1.0 [19] was released in September 1997. Since then, the current specification 2.2[20] has evolved to be large and complex with many constructs.

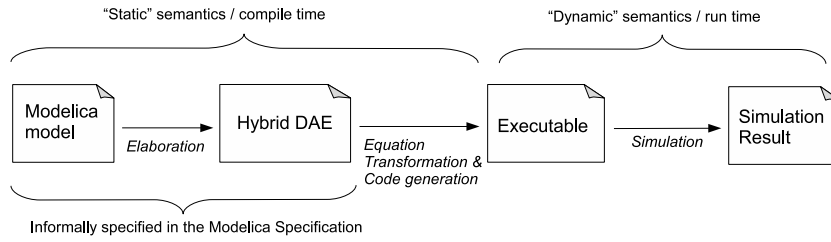
During these past ten years, the user community has grown fairly large and the Modelica Standard Library has evolved to include several physical domains. The dominating Modelica tool has for a long time been the commercial tool Dymola [4]. However, during recent years, alternative tools have emerged; both open source (OpenModelica [7, 21]) and commercial environments (e.g., MathModelica System Designer [16], MOSILAB [5], and SimulationX[12]).

The rapidly growing user community and increasing number of tool vendors augment the demand of the language specification being precise so that different

tools will be compatible. Hence, the Modelica Association, who is responsible for the language specification, has defined the goals for the next language version both to make the specification clearer and to simplify the language itself.

### 1.1 Specification of the Modelica Simulation process

Modelica’s compilation and simulation process can be divided into several stages or sub-processes. Consider Fig. 1, where a Modelica model is *elaborated*<sup>1</sup> into a Hybrid Differential Algebraic Equation (Hybrid DAE) and then transformed into an executable, which after execution produces a simulation result.



**Fig. 1.** Overview of a typical Modelica compilation and simulation process.

The syntax and semantic analysis take place at compile time and the generation of simulation output is produced at run-time.

In the current specification 2.2 [20], the concrete syntax is stated formally using Extended Backus-Naur Form (EBNF), but only the semantics of the first part of the process is informally described using natural language backed up with concrete source code examples.

Due to the fact that output of this process is not precisely defined, and that the semantics is described informally using natural language, the current specification is to a high degree open for interpretation.

### 1.2 Unambiguous and Understandable Language Specification

The natural goal of a language specification is to be *unambiguous*, so that tool implementors interpret the specification in exactly the same way. At the same time, it is important that the specification is *easy to understand* for the intended

<sup>1</sup> In this paper, we call this process *elaboration*. In the Modelica specification 2.2, this process is called *instantiation*. Sometimes, this transformation is also referred to as the *flattening* phase, since it creates a flat system of equations. However, we think that both these terms are misleading. The former, since it is performed at compile time and is not allocating memory analogous to instance creating in standard programming languages. The latter, since the final equation system does not need to be flat - it can still be represented in a hierarchical structure.



audience. Unfortunately, it is not that easy to meet both of these goals when describing a large and complex modeling language such as Modelica. There are several specification approaches with different pros and cons. Hence, the overall problem is to find an approach that satisfies the specification goals in the best possible way.

If the language is described using *formal semantics*, e.g., structured operational semantics [25], the language semantics is precise and can in some cases be proved to have certain properties, such as type safety [24, 26]. However, to understand and interpret a formal language specification require a rigorous theoretical computer science knowledge. Furthermore, even if great effort has been spent during the last decades in formalizing mainstream programming languages, only a few, e.g., Standard ML [18], are actually fully formally specified. Accordingly, it turns out to be a very hard task to specify an understandable and concise formal specification of an existing complex language.

Alternatively, if the language semantics is described using *natural languages*, e.g., plain English text describing the semantics, it might be easy for software engineers to understand the specification. Many languages are described in this way, for example Java [9], C++ [11], and Modelica [20]. However, ease of understanding does not imply that different individuals interpret the specification in the same way. It is a well known fact that it is very hard to write unambiguous natural language specifications, and perhaps even harder to verify their consistency.

### 1.3 Previous Specification Attempts

Several previous attempts have been made to formalize and improve the specification of the Modelica language. The most obvious one is the further development of the official language specification itself, conducted by the Modelica Association. The work on the next language specification includes substantial restructuring and a more detailed description of the semantics of the language. However, it is not planned to include any formal descriptions, apart from an appendix containing one possible definition of Modelica abstract syntax.

**Natural Semantics.** Already in 1998 Kågedal and Fritzson [14, 15], created a formal specification for a subset of the Modelica language, influenced by the language specification examples described in the 1997 version of [6]. The specification was using *Natural Semantics* [13] and the executable specification language Relational Meta Language (RML) [22]. This work influenced the design of the language and the official Modelica specification. The executable specification has gradually evolved and is now the code basis for the OpenModelica project [21]. In 2006, the code base was converted from RML to Meta-Modelica [8] with the purpose of making it more accessible for software engineers in the Modelica community. Hence, today the project is more intended to be a complete implementation of the language than a specification itself. One lesson learned from this specification project was that for an almost complete specification of

an early Modelica language version, the formal specification became hard to get an overview of, since it grew to be very large.

**Elaboration.** Jakob Mauss has made several contributions to formally describe the elaboration process (called *instance creation* in his work) of a subset of Modelica, i.e., the translation process from a Modelica model into a system of equations. The published work [17] describes an algorithmic specification approach, which focuses on Modelica’s complex lookup rules and modification semantics; including redeclaration of classes and components. Semantics for describing restrictions on validity of a model, such as types, restricted classes, and most prefixes are not considered. It exists also a refined version of this work, which uses a more compact notation. However, this work is still unpublished.

**Modelica Types.** In our previous work on types in the Modelica language[2], we concluded that the type concept is only implicitly defined in the Modelica language specification. In that work, we proposed a concrete syntax of specifying Modelica types and gave a suggestion for constraining information of element prefixes in the types. Furthermore, it was emphasized that Modelica has a *structural type system*, which implies that a *class* and a *type* are two separate language concepts. In this paper, we will not cover types, even though parts of a specification can also be described using type rules.

A common dominator for all these isolated formal specification attempts is that they have been conducted in parallel with the official language specification. Even if a proposed alternative specification covers large portions of the language, it will not be used as a specification by the community if it is not replacing the official specification. If there are two specifications of the same concept, how do we then know which one is valid if they are not consistent? Nevertheless, these formal specification attempts are still very important to promote understanding and discussion about the informal semantics. It is of great importance that these works gradually find their way into the official specification. The question is how to make this possible in practice, since all attempts so far only model subsets of the real language.

#### 1.4 Abstract Syntax as a Middle-Way Strategy

Improving the natural language description of the Modelica specification is an obvious way of increasing the understandability and removing ambiguity. However, since this process is tedious and error prone, it is very hard to ensure that the ambiguity decreases. Moreover, previous work on formalization of the complete semantics of subsets of the language has shown to be complex and resulting in very large specifications. Hence, there is a concrete and practical need to find a “middle-way” strategy to improve the clarity of the complete language, not just subsets. This strategy must be simple enough to not require in depth theoretical computer science knowledge of the reader, but still precise enough to avoid ambiguities.

When a compiler parses a model, the result is normally stored internally as an *Abstract Syntax Tree (AST)*. Hence, one particular model results in a specific AST, which can be seen as an instance of the language’s abstract syntax. The abstract syntax can be specified using a *context-free grammar*, and an AST can also have a corresponding textual representation.

The internal representation of an AST is often seen as a tool implementation issue, and not as something that is defined in a language specification. Nevertheless, in this paper we propose that the intermediate representations between the transformation steps (recall Fig. 1) should be described by specifying its abstract syntax.

However, specifying different forms of abstract syntax *cannot* replace the semantic specification need in the transformation process, since the syntax only describes the *structure* of a model, while the semantics states the *meaning* of it. Hence, in the short term, this specification *complements* the current informal specification, by clarifying exactly what both the input and the output structure of a transformation are.

By following this *evolutionary* strategy, the semantic description may then be gradually more described using techniques such as Syntax-Directed Translation Schemes (SDT)[1] or different forms of operational semantics. However, as earlier described, this is not straight forward when considering the whole Modelica language. The main purposes of including abstract syntax definitions in the specification can be summarized to be:

1. **Specifying Valid Input.** Increase the clarity of what valid Modelica actually is, i.e, to make sure that different tools reject the same models.
2. **Specifying Expected Output.** Remove confusion of what the actual outcome of executing a Modelica model is.
3. **Promoting Language Simplification.** The Modelica language has been identified to be sometimes more complicated than necessary (e.g., relations between the general class and restricted classes). An abstract syntax formulation can be used as a guidance tool for identifying the most useful reformulations needed.

Part of the first item is already specified using the concrete grammar. To increase the level of details that can be specified of the abstract syntax, we will later in the paper suggest an informal approach to include context-sensitive information in the abstract grammar specification. This rules out parts of the informal semantics used for rejecting invalid models. However, great parts of the rejecting semantics must still be described using another semantic specification form.

In the following sections, we will gradually introduce more motivations and descriptions of the abstract syntax approach. Section 2 gives an overview of different aspects of specifying a language specification in the context of Modelica. The discussion on different specification alternatives and aspects forms the basis for Section 3, which more concretely elaborates on our proposal. Finally, in Section 4 concluding remarks are stated and future work is outlined.

## 2 Specifying the Modelica Specification

Defining a new language from scratch with an unambiguous and understandable language specification is a difficult and time consuming task. Developing and enhancing a language over many years and still being able to keep the language backwards compatible and the specification clear, is perhaps an even more challenging mission. In the previous section, we described this problem with the current specification, motivated the need for improvement, and briefly introduced a proposed strategy. In the beginning of this section, we will focus on the question *what* should actually be specified in the Modelica specification. At the end of the section, we will discuss *how* this specification can be achieved by surveying some different specification approaches and compare how they relate to the abstract syntax approach.

At a high level, the syntax and semantics of Modelica can be divided into two main aspects:

- *Transformation*, i.e., the process of transforming a Modelica source code model into a well defined result. Depending on the purpose, the result can either be an intermediate form of a Hybrid Differential Algebraic Equations (Hybrid DAE), or the final simulation result.
- *Rejection*, i.e., rules describing what a valid Modelica model actually is. These rules should unambiguously describe when a tool should reject the input model as invalid.

Both these aspects are important for a clear-cut result, so that tool vendors can create compatible tools.

### 2.1 Transformation Aspects - *What* is Actually the Result of an Execution?

In the introduction section of the Modelica specification 2.2 [20], it is stated that the scope of the specification is to define the semantics of the translation to a flat Hybrid DAE and that it does not define the result of a simulation. A mathematical notation of the hybrid DAE is given, but no precise and complete output is defined.

However, many constructs given in the specification are not handled during this translation to a Hybrid DAE. Hence, the semantics of these constructs (e.g., when-equations, algorithm sections), are implicitly defined, even if the specification states that this should not be the case.

So, the questions arise: what is actually the transformation process? What is the expected result of the execution? We would argue that the answer to these questions would differ depending on who you ask, since the current specification is open for interpretation. In this subsection, we give our view of a typical Modelica transformation process.

Recall Fig. 1, where the high-level view of a typical Modelica compilation and simulation process is outlined. The translation process is divided into three sub-processes, each having an artifact as input and output.

**Elaboration.** The *elaboration* process (also called *instantiation* and sometimes *flattening*) takes as input a source code Modelica model and transforms it into a Hybrid DAE. This is the main part described in the Modelica specification, which includes among other things parsing, type checking, redeclarations, connection elaboration, and generation of equations. The output is the Hybrid DAE, which includes items such as equations, function calls, algorithm sections, declaration of variables etc.

**Equation Transformation and Code Generation** The Hybrid DAE is simplified and transformed (index reduction, generation of Block Lower Triangular form (BLT)). Finally, target code is generated (typically C-code), which is linked together with a numerical solver, such as DASSL[23].

**Simulation** The final transformation step is basically running the executable, where the actual simulation takes place. During this step, numerical integration of the continuous system and discrete event handling occurs.

**Static vs. Dynamic.** In the example above, it was assumed that the process was *compiled* and not *interpreted*. This is not a specification requirement, even if it is common that tools are implemented as compilers. The definitions of static and dynamic semantics are often confusing in relation to compile-time and simulation-time. Some people will argue that the dynamic semantics is only the simulation sub-process and that the elaboration and equation transformation as well as the code generation phases are the static semantics. If the tool is implemented as an interpreter, the distinction becomes less clear. In such a case, it is natural to view all three processes as the dynamic semantics. Even if this is only a matter of definitions, it becomes significantly important when reasoning about type checking and separate compilation.

From the discussion above, it is clear that we need to have a precise definition of the input and the output of the elaboration process. Whether the two last sub-processes should be part of the specification is an open design issue, but it is obviously important that the decision is made if it should be completely included or removed.

## 2.2 Rejection Aspects - *What is actually a Valid Modelica Model?*

In the current specification, it is hard to interpret what valid Modelica input is, i.e., it is difficult for a tool implementor to know which models that should be rejected as invalid Modelica. A restrictive abstract syntax definition can help clarifying several issues.

Besides specifying the translation semantics of a model, a language specification typically describes which models that should be treated as valid, and which

should not. By an *invalid model* we mean an transformation that should result in an error report by the tool. In order for different tool vendors to be able to state that exactly the same models are invalid, *when* and *how* to detect model faults must be clearly and precisely described in the language specification. Unfortunately, this is not as easy as it might seem.

Basically, rules in a specification for stating a valid model can be specified by using one of the following strategies, or a combination of both:

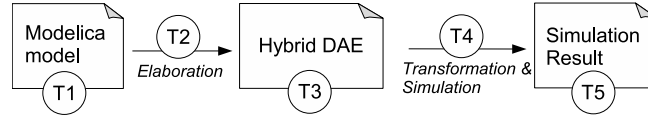
- Specify rules that indicate valid models. All models that do not fit to these rules are assumed to be invalid.
- Assume that all models are valid. Explicitly state exceptions where models are *not* valid.

The current Modelica specification mostly follows the latter approach. Here the concrete syntax constrains the set of legal models at a syntactic level. Then, informal rules given in natural language together with concrete examples state when a model can be legal or illegal.

The problem with this approach is that it is very hard for a tool vendor to be sure that it is compliant with the specification.

**Time of checking.** Detecting that a model is invalid can take place at different points in time during the compilation and simulation phase. Even if this can be regarded as a tool issue and not a language specification detail, the checking time have great implications on the tools ability to guarantee detection of invalid models.

Fig. 2 outlines a simplified view of the earlier described compilation and simulation process, where sub-processes of equation-transformation, code generation and simulation are combined into one transformation step.



**Fig. 2.** Possible checking-time during the process

The figure shows five (T1 - T5) conceptual points in time where the checking and rejection of models can take place. Starting from the end, T5 illustrates the final step of checking that the simulation result data is correct according to some requirements. This checking can normally not be conducted by a tool, but only by humans who have the domain knowledge.

The checking at point T4 takes place during simulation of the model. This is what many would refer to as *dynamic checking*, since it is performed during run-time. Errors which can occur here are for example numerical singularities after events or array out-of-bound errors. Since Modelica does not have an exception

handling mechanism, it is implicitly assumed that the tool exits with an error statement. Checking point T3 is performed after the elaboration phase. This can for example concern the control that the number of equations equals the number of unknowns.

Even if it is not stated in the Modelica specification, T2 is our interpretation of the specification where the type checking takes place. Here, the naming of this kind of checking is often a source of confusion. If the elaboration phase is regarded as the *static semantics*, some people call this *static type checking*. However, since the elaboration phase is the major part of the semantics described in the specification, and it involves complex transformation semantics, this can be viewed as something dynamic from an interpretive semantics point of view, or as something static from a translational semantics point of view. Using an interpretive semantics style, T2 would involve *dynamic type checking*.

Following this argumentation, then T1 would represent *static type checking*, i.e., the types in the language are checked *before* elaboration. This reasoning is analogous to dynamic checking in languages such as PHP and Common LISP, compared to static type checking in Haskell, Standard ML, or Java. Even if the Modelica specification does not currently support this kind of static checking, it has a major impact on the ability to detect and isolate for example over- and under-constrained systems of equations[3] or to enable separate compilation.

### 2.3 Specification Approaches - *How* can we state what it's all about?

When it is clear *what* to specify, the next obvious question is *how* to specify it. There are several specification approaches, and we have briefly mentioned some of them earlier in this paper.

As evaluation criteria, it is natural to use the specification goals of *understandability*<sup>2</sup> and *unambiguity*. Furthermore, it is also of interest to estimate the *expressiveness* of the approach, i.e., how much of the intended specification task can be covered by the approach.

In the following table, a number of possible specification approaches are listed, with our judgements of the evaluation criteria.

A natural language specification can be understandable and expressive, depending on the size and quality of the text, but easily leads as we have discussed earlier to ambiguous specifications. Using a formal type system together with formal semantics [24] is here seen as having low understandability, since it requires high technical training. It is however very precise and fairly expressive.

The expressiveness of the abstract syntax is stated as higher than the concrete syntax, since we can introduce context dependent information in the grammar using meta-variables. An example of this will be given in the next section.

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<sup>2</sup> Understandability is of course a very subjective measurement. In this context, we have chosen to also include the level of needed knowledge to understand the concept, i.e., a concept requiring an extensive computer science or mathematical background results in lower understandability rating.

Approach	Understandability	Expressiveness	Unambiguous
Natural language description	High-Medium	High	Low
Formal semantics	Low	Medium	High
Abstract Syntax Grammar	Medium	Medium	High
Concrete Syntax Grammar	Medium	Low	High
Test suite	High	High	Low
Reference Implementation	Low	High	High

**Table 1.** Possible specification approaches with estimated evaluation criteria.

We have also, for the sake of completeness, included related approaches such as the use of a test suite and reference implementation. The approach to use a test suite as a specification can be an interesting complement to abstract syntax and informal semantics. However, it is very important to state which description that has precedence if ambiguities are discovered. Finally, a reference implementation can also be seen as a specification, even if it is hard to get an good overview and reason about it.

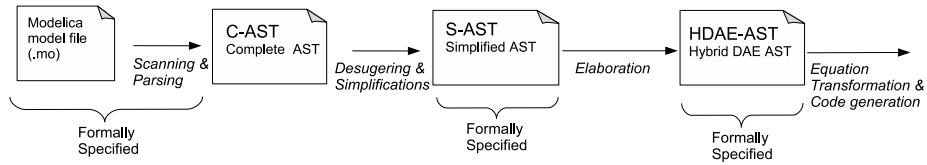
### 3 An Abstract Syntax Specification Approach

In the following section we will go into more details about the proposal to use abstract syntax as part of the Modelica specification. Initially, the different abstract syntax representations are outlined in relation to the transformation process described in Section 2.1, followed by a discussion about the specification and representation of the syntax. Finally a small example of abstract syntax grammar is given and discussed.

#### 3.1 Specifying the Elaboration Process

An *Abstract Syntax Tree* (AST) can be seen as a specific instance of an abstract syntax. Transformation processes inside an compiler can be defined as transformations from one intermediate representation to another. ASTs are a natural form of intermediate representation.

Consider Fig. 3, where the elaboration process is shown with surrounding ASTs. The first step in the process is the ordinary scanning and parsing step,



**Fig. 3.** Modelica’s compilation process divided into intermediate representations in the form of abstract syntax trees (ASTs).



which is formally defined in the specification using lexical definitions and concrete syntax definitions using Extended BNF.

**Complete AST (C-AST).** This step transforms into the first tree called *Complete AST (C-AST)*, which is a direct mapping of the concrete syntax. Although this is a natural step in a compiler implementation, it is of minor interest from a specification perspective.

**Simplified AST (S-AST).** From the C-AST, a simplification transformation translates the C-AST into a simplified form called *Simplified AST (S-AST)*. This transformation's goals are:

- *Desugaring* : The process of removing so called *syntactic sugar*, which is a convenient syntactic extension for the modeling engineer, but with no direct implication on the semantics. Example of such desugaring of a model is to collect all equation sections into one list, since the Modelica syntax allows several algorithm and equation sections to be defined in a model.
- *Canonical Transformations* Minor transformations and operations that help the S-AST to be a canonical form which is more suitable as input to the elaboration process. For example assigning correct prefixes to subelements (e.g., Section 3.2.2.1 in [20]).
- *Checking model validity*. One of the purposes with S-AST is that it is more restrictive than the C-AST. Hence, some C-AST are not valid S-AST. This restriction gives the possibility to ensure certain model properties, which in the current Modelica specification is described using informal natural languages. For example, which kind of restricted classes is the record class allowed to contain as its elements?

The S-AST can be seen as a simplified internal language analogously to the *bare* language of Standard ML[18]. However, initially, we do not see a similar short and precise way of specifying the transformation from C-AST to S-AST, as the transformation rules are given in the Standard ML specification.

**Hybrid DAE AST (HDAE-AST).** Besides S-AST, the output of the elaboration phase called Hybrid DAE AST (HDAE-AST) is proposed to be specified formally in the specification. The HDAE-AST must not just be a high-level mathematical description of an Hybrid DAE, but an explicit syntax description describing a complete specification of what the actual output of the elaboration phase is. This does not only include equations and variables, but function definitions, algorithm sections, when-equations and when-statements. Even if this information is possible to derive from the current specification, it would be a

great help for the reader to actually know what the output is, not just assume it.

Note that our approach suggests that the language specification should initially include a precise description of the possible *structures* of the ASTs; specifying input and output to the transformation process. The semantics of the transformation must still be described using another approach.

### 3.2 Specifying the Abstract Syntax

The specification of the syntax must be described using some kind of *grammar*, or data type construct in a language such as in Haskell, Standard ML, or MetaModelica [8].

The syntax can be specified using a *context-free* grammar, e.g. in Backus-Naur Form (BNF). However, we propose a more abstract definition of a grammar, where certain *meta-variables* range over names and identifiers. The notation has to some extent similarities to and is inspired by the abstract syntax definition of Featherweight Java[10].

For example, by stating that a meta variable  $R_r$  ranges over *names* (identifiers with possible dot-notation) referencing a `record`, we have introduced a contextual dependency in the grammar. The grammar declaratively states the requirement that this name must after lookup be a record, without stating *how* the name lookup should be performed. The latter must of course also be described in the specification, but in this way the different issues are separated. Consequently, this grammar is not intended to be used directly by a parser generator tool such as Yacc, but as a high-level specification which is less open for interpretation.

### 3.3 The Structure of an Abstract Syntax

Depending on the purpose and language for an abstract syntax, the structure of the syntax itself can be very different.

When specifying a simple functional languages, it is common that the grammar of the abstract syntax only has one non-terminal, namely a *term* [24]. Hence, all evaluation semantics is performed on this node type only, and all terms can be nested into each other. This gives a very expressive language, but the constraining rules ensuring the validity of an input program must be given in another form. This form is normally a formal *type system*, describing allowed terms.

Another method is to describe the abstract syntax with many non-terminals; more than needed for a production compiler. In for example the Modelica case, the different restricted classes: `model`, `block`, `connector`, `package`, and `record` would not be represented as one non-terminal *class*, but as different non-terminals. This structure would be more verbose, but also give the possibility of more precisely describing relations between restricted classes.

Somewhere inbetween those two extremes is for example the SCODE representation used in the earlier RML specification[14] and the current OpenModelica implementation.

```

connector ::= Connector(
    {Extends(Cr conModification)}
    {DeclCon(modifiability outinner Cd connector)}
    {DeclRec(modifiability outinner Rd record)}
    {CompCon(conconstraint Cr cd conModification)}
    {CompRec(conconstraint Rr rd recModification)}
    {CompInt(conconstraint xd)}
    {CompReal(conconstraint flowprefix yd)}
)

access ::= Public | Protected
modifiability ::= Replaceable | Final
outinner ::= Outer | Inner | OuterInner | NotOuterInner
conconstraint ::= Input | Output | InputOutput
flowprefix ::= Flow | NonFlow

```

**Fig. 4.** Example of a grammar for the connector non-terminal.

For the specification purpose, we suggest to use the most verbose alternative, i.e. the second alternative using many non-terminals. The rational for this choice is basically that this more restrictive form gives more information about what the actual input and output of the elaboration processes are.

### 3.4 A Connector S-AST Example with Meta-Variables

To give a concrete example where a grammar for S-AST can improve the clarity compared to the current informal specification, we take the restricted class `connector` as an example. In the Modelica specification it is stated that for a connector *"No equations are allowed in the definition or in any of its components"*. What does this mean? That no equations are allowed at all? Are declaration equations allowed, for example `Real x = 4`? Obviously, it is not allowed to have instances of models that contain equations, but is it allowed to have models that do not contain equations? Is it only allowed to have connectors inside connectors, or can we also have records in connectors, since these are not allowed to have equations either? These questions are not easy to answer with the current specification, because it is open for interpretation.

Consider Fig. 4, where an example of the non-terminal for a `connector` is listed using a variant of Extended BNF<sup>3</sup>. As usual, alternatives are separated using

<sup>3</sup> The following example grammar is not intended to exactly describe the current Modelica specification. The aim is only to outline the principle of such grammar in order to describe the abstract syntax approach.

the `'|'` symbol, and curly brackets  $(\{ \dots \})$  denote that the enclosing elements can be repeated zero or more times.

The grammar is extended with a more abstract notation of *metavariables*, which range over names or identifiers. Metavariables  $C_d$  and  $R_d$  range over identifiers declaring a new connector respectively record;  $C_r$  and  $R_r$  range over connector and record names referencing an already declared connector or record. Metavariables  $c_d$ ,  $r_d$ ,  $x_d$ , and  $y_d$  range over *component* identifiers having the type of connector, record, Integer, and Real. All bold strings denote a node in the AST. If the AST is given in a concrete textual representation, these keywords are used when performing a pre-order traversal of the tree.

In the example, *connector* can hold zero or many `extends` nodes, referencing the meta-variable  $C_r$ , denoting all names that reference a declared connector. Hence, using this meta-variable notation, this rule states that a connector is only allowed to inherit from another connector.

Furthermore, the example shows that a connector is allowed to have two kinds of local classes: Connector and Record (nodes `DeclCon` and `DeclRec`). `CompCon` and `CompRec` state that a connector can have both connector and record components.

For each of the different kinds of elements, it is stated exactly which prefixes that are allowed. This description is more restrictive than the concrete syntax, which basically allows any prefix. In the current specification these restrictions are stated in natural languages, spread out over the specification. For example, on one page it is stated *"Variables declared with the flow type prefix shall be a subtype of Real"*. Such a text is superfluous when the grammar for S-AST is specified (note that *flowprefix* is only available in the `CompReal` node).

### 3.5 What can and should be specified by the abstract syntax?

In the previous sections we have briefly outlined how an abstract syntax grammar can specify the structure of input and output of a transformation, but also as a method for specifying context-dependent information about rejection of illegal models. The question then arise: what should be specified using this grammar approach, and what should be addressed with other semantic rules?

The proposed grammar approach with meta-variables is declarative in the sense that it does not state information about how the rejecting rules should be implemented. Hence, it is less formal compared to e.g. a formal type system. However, it is still more precise than giving the rules using natural languages.

We believe that as long as the alternative semantic description is using natural languages, the abstract syntax approach can both be easier to understand and less ambiguous. Furthermore, if it can be complemented with aspects which are more precisely described, e.g. the lookup-process, it can clarify the specification even more. However, several parts of the rejection aspect, e.g. subtyping rules, cannot be described with the abstract syntax grammar. The other aspect of transformation semantics can of course not be specified with this approach.

The concept is still at a very early stage, and further investigations need to be performed, to see if this approach can cover the current Modelica language.

## 4 Conclusion

In this paper we have given an overview of different aspects of defining a modeling language; using the Modelica language's syntax and semantics.

Furthermore, we have argued that an approach which uses *abstract syntax* to describe both the input to Modelica's elaboration process (S-AST) as well as its output (HDAE-AST) can both clarify the transformation process as well as the rejection of invalid models. Furthermore, while developing the language, this approach promotes the focus on semantic issues, to avoid getting trapped in the common syntax pitfall.

The obvious next step for future work would be to design and implement the S-AST and HDAE-AST, and to verify that the ASTs meets most of the current code base publicly available.

We have described this as an evolutionary approach, which is intended to be practical in the short-term. However, in the long term, we still think that it is important that a formal semantics is given for the Modelica language.

## Acknowledgments

We would like to thank the anonymous reviewers for their suggestions and Johan Åkesson and Åsa Broman for useful comments and feedback.

This research work was funded by CUGS (the Swedish National Graduate School in Computer Science), by SSF under the VISIMOD project, and by Vinnova under the NETPROG Safe and Secure Modeling and Simulation on the GRID project.

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# Physical Modeling with ModelVision, a DAE Simulator with Features for Hybrid Automata

## Abstract for short talk

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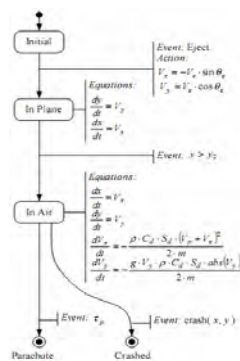
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**Abstract.** This contribution presents the modeling capabilities of the fully hybrid simulator ModelVision. Development of ModelVision started in the 1990ies, in autumn 2007 an English version is to be released. Basis of ModelVision are hybrid statecharts, allowing any parallel, serial, and conditional combination of continuous models, described by DAEs. State models itself are objects to be instantiated in various kinds, so that structural-dynamic systems of any kind can be modeled. DAE modeling is supported by an editor capable of editing mathematical formula.

Project representation can be done in visual and textual form, respectively. These representations can be restored from each other.

A subset of UML for Real Time was chosen and extended to incorporate continuous behavior. The modeling language implemented in our tool supports two types of UML diagrams: collaboration diagrams and state chart (state machine) diagrams with some changes. In collaboration diagrams we have added unidirectional continuous connections between objects (capsules in UML-RT) and the corresponding interface elements – input and output variables. UML state charts are made hybrid: a system of algebraic-differential equations over variables (interface or object's internal ones) can be associated with each simple or composite state. To make such an UML-based model fully executable we have taken Java as a reasonably high-level language for defining data types and data transformation.

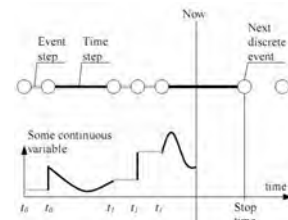
As an example we consider an airplane ejection process. This model has a hybrid structure. The equations and states can be represented in our notation like in the lower left figure.



The catapulting will start when external signal 'Eject' occurs. The first state of catapulting is track moving with fixed velocity. The second state is free flight which will start after an arm-chair has left airplane and will stop either by parachute opening after delay  $TauP$ , or by arm-chair and airplane collision and destruction. The B-Chart of the device is shown in the left figure. The solution is calculated following the hybrid event calendar in the right figure.

Users are recommended to construct a Behavior-Chart. Thus, the solved system has fixed structure and smooth right-hand sides.

Three Equation Solvers may be suggested, which are ODE-solver, DAE-solver and AE-solver. By Equation Solver we mean a heuristic algorithm that chooses the simplest numerical software for solving a problem with user prescribed tolerance. All our Solvers are based on numerical software available in the Web, namely ODEPACK, Hairer, Norsett, Wanner and Hairer, Wanner collection, DASSL and some others programs.







# An Approach to the Calibration of Modelica Models

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**Abstract.** An approach to the calibration of Modelica models using genetic algorithms (GA) is presented. The functions required to perform the model calibration have been programmed in the Modelica language and structured in a Modelica library, called *GAPLib*. This Modelica library is intended for parameter estimation in any Modelica model, supporting simple-objective optimization. Model calibration with *GAPLib* does not require to perform model modifications. During the algorithm run, the user can interactively change the value of the GA parameters. In addition, *GAPLib* supports parameter sensitivity analysis, and it is well suited for parallel computing. *GAPLib* is a free library (available on <http://www.euclides.dia.uned.es/GAPLib>) that can be easily used, modified and extended.

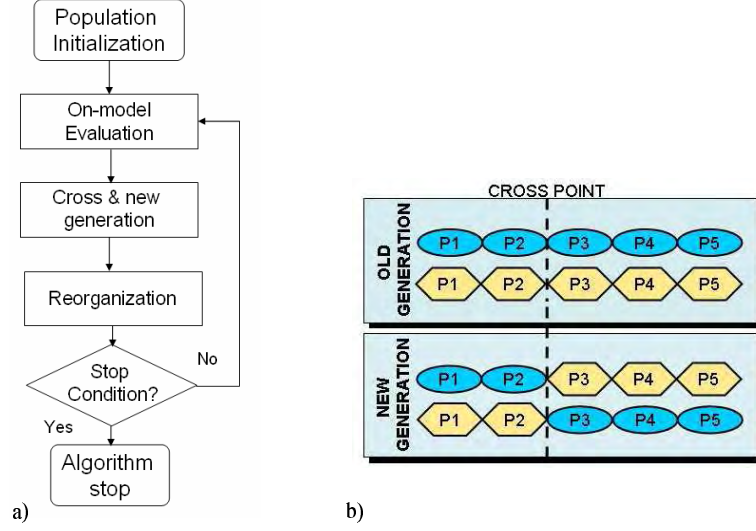
The design, implementation and use of *GAPLib* are discussed in this manuscript. Its use is illustrated by means of a case study: the estimation of electrochemical parameters in fuel cell models, which have been composed using *FuelCellLib* Modelica library.

## 1 Introduction

Genetic algorithms (GA) are used for model parameter estimation from experimental data. The main advantage of this technique lies in its robustness and simplicity. GA can be successfully applied for finding solutions in high-dimensional search spaces. The search range of the parameters can be changed during the algorithm run [1]. In addition, parallel implementations of genetic algorithms, intended to reduce the computation time, have been developed.

The use of GA for parameter estimation in Modelica models has been previously proposed by [2]. However, those authors implemented and ran the GA using Matlab/Simulink. As a consequence, those author's approach requires the combined use of Modelica/Dymola and Matlab/Simulink.

The lack of a freely-available Modelica library implementing GA, suited for parameter estimation in Modelica models, has motivated the implementation of the *GAPLib* library [3]. Two key advantages of *GAPLib* are its simplicity of use and generality: it can be applied for parameter estimation in any Modelica model, without needing to modify the model. The *GAPLib* library is freely available and can be downloaded from <http://www.euclides.dia.uned.es/GAPLib>



**Fig. 1.** a) Genetic algorithm supported by *GAPLib*; b) New generation obtained by crossover.

The fundamentals of the GA supported by the *GAPLib* library are briefly explained and the library structure is described. A new feature introduced since *GAPLib* version 1.0 [3] is discussed: the capability of changing the search range of the parameters during the GA execution. A procedure to compare the relative sensibility on the parameters is proposed. Also, a future development is discussed: support for parallel implementation of the GA. Finally, the use of *GAPLib* is illustrated by means of a case study: the estimation of electrochemical parameters in fuel cell models, which have been developed using *Fuel-CellLib* Modelica library [4].

## 2 Model Calibration Using GA

The GA supported by the *GAPLib* library is schematically represented in Figure 1a [5–7]. The application of this algorithm will be illustrated by means of the simple model shown in Eq. (1).

$$y = a \cdot x^3 + b \cdot x^2 + c \cdot x + d \quad (1)$$

The GA is used to estimate the four parameters of the model (i.e.,  $a$ ,  $b$ ,  $c$  and  $d$ ) from the following set of experimental data:

$$\{x_i, y_i\} \quad \text{for } i : 1, \dots, N \quad (2)$$

The GA starts with an initial population, composed of  $N_{POPULATION}$  individuals, which are randomly selected from the search space. Each individual of the population is formed by a group of chromosomes, which represents a solution to the problem. In case of the model shown in Eq. (1), each individual consists of a specific value of the parameters  $a$ ,  $b$ ,  $c$  and  $d$ . The  $j$ -th individual of the population is  $I_j = \{a_j, b_j, c_j, d_j\}$ . These initial values are randomly selected from the parameter search ranges.

Each individual of this initial population is evaluated by using a cost function. This function is used to calculate the validity of the population members. The cost function, evaluated for the  $j$ -th individual of the population, is the following:

$$f_j = \sum_{i:1}^N (y_i - \hat{y}_{i,j})^2 \quad (3)$$

where

$$\hat{y}_{i,j} = a_j x_i^3 + b_j x_i^2 + c_j x_i + d_j \quad (4)$$

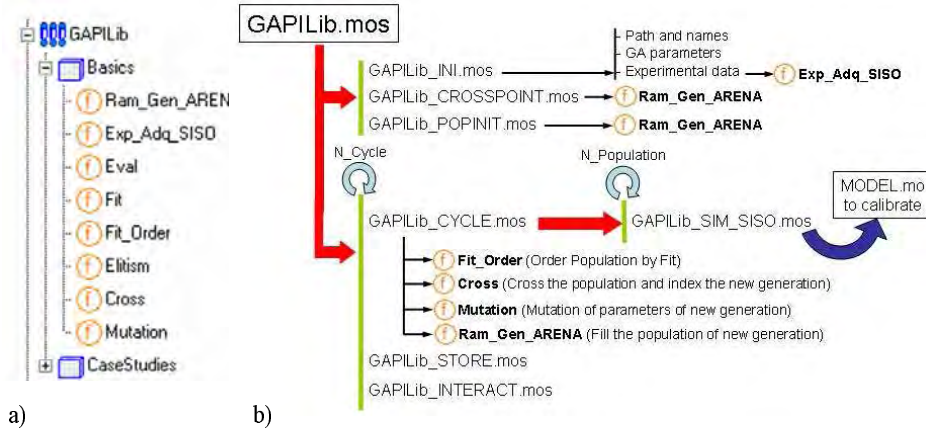
The population members (i.e.,  $\{I_j\}$ , with  $j = 1, \dots, N_{POPULATION}$ ) are sorted according to this criterion (i.e., the smaller  $f_j$ , the better). The sorted individuals can be represented as  $I(1), I(2), \dots, I(N_{POPULATION})$ , where  $I(1)$  is the best one (i.e., that with the smallest cost function).

- The  $N_{ELITISM}$  best individuals (i.e.,  $I(1), I(2), \dots, I(N_{ELITISM})$ ) pass unchanged to the following generation.
- The next  $N_{PARENTS}$  individuals (i.e.,  $I(N_{ELITISM} + 1), I(N_{ELITISM} + 2), \dots, I(N_{ELITISM} + N_{PARENTS})$ ) go through the *crossover* (see Figure 1b) and mutation processes.
- The remaining individuals of the population (i.e.,  $I(N_{ELITISM} + N_{PARENTS} + 1), \dots, I(N_{POPULATION})$ ) are discarded.

The new generation is composed of the  $N_{ELITISM}$  best individuals of the previous generation, the  $N_{PARENTS}$  individuals obtained from the crossover and mutation processes, and  $N_{POPULATION} - N_{ELITISM} - N_{PARENTS}$  new members, which are randomly selected from the search space. The individuals of this new generation are evaluated using the cost function, sorted, etc. The algorithm steps are repeated until the stop condition is reached (see Figure 1a).

The GA supported by *GAPLib* includes several processes intended to improve the algorithm performance, such as *elitism* and *mutation*.

- *Elitism* ensures that the most valid individuals pass on to the next generation without being altered by genetic operators. It guarantees that the best solution is never lost from one generation to the next.
- *Mutation* introduces random changes on the individuals, maintaining genetic diversity from one generation of the population of chromosomes to the next. The purpose of mutation is to allow the algorithm to avoid local minima.



**Fig. 2.** *GAPILib* library: a) Functions; b) Script files with the function calls signaled.

### 3 *GAPILIB* Architecture

*GAPILib* has been implemented by combining the use of the scripting Modelica language and the use of functions written in the Modelica language. The functions, which are stored within the *GAPILib.Basics* package, are listed in Figure 2a. A detailed description of these functions can be found in [3].

*GAPILib* contains a set of script files, written in scripting Modelica language (.*mos* files in Figure 2b), that implement the GA. The GA execution is started by running the script file *GAPILib.mos* (see Figure 2b). This file contains the sentences required to execute the script files that set the GA initial conditions:

- *GAPILib\_INI.mos* carries out the initialization of the GA parameters, including the number of individuals of the population ( $N_{POPULATION}$ ), elitist individuals ( $N_{ELITISM}$ ), parents ( $N_{PARENTS}$ ) and cross points (see Figure 1b). Also, the mutation probability, the stop condition of the GA, the path of the file containing the experimental data, the Modelica model, the start and stop times for the Modelica model simulation, etc. are set in this script file.
- *GAPILib\_POPINIT.mos* randomly generates the initial population.
- *GAPILib\_CROSSPOINT.mos* randomly sets the initial value of the cross point, which is used in the crossover process (see Figure 1b).

The *Ram\_Gen\_arena* function, which is a pseudo-random number generator, is called by *POPINIT* and *CROSSPOINT* script files.

Next, *GAPILib.mos* executes a loop until the GA stop condition is satisfied. The stop condition shown in Figure 2b is of the type: “ $N_{Cycle}$  generations have been obtained”. Other stop conditions are possible, e.g., “the calculated fitness

value is smaller than a given value”. The loop statements launch the execution of the following script files (see Figure 2b):

- *GAPILib\_CYCLE.mos* performs the operations required to obtain the next generation.
- *GAPILib\_STORE.mos* logs the results to a file. The results, which are stored in the Matlab format, can be accessed by the user during the GA run.
- *GAPILib\_INTERACT.mos* allows the user to change interactively (i.e., during the GA run) the GA parameters.

The script file *GAPILib\_CYCLE.mos* contains the required function calls to perform the following tasks (see Figure 2b):

1. To execute the script file *GAPILib\_SIM\_SISO*, that performs the simulation of the Modelica model, with the parameter values corresponding to each individual of the population. The model is simulated as many times as individuals are in the population. The simulation results are stored and compared with the experimental data. The *Eval* and *Fit* functions are used. All the population individuals are evaluated.
2. To sort the population individuals according to the fitness values previously calculated. The *Fit\_Order* function is used.
3. To pass on the elitist individuals to the next generation. These individuals are not altered by crossover and mutation.
4. To apply the crossover process to the parents, using the cross point calculated from *GAPILib\_CROSSPOINT*. The *Cross* function is used. The algorithm implemented is shown in Figure 1b.
5. To apply the *Mutation* function. The mutation factor is the probability used to mutate any chromosome of an individual.
6. The new population is completed with random elements. *Ram\_Gen ARENA* function is called.

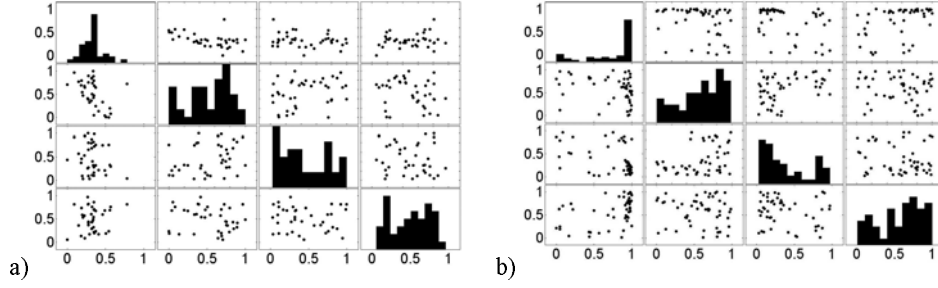
## 4 GAPILIB Use

Three practical aspects of *GAPILib* use are discussed in this section: (1) the set up of the model calibration; (2) the runtime monitoring of the algorithm convergence and the interactive change of the GA parameters; and (3) the analysis of the parameter sensitivity. Finally, a new capability that will soon be available is described: support for parallel computing of the GA.

### 4.1 Set up of the Model Calibration Study

The initial conditions of the model calibration study need to be provided by the user. They are defined by giving values to the parameters of the *GAPILib\_INI* script file. This set up information includes:

- The name of the Modelica model and the parameters to fit.



**Fig. 3.** Parameter sensitivity: a) Calibration of the model described by Eq. (1) (normalized  $a$ ,  $b$ ,  $c$  and  $d$  parameters of the 50-th generation); b) Calibration of a fuel cell model (normalized parameters of the 20-th generation).

- The name and path of the input data file (i.e., the file containing the experimental data) and the output file.
- The GA parameters, including the parent number ( $N_{PARENT}$ ), the mutation factor, the number of elitist individuals ( $N_{ELITISM}$ ), the number of cross points for the crossover process, the search space and the stop condition of the algorithm.

## 4.2 Runtime Monitoring of the Algorithm Convergence

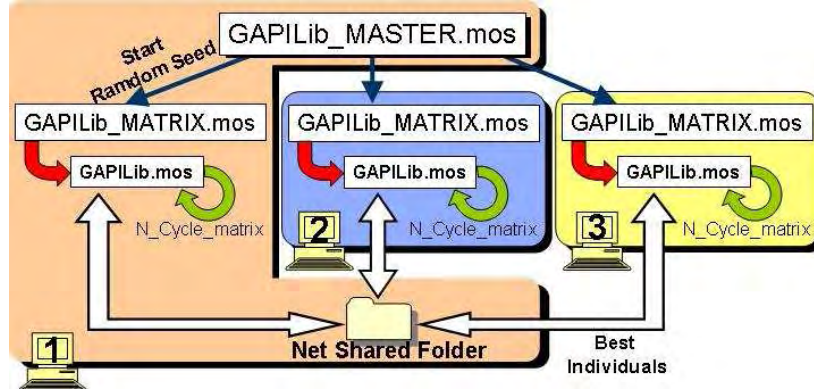
*GAPILib* supports runtime monitoring of the algorithm. The data of the population chromosomes, the cost function of the individuals, etc. is saved to a file during the algorithm run. This information allows the user to monitor the algorithm convergence and to decide whether he has to interactively modify the GA parameters. The GA parameters can be modified during the algorithm run.

## 4.3 Parameter Sensitivity

*GAPILib* assists in the analysis of the parameter sensitivity. It provides a Matlab function (i.e., *GAPILib\_SENSt.m*) that helps to estimate the relative sensitivity of the fitted parameters. This estimation is made considering the dispersion in the chromosome value of the population members with respect to the chromosome value of the best individual. The greater the dispersion, the smaller the parameter sensitivity.

An example of parameter sensitivity analysis is shown in Figure 3a. The 50-th generation of the model described by Eq. (1) is analyzed by plotting the normalized value of the  $a$ ,  $b$ ,  $c$  and  $d$  parameters. The diagonal plots show the relative frequency histograms of  $a$ ,  $b$ ,  $c$  and  $d$  parameters. As the  $a$ -parameter histogram exhibits the smaller dispersion, this is the most sensitive parameter.

Another example is the fitness of the fuel cell voltage in response to step changes in the load. The details of this model calibration will be discussed in



**Fig. 4.** Parallel computing of the GA using *GAPILib*.

Section 5. The goal is to fit the four parameters shown in Table 2. The relative frequency histograms of the 20-th generation chromosomes are plotted in Figure 3b. The first and third parameters (i.e.,  $R_{inf}$  and  $C_{dl}$ ) exhibit less dispersion than the other two parameters (i.e.,  $R_{sup}$  and  $k_s$ ). As a consequence,  $R_{inf}$  and  $C_{dl}$  are more sensitive than  $R_{sup}$  and  $k_s$ .

#### 4.4 Parallel Computing of the GA

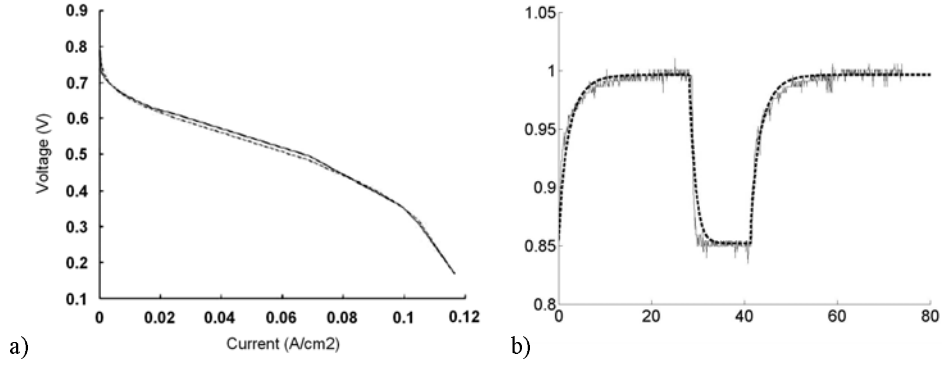
Parallel computing allows reducing the time required to complete the model calibration study. Next version of *GAPILib* will support the parallelization of the GA. The architecture is shown in Figure 4. *GAPILib* needs to be installed in all the computers, which have to be connected (e.g., using TCP/IP).

Initially, the user starts the *GAPILib\_MASTER* script file in the master computer. This script file sends random seeds to the other computers, in order to guarantee that the sequences of pseudo random numbers used in the different computers are independent.

Then, the GA is run independently in each computer during certain number of generations (*N\_Cycle\_matrix*). Periodically, the chromosomes of the best individuals obtained in each computer are saved in a shared folder (see Figure 4) and they are included in the next generation of all the computers.

### 5 Case Study: Calibration of Fuel Cell Models using *GAPILib*

*GAPILib* has been successfully applied to the estimation of electrochemical parameters in fuel cell models composed by using *FuelCellLib* Modelica library [4]. The obtained models can be used to simulate the steady-state and the dynamic



**Fig. 5.** Experimental (—), simulated using *FuelCellLib* (---): a) Fuel cell polarization curve; b) Fuel cell voltage in response to step changes in the load, Voltage [V] vs. Time [s].

behavior of the fuel cells along their complete range of operation [8–10]. Three model calibrations have been performed, in order to fit the model to:

1. The experimental polarization curve (I-V) of a fuel cell.
2. The experimental data of the fuel cell voltage obtained in response to step changes in the load.
3. The experimental data of the water long-term effect. The fitted model reproduces:
  - (a) The slow voltage rise due to the membrane hydrate.
  - (b) The voltage fall due to the water flooding of the cathode.

### 5.1 Fitness of the Polarization Curve

In order to obtain the polarization curve, the model has to be simulated, for each of the operation points composing the curve, until the steady-state is reached. The parameters estimated and the obtained values are shown in Table 1. One cross point was used. The fitness function was defined as the sum of the quadratic differences between the experimental and the simulated values of the variable.

The GA parameters were set to the following values: the stop condition is satisfied after 5000 generations; the population was composed of 100 individuals; the mutation factor was 0.25;  $N_{PARENT} = 70$ ; and  $N_{ELITISM} = 1$ . The experimental data of the fuel cell polarization curve and the simulation results of the calibrated model are shown in Figure 5a.



**Table 1.** Model parameters and their fitted values.

Parameter		Value	Unit
$A$	Tafel Slope	0.0390	$V$
$I_n$	Internal current density	$1.4 \cdot 10^{-3}$	$A \cdot cm^{-2}$
$I_0$	Exchange current density	$1.5856 \cdot 10^{-6}$	$A \cdot cm^{-2}$
$B$	Mass Transfer slope	0.0918	$V$
$R$	Internal specific resistance	$7.2860 \cdot 10^{-4}$	$\Omega \cdot cm^{-2}$
$I_{lim}$	Limiting internal current density	0.2265	$A \cdot cm^{-2}$

**Table 2.** Model parameters and their fitted values.

Parameter		Value	Unit
$R_{inf}$	Low value of the load	0.03315	$\Omega \cdot m^{-2}$
$R_{sup}$	High value of the load	5.1	$\Omega \cdot m^{-2}$
$C_{dl}$	Double layer capacitance	10.12	$F \cdot m^{-2}$
$k_s$	Electrical conductivity of the solid	0.01	$S \cdot m^{-1}$

### 5.2 Fitness of the Fuel Cell Voltage in Response to Step Changes in the Load

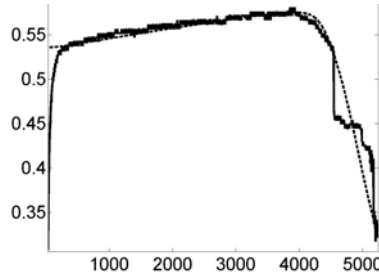
*GAPLib* is used to estimate the values of the four parameters shown in Table 2. The GA parameters are set to the following values: the stop condition is satisfied after 200 generations; the population contains 150 individuals; a factor of mutation of 0.25 was applied;  $N_{PARENT} = 100$ ; and  $N_{ELITISM} = 1$ . The experimental data of the fuel cell response to step changes in the load and the simulation results of the calibrated model are shown in Figure 5b.

### 5.3 Fitness of the Long Term Effect of Water on the Fuel Cell Voltage with Constant Resistance Load

The fuel cell model was modified in order to reproduce the variation of the membrane conductivity. The parameters used to fit the model are shown in Table 3. The GA parameters were set to the following values: the stop condition was satisfied after 700 generations; the population was composed of 70 individuals; a factor of mutation of 0.15 was applied;  $N_{PARENT} = 50$ ; and  $N_{ELITISM} = 1$ . The experimental data of the cathode flooding process and the simulation results of the calibrated model are shown in Figure 6.

**Table 3.** Model parameters and their fitted values.

Parameter		Value	Unit
$d_{a(Act)}$	Width of active layer	$6 \cdot 10^{-8}$	$m$
$\epsilon_g$	Volume fraction of pore	0.05	
$D_{12}$	Binary diffusion coefficient	$5 \cdot 10^{-9}$	$m^2 \cdot s^{-1}$
$d_{a(Mem)}$	Width of membrane layer	$1.6 \cdot 10^{-5}$	$m$
$R_{mem}$	Resistance of membrane layer	$1.42 \cdot 10^{-3}$	$\Omega \cdot m^{-2}$

**Fig. 6.** Experimental (—), simulated using *FuelCellLib* (---): long-term effect of the water with a constant load applied, Voltage [V] vs. Time [s].

## 6 Conclusions

The design, implementation and use of *GAPLib* has been discussed. The *GAPLib* library is an effective tool for parameter identification in Modelica models using GA. It is completely written in the Modelica language, which facilitates its use, modification and extension. *GAPLib* can be used for parameter identification in any Modelica model and the estimation process does not require to perform model modifications. *GAPLib* has been successfully applied to the estimation of electrochemical parameters in fuel cell models, which have been composed by using *FuelCellLib* library.

## Acknowledgements

This work has been supported by the Spanish CICYT, under DPI2004-01804 grant, and by the IV PRICIT (Plan Regional de Ciencia y Tecnología de la Comunidad de Madrid, 2005-2008), under S-0505/DPI/0391 grant.

The fuel cell experimental data used in this work has been obtained in the Laboratory of Renewable Energy of the IAI-CSIC in Madrid (Spain).

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# Dynamic Optimization of Modelica Models – Language Extensions and Tools

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**Abstract.** The Modelica language is currently gaining increased interest, both in industry and in academia. Modelica is an object-oriented, general purpose modeling language, targeted at modeling of complex physical systems. While the main usage of models developed in Modelica is simulation, several other usages emerge. Examples of such usages are dynamic optimization, model reduction, calibration, verification and code generation for embedded systems. This paper reports the current status of the JModelica project, in which an extensible, Java-based Modelica compiler is being developed. In addition, an extension of the Modelica language directed towards dynamic optimization, Optimica, is discussed.

## 1 Introduction

High-level modeling languages are receiving increased industrial and academic interest within several domains, such as chemical engineering, thermo-fluid systems and automotive systems. One such modeling language is Modelica, [8]. Modelica is an open language, specifically targeted at multi-domain modeling and model re-use. Key features of Modelica include object oriented modeling, declarative equation-based modeling, and a component model enabling acausal connections of submodels, as well as support for hybrid/discrete behaviour. These features have proven very applicable to large-scale modeling problems in various fields.

While there exist very efficient software tools for simulation of Modelica models, tool support for static and dynamic optimization is generally weak. Furthermore, specification of optimization problems is not supported by Modelica. Since Modelica models represent an increasingly important asset for many companies, it is of interest to investigate how Modelica models can be used also for optimization.

This contribution gives an overview of a project, entitled JModelica, targeted at *i)* defining an extension of Modelica, Optimica, which enables high-level formulation of optimization problems, *ii)* developing prototype tools for translating a Modelica model and a complementary Optimica description into a

representation suited for numerical algorithms, and *iii*) performing case studies demonstrating the potential of the concept.

The project integrates dynamic modeling and optimization with computer science and numerical algorithms. One of the main benefits of the suggested approach is that the high-level descriptions are automatically translated into an intermediate representation by the compiler front-end. This intermediate representation can then be further translated to interface with different numerical algorithms. The user is therefore relieved from the burden of managing the often cumbersome APIs of numerical algorithms. The flexibility of the architecture also enables the user to select the algorithm most suitable for the problem at hand.

## 2 Software Tools

In order to demonstrate the proposed concept, prototype software tools are being developed. In essence, the task of the software is to read the Modelica and Optimica source code and then translate, automatically, the model and optimization descriptions into a format which can be used by a numerical algorithm. The core of the software is a compiler front-end, referred to as the JModelica compiler, which translates a subset of Modelica into a flat model description. In addition, an extended front-end, based on the JModelica compiler, supporting a first prototype of the Optimica extension has been developed. The extended compiler is referred to as the Optimica compiler. In addition, a back-end for generation of efficient code for dynamic optimization has been developed.

### 2.1 Development Environment

The JModelica compiler is developed using the Java-based compiler construction tool JastAdd, [7]. JastAdd is a development environment targeted at implementation of the semantics of computer programming languages, and has also been explicitly designed with modular and extensible compiler construction in mind. The core concepts used in JastAdd are object orientation, static aspect orientation, and reference attributed grammars [6].

The JastAdd system is based on an object oriented specification of an abstract grammar (AG), from which standard Java classes are generated. Semantic behaviour is added in *aspects*, which are useful for organizing cross-cutting behaviour. It is natural to structure the implementation of different semantic functions, such as name analysis (the task of binding identifiers to declarations) and type analysis (e.g. computation of the types of expressions), into separate modules. However, since the implementation of, for example, name analysis, typically affects a large number of classes, the object-oriented paradigm does not inherently offer support for this kind of modularization. In JastAdd, this problem is overcome by allowing definition of behaviour, in the form of inter-type declarations, in separate aspects, which are then *woven* into the AG classes. The

resulting classes contain only Java code, and can be compiled by a standard Java compiler.

The choice of JastAdd is natural in this project, since its main focus is extensions of the Modelica language. In particular, the methodology adopted by JastAdd enables the implementations of the core language compiler and the extensions to be separated. It is then possible to build the core compiler alone, or with one or more extensions. As a notable example, a full Java 1.4 compiler, and a fully modular extension to also support Java 1.5 have been implemented in JastAdd, [3]. For an overview of the JModelica compiler implementation, including some performance benchmarks, see [1].

## 2.2 Code Generation to AMPL

Currently, the front-end of the JModelica/Optimica compiler supports a subset of Modelica and a basic version of Optimica. In addition, a code-generation back-end for AMPL, [4], has been developed. AMPL is a language intended for formulation of algebraic optimization problems. Accordingly, the compiler performs automatic transcription of the original continuous-time problem into an algebraic formulation which can be encoded in AMPL. In the transcription procedure, the problem is discretized by means of a simultaneous optimization approach based on collocation over finite elements, see for example., [2] for an overview. Finally, the automatically generated AMPL description may be executed and solved by a numerical NLP algorithm. For this purpose we have used IPOPT, [9].

## 2.3 Project Status

This paper describes the current status of the JModelica project, as of June 2007. Currently, the JModelica compiler supports a limited subset of Modelica, which includes classes, components, inheritance, value modifications, connect-clauses and partial support for arrays. The functionality of the Optimica compiler will be described in detail in the next section.

## 3 Optimica

A key issue is the definition of syntax and semantics of the Modelica extension, Optimica. Optimica should provide the user with language constructs that enable formulation of a wide range of optimization problems, such as parameter estimation, optimal control and state estimation based on Modelica models.

At the core of Optimica are the basic optimization elements such as cost functions and constraints. It is also possible to specify bounds on variables in the Modelica model as well as marking variables and parameters as optimization quantities, i.e., to express what to optimize over. While this type of information represents a canonical optimization formulation, the user is often required to supply additional information, related to the numerical method which is used to

solve the problem. In this category we have e.g., specification of transcription method, discretization of control variables and initial guesses. Optimica should also enable convenient specification of these quantities.

The current preliminary specification of the Optimica language admits formulation of dynamic optimization problems on the following form:

$$\begin{aligned}
& \min_{u(t), p} \int_0^{t_f} L(x(t), u(t), p) dt + \phi(x(t_f)) \\
& \text{subject to} \\
& f(\dot{x}, x, u, p) = 0 \\
& c_i(x(t), u(t), p) \leq 0, \quad c_e(x(t), u(t), p) = 0 \\
& c_{fe}(x(t_f), u(t_f), p) = 0, \quad c_{fi}(x(t_f), u(t_f), p) \leq 0 \\
& c_{0e}(x(0), u(0), p) = 0, \quad c_{0i}(x(0), u(0), p) \leq 0
\end{aligned} \tag{1}$$

The dynamic constraint  $f(\dot{x}, x, u, p) = 0$  is expressed using Modelica, and Optimica is used for everything else.

### 3.1 The Optimica Extension

The anatomy of an Optimica description of an optimization problem is similar to a simple Modelica model, and consists of three sections. In the first section, information relevant for formulation of the optimization problem may be superimposed on elements in the Modelica model. For example, variable bounds and initial guesses can be specified. In addition, it is possible to mark Modelica parameters and initial conditions of dynamic variables as free optimization variables. In the second section, referred to as **optimization**, the cost function and the optimization horizon can be specified. In the third section, referred to as **subject to** the constraints of the problem is given.

In the current version of Optimica, the content of a Modelica class is implicitly assumed to be present in the scope of an Optimica class. This is equivalent to the Optimica class extending from the corresponding Modelica class. In future versions of Optimica, this implicit assumption will be removed in favor of allowing explicit extends statements as well as component declarations in the Optimica description.

In essence, Optimica supports four constructs:

- **Superimpose information on Modelica variables.** Commonly, it is desirable to superimpose optimization-related information on variable declarations in the Modelica model. For this purpose, a new construct is introduced:

```
[oq] component_access [modification]
```

where the name **component\_access** binds to a name in the corresponding Modelica model. In addition, the optional prefix **oq**, see below, and a modification construct can be specified. Notice that this is not a component declaration, but should be seen as a mechanism for adding information to



an existing declaration; modifications given in this construct are merged with those of the original declaration. In Modelica, this construct corresponds to a `redeclare` modification, which may change the prefix of a variable as well as add modifications. This new construct can therefore be viewed as a simplified and shorthand alias for a `redeclare` modification. The introduction of a new language construct is motivated by the need for a compact and efficient way to superimpose information on variables, without having to use the more involved component redeclaration mechanism. In addition, the current version of Optimica does not support component declarations, which makes the proposed construct convenient.

Bounds on variables, both inputs and states, and parameters can be expressed using the construct

```
[oq] varName(lowerBound=-1,upperBound=1);
```

where `varName` refers to a variable or parameter in the Modelica model. The optional prefix `oq` (Optimization Quantity) is used to let a Modelica parameter or variable be free in the optimization. The effect of using the `oq` prefix for a variable is that the binding expression, if any, of the corresponding declaration is removed.

It is also possible to specify an initial guess for a variable or parameter in Optimica:

```
varName(lowerBound=-1,upperBound=1,initialGuess=0);
```

The initial guess is a constant expression, which is used to initialize variables and optimization parameters. If an initial guess file is supplied upon compilation, the initial guess in the Optimica description has priority over the one in the file. Also notice that the initial guess has no effect for a Modelica parameter if the `oq` prefix is not specified.

It is also possible to specify bounds and initial guess for derivatives of variables:

```
der(varName)(lowerBound=-1,upperBound=1,initialGuess=0.3);
```

Dynamic variables by default have fixed initial conditions, specified by the `start`-attribute given in the corresponding Modelica variable declaration. The following construct enables free initial conditions:

```
varName(freeInitial(lowerBound=[-0.01;-0.001;-0.01;-0.001],
                    upperBound=[0.01;0.001;0.01;0.001],
                    initialGuess=[0.001;0;0;0])=true);
```

where there the variable `varName` in this case is an array variable. Notice that upper and lower bound as well as initial guess (optional) for the variable can be given in the same construct:

```
varName(lowerBound=-3,upperBound=3,initialGuess=1,
        freeInitial(lowerBound=-2,upperBound=2,initialGuess=0)=true);
```

- **Specification of grid.** The solution of the optimization problem is defined on a grid, consisting of a number of time points. The accuracy (and usually execution time) is increased if a grid with more points is used. Due to the nature of the transcription scheme used in the Optimica compiler, it is more natural to specify the number of *elements* of the grid. The number of points is then given by three times the number of elements, since a third order collocation method is used. A grid with fixed final time is specified by the construct

```
grid(finalTime=fixedFinalTime(finalTime=tf),nbrElements=n_el);
```

and a grid with free final time is specified by

```
grid(finalTime = openFinalTime(initialGuess=tf_ig,lowerBound=tf_lb,
                                upperBound=tf_ub),nbrElements=n_el);
```

By specifying a free final time, it is possible to formulate minimum time problems.

A static optimization problem is defined by using the construct:

```
grid(static=true);
```

In this case, all **der**-operators in the model are replaced by zero.

Notice that the **grid** construct must reside in an **optimization** section.

- **Definition of cost function.** The cost function is specified in the **optimization** section using the construct

```
minimize(lagrangeIntegrand=li_exp,terminalCost=tc_exp);
```

The argument **lagrangeIntegrand** corresponds to the integrand expression in the Lagrange cost function,  $L$  and **terminalCost** corresponds to  $\phi$ .

- **Specification of constraints** In the **subject** to section, path, initial and terminal constraints can be specified. A terminal constraint is introduced using the prefix **terminal** and an initial constraint is introduced by the prefix **initial**. Examples of constraints are

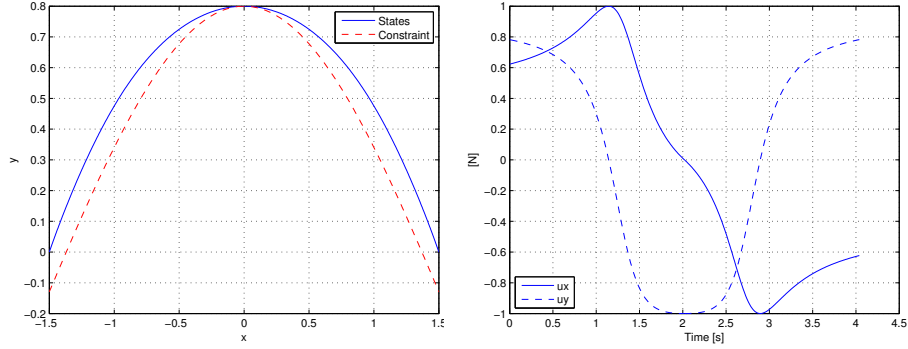
```
y<=x^2; // Path constraint
initial cos(x)>=0.4 // Initial constraint
terminal y=4; // Terminal constraint
```

### 3.2 2D Double Integrator Example

Consider the following model of a two dimensional double integrator:

$$\begin{aligned}\ddot{x}(t) &= u_x(t) \\ \ddot{y}(t) &= u_y(t)\end{aligned}\tag{2}$$

We would like to find trajectories that transfer the state of the system from  $(-1.5, 0)$  to  $(1.5, 0)$  in shortest possible time. In addition, we would like to impose



**Fig. 1.** Resulting optimization profiles for the minimum time case.

the path constraint  $y \geq \cos x - 0.2$  and  $u_x^2 + u_y^2 \leq 1$ . The latter constraint ensures that the resulting force has a magnitude equal to or less than 1. This gives us the following optimal control formulation

$$\begin{aligned}
 & \min_u \int_0^{t_f} 1 dt \\
 & \text{subject to} \\
 & \quad \ddot{x}(t) = u_x(t) \\
 & \quad \ddot{y}(t) = u_y(t) \\
 & \quad x(0) = -1.5, \quad x(t_f) = 1.5, \quad y(0) = 0, \quad y(t_f) = 0 \\
 & \quad \dot{x}(0) = 0, \quad \dot{x}(t_f) = 0, \quad \dot{y}(0) = 0, \quad \dot{y}(t_f) = 0 \\
 & \quad y(t) \geq \cos x(t) - 0.2 \\
 & \quad 1 \geq u_x(t)^2 + u_y(t)^2
 \end{aligned} \tag{3}$$

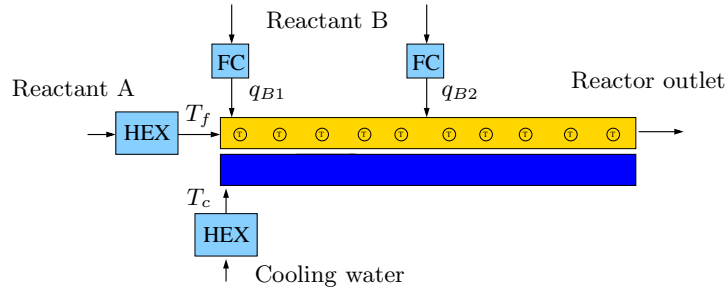
The dynamics of the double integrator system is given by the following Mod-  
elica model:

```

model DoubleIntegrator2d
  input Real ux;
  input Real uy;
  Real x(start=-1.5), vx(start=0);
  Real y(start=0), vy(start=0);
equation
  der(x)=vx; der(vx)=ux;
  der(y)=vy; der(vy)=uy;
end DoubleIntegrator2d;

```

and the Optimica description of the optimization problem is given by:



**Fig. 2.** The reactor shown as a schematic tubular reactor. There are four inflows to the process and there is one manipulated variable for each inflow;  $q_{B1}$ ,  $q_{B2}$ ,  $T_f$  and  $T_c$ . Each inflow has an actuator subsystem that provides flow control (FC) or temperature control through heat exchangers (HEX). The circles with  $T$  represents internal temperature sensors.

```

class optDI2d
optimization
    grid(finalTime = openFinalTime(initialGuess=4.5,lowerBound=3,
                                   upperBound=tf_ub),nbrElements=5);
    minimize(lagrangeIntegrand=1);
subject to
    terminal x=1.5; terminal vx=0;
    terminal y=0;   terminal vy=0;
    ux^2+uy^2<=1;  y>=cos(x)-0.2;
end optDI2d;

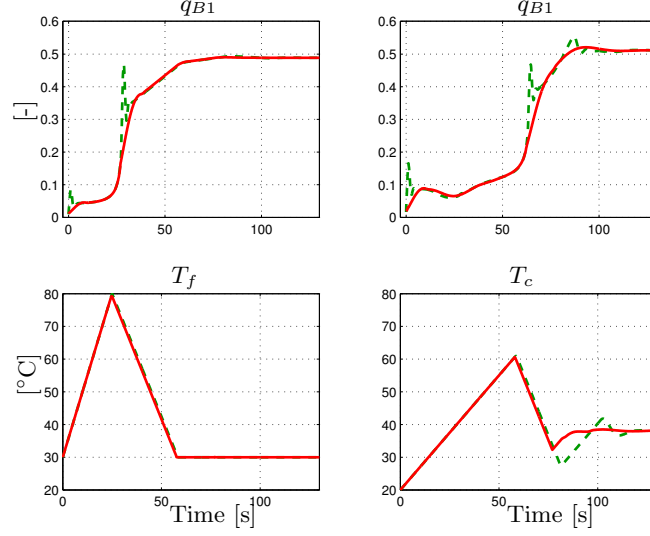
```

Notice that the initial conditions are expressed in the Modelica model using the **start** attribute, whereas the terminal constraints are given in the **subject to** clause in the Optimica model. The resulting time optimal trajectories are shown in Figure 1.

## 4 A Case Study

The Optimica compiler has been used to formulate and solve a start-up problem for a plate reactor system. The plate reactor is conceptually a tubular reactor located inside a heat exchanger, and offers excellent flexibility, since it is reconfigurable and allows multiple injection points for chemicals, separate cooling/heating zones and easy mounting of temperature sensors. In this case study, an exothermic reaction,  $A + B \rightarrow C$ , was assumed. The reactor was fed with a fluid with a specified concentration of the reactant  $A$ . The reactant  $B$  was injected at two points along the reactor. The control variables of the system were the temperatures of the inlet flow, the temperature of the cooling flow and the injection flow-rates of the reactant  $B$ , see Figure 2.

The primary objective of the start-up sequence was to transfer the state of the reactor from an operating point where no reaction takes place, to the desired



**Fig. 3.** Optimal control profiles. The dashed curves correspond to a case with a small high frequency penalty on the inputs, whereas the solid curves represents a case with a larger high frequency penalty, resulting in smoother control profiles.

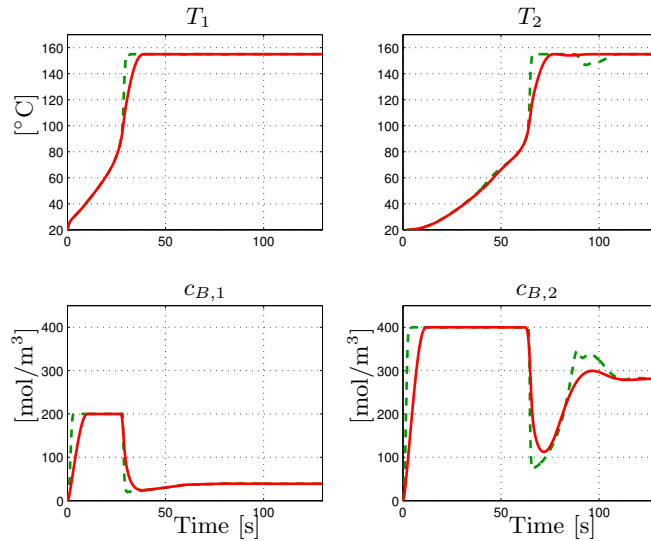
point of operation. This problem is challenging, since the dynamics of the system is fast and unstable in some operating conditions. Also, the temperature in the reactor must be kept below a safety limit, in order not to damage the hardware.

A Modelica model, containing 131 states and 71 algebraic variables, was used to represent the dynamics of the system. Optimal control and state profiles were calculated off-line and then used as feedforward and feedback signals in a PID-based mid-ranging control system. The resulting optimization problem contained approximately 160,000 variables. The optimal control profiles,  $q_{B1}$ ,  $q_{B2}$ ,  $T_f$  and  $T_c$  are shown in Figure 3, and the corresponding output temperature and concentration profiles,  $T_1$ ,  $T_2$ ,  $c_{B,1}$  and  $c_{B,2}$  are shown in Figure 4.

The experiences from using the Optimica compiler in this project are promising, in that the tools enable the user to focus on *formulation* of the problem instead of, which is common, *encoding* of the problem. For more details on this case study, see [5].

## 5 Summary

This contribution gives an overview of the JModelica project, which is targeted at extending the Modelica language to also support optimization. The goals of the project include specification of the language extension Optimica, development of prototype software tools and case studies. A preliminary specification of Optimica, offering basic support for formulation of dynamic optimization problems based on Modelica models has been presented.



**Fig. 4.** Optimal profiles for reactor temperature and concentration of substance  $B$ . The left plots correspond to the first injection point, whereas the right plots correspond to the second injection point.

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# Robust Initialization of Differential Algebraic Equations

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(Previously published in Modelica'2006, Vienna, Sept 4-5, 2006. [www.modelica.org](http://www.modelica.org))

**Abstract.** This paper describes a new solution method applied to the problem initializing DAEs using the Modelica language. Modelica is primarily an object-oriented equation-based modeling language that allows specification of mathematical models of complex natural or man-made systems. Major features of Modelica are the multi-domain modeling capability and the reusability of model components corresponding to physical objects, which allow to build and simulate highly complex systems. However, initializing such models has been quite cumbersome, since initial equations have to be provided at the system level, where the user needs to know details on the underlying transformation and index-reduction algorithms, that in general are applied to simulate a Modelica model. .

## 1 Introduction

So far, using model initialization in Modelica has only been possible for higher-index problems if the user formulates the initial equations globally. This was also the case, e.g. when using the OpenModelica compiler which is an open source implementation developed at PELAB, Linköping University. In order to do such a global formulation successfully, the user needs to know about index reduction, at least the number of freedom left after applying the dummy derivative method is necessary. Therefore, only advanced users have been able to use this feature in the Modelica language, when higher index problems occur (which is very common). In order to provide a more complete simulation environment, we have started to add robust initialization techniques to the OpenModelica compiler.

## 2 Flattening of a Modelica Model to a Hybrid DAE

A Modelica model is typically translated to a basic mathematical representation in terms of a flat system of *differential and algebraic equations* (DAEs) before being able to simulate the model. This translation process elaborates on the internal model representation by performing analysis and type checking, inheritance and expansion of base classes, modifications and redeclarations, conversion of connect-equations to basic equations, etc. The result of this analysis and translation process is a flat set of equations, including conditional equations, as well as constants, variables, and function definitions. By the term *flat* is meant that the object-oriented structure has been broken down to a flat representation where no trace of the object hierarchy remains apart from dot notation (e.g. `Class.Subclass.variable`) within names.

## 3 Mathematical Formulation of Hybrid DAEs

### 3.1 Summary of notation

Below we summarize the notation used in the equations that follow, with time dependencies stated explicitly for all time-dependent variables by the arguments  $t$  or  $t_e$ :

- $p = \{p_1, p_2, \dots\}$ , a vector containing the Modelica variables declared as parameter or constant i.e., variables without any time dependency.
- $t$ , the Modelica variable time, the independent variable of type `Real` implicitly occurring in all Modelica models.
- $x(t)$ , the vector of state variables of the model, i.e., variables of type `Real` that also appear differentiated, meaning that `der()` is applied to them somewhere in the model.
- $\dot{x}(t)$ , the differentiated vector of state variables of the model.
- $u(t)$ , a vector of input variables, i.e., not dependent on other variables, of type `Real`. These also belong to the set of algebraic variables since they do not appear differentiated.
- $y(t)$ , a vector of Modelica variables of type `Real` which do not fall into any other category. Output variables are included among these, which together with  $u(t)$  are algebraic variables since they do not appear differentiated.
- $q(t_e)$ , a vector of discrete-time Modelica variables of type `discrete Real`, `Boolean`, `Integer` or `String`. These variables change their value only at event instants, i.e., at points  $t_e$  in time.
- $q_{pre}(t_e)$ , the values of  $q$  immediately before the current event occurred, i.e., at time  $t_e$ .
- $c(t_e)$ , a vector containing all `Boolean` condition expressions evaluated at the most recent *event* at time  $t_e$ . This includes conditions from all if-equations/statements and if-expressions from the original model as well as those generated during the conversion of when-equations and when-statements.



- $rel(v(t)) = rel(cat(1, x, \dot{x}, u, y, \{t\}, q(t_e), q_{pre}(t_e), p))$ , a Boolean vector valued function containing the relevant elementary relational expressions from the model, excluding relations enclosed by `noEvent()`. The argument  $v(t) = \{v_1, v_2, \dots\}$  is a vector containing all elements in the vectors  $x, \dot{x}, u, y, \{t\}, q(t_e), q_{pre}(t_e), p$ . This can be expressed using the Modelica concatenation function `cat` applied to these vectors;  $rel(v(t)) = \{v_1 > v_2, v_3 \geq 0, v_4 < 5, v_6 \leq v_7, v_{12} = 133\}$  is one possible example.
- $f(\dots)$ , the function that defines the differential equations  $f(\dots) = 0$  in (1a) of the system of equations.
- $g(\dots)$ , the function that defines the algebraic equations  $g(\dots) = 0$  in (1b) of the system of equations.
- $f_q(\dots)$ , the function that defines the difference equations for the discrete variables  $q := f_q(\dots)$ , i.e., (2) in the system of equations.
- $f_e(\dots)$ , the function that defines the event conditions  $c := f_e(\dots)$ , i.e., (3) in the system of equations.
- $f_x(\dots)$ , the function that defines the reinitialization values for the continuous variables  $x(t_e) := f_x(\dots)$  at events.

In the context of hybrid DAE:s the *state* of a system is not only made up of the values of the set of variables that occur differentiated in the model. The overall *state* of a system may also include values of discrete variables. In this paper the word *state* is used in this sense, including the state of the discrete part of the system.

### 3.2 Continuous-Time Behavior

Now we want to formulate the continuous part of the *hybrid DAE* system of equations including discrete variables. This is done by adding a vector  $q(t_e)$  of *discrete-time variables* and the corresponding predecessor variable vector  $q_{pre}(t_e)$  denoted by `pre(q)` in Modelica. For discrete variables we use  $t_e$  instead of  $t$  to indicate that such variables may only change value at event time points denoted  $t_e$ , i.e., the variables  $q(t_e)$  and  $q_{pre}(t_e)$  behave as constants between events.

We also make the constant vector  $p$  of *parameters and constants* explicit in the equations, and make the time  $t$  explicit. The vector  $c(t_e)$  of condition expressions, e.g. from the conditions of `if` constructs and `when` constructs, evaluated at the most recent event at time  $t_e$  is also included since such conditions are referenced in conditional equations. We obtain the following *continuous DAE* system of equations that describe the system behavior *between* events:

$$\begin{aligned} f(x(t), \dot{x}(t), u(t), y(t), t, q(t_e), q_{pre}(t_e), p, c(t_e)) &= 0 & (a) \\ g(x(t), u(t), y(t), t, q(t_e), q_{pre}(t_e), p, c(t_e)) &= 0 & (b) \end{aligned} \quad (1)$$

### 3.3 Discrete-Time Behavior

Discrete time behavior is closely related to the notion of an event. Events can occur asynchronously, and affect the system one at time, causing a sequence of state transitions.

An event occurs when any of conditions  $c(t_e)$  (defined below) of conditional equations changes value from `false` to `true`. We say that an event becomes *enabled* at the time  $t_e$ , if and only if, for any sufficiently small value of  $\varepsilon$ ,  $c(t_e - \varepsilon)$  is `false` and  $c(t_e + \varepsilon)$  is `true`. An enabled event is *fired*, i.e., some behavior associated with the event is executed, often causing a discontinuous state transition.

Firing of an event may cause other conditions to switch from `false` to `true`. In fact, events are fired until a stable situation is reached when all the condition expressions are `false`.

However, there are also state changes caused by equations defining the values of the *discrete* variables  $q(t_e)$ , which may change value *only* at events, with event times denoted  $t_e$ . Such discrete variables obtain their value at events, e.g. by solving equations in when-equations or evaluating assignments in when-statements. The instantaneous equations defining discrete variables in when-equations are restricted to particularly simple syntactic forms, e.g.  $var = expr$ . These restrictions are imposed by the Modelica language in order to easily determine which discrete variables are defined by solving the equations in a when-equation.

Such equations can be directly converted to equations in assignment form, i.e., assignment statements, with fixed causality from the right-hand side to the left-hand side. Regarding algorithmic when-statements that define discrete variables, such definitions are always done through assignments. Therefore we can in both cases express the equations defining discrete variables as *assignments* in the vector equation (1a), where the vector-valued function  $f_q$  specifies the right-hand side expressions of those *assignments to discrete variables*.

$$q(t_e) := f_q(x(t_e), \dot{x}(t_e), u(t_e), y(t_e), t_e, q_{pre}(t_e), p, c(t_e)) \quad (2)$$

The last argument  $c(t_e)$  is made explicit for convenience. It is strictly speaking not necessary since the expressions in  $c(t_e)$  could have been incorporated directly into  $f_q$ . The vector  $c(t_e)$  contains all `Boolean` condition expressions evaluated at the most recent *event* at time  $t_e$ . It is defined by the following vector assignment equation with the right-hand side given by the vector-valued function  $f_c$ . This function has as arguments the subset of the discrete variables having `Boolean` type, i.e.,  $q^B(t_e)$  and  $q_{pre}^B(t_e)$ , the subset of `Boolean` parameters or constants,  $p^B$ , and a vector  $rel(v(t))$  evaluated at time  $t_e$ , containing the elementary relational expressions from the model. The vector of condition expressions  $c(t_e)$  is defined by the following equation in assignment form:

$$c(t_e) := f_c(q^B(t_e), q_{pre}^B(t_e), p^B, rel(v(t_e))) \quad (3)$$

The argument  $v(t) = \{v_1, v_2, \dots\}$  is a vector containing all scalar elements of the argument vectors. This can be expressed using the Modelica concatenation function `cat` applied to the vectors, e.g.  $v(t) = cat(1, x, \dot{x}, u, y, \{t\}, q(t_e), q_{pre}(t_e), p)$ . For exam-

ple, if  $rel(v(t)) = \{v_1 > v_2, v_3 \geq 0, v_4 < 5, v_6 \leq v_7, v_{12} = 133\}$  where  $v(t) = \{v_1, v_2, v_3, v_4, v_6, v_7, v_{12}\}$ , then it might be the case that  $c(t) = \{v_1 > v_2 \text{ and } v_3 \geq 0, v_{10}, \text{not } v_{11}, v_4 < 5 \text{ or } v_6 \leq v_7, v_{12} = 133\}$ , where  $v_{10}, v_{11}$  are Boolean variables and  $v_1, v_2, v_3, v_4, v_6, v_7$  might be Real variables, whereas  $v_{12}$  might be an Integer variable.  $rel(v(t)) = rel(\text{cat}(1, x(t), \dot{x}(t), u(t), y(t), t, q(t_e), q_{pre}(t_e), p))$ , is a Boolean-typed vector-valued function containing the relevant elementary *relational expressions* from the model, excluding relations enclosed by `noEvent()`.

Discontinuous changes of continuous dynamic variables  $x(t)$  can be caused by so-called `reinit` equations in Modelica. As in the case of discrete variables, such discontinuous changes can only occur at events. The effect of a `reinit`-equation that is activated at  $t_e$  is an assignment to the continuous variable at time  $t_e$  of the form:

$$x(t_e) := f_x(x(t_e), \dot{x}(t_e), u(t_e), y(t_e), t_e, q_{pre}(t_e), p, c(t_e)) \quad (4)$$

For all variables in  $x(t_e)$  that are not affected by an `reinit`-equation  $f_x(\dots)$  takes the value of  $x(t_e)$ , leaving the variable unchanged.

### 3.4 The Complete Hybrid DAE

The total equation system consisting of the combination of (1), (2), (3) and (4) is the desired *hybrid DAE* equation representation for Modelica models, consisting of *differential*, *algebraic*, and *discrete* equations.

This framework describes a system where the state evolves in two ways: continuously in time by changing the values of the state vector  $x(t)$ , and instantaneously during events triggered when some of the conditions  $c(t_e)$  change value from `false` to `true`. The set of *state variables* from which other variables are computed is selected from the set of differentiated variables  $x(t)$ , algebraic variables  $y(t)$ , and discrete-time variables  $q(t)$ .

## 4 Simulation of Models Represented by Hybrid DAEs

### 4.1 Well-defined problem description

A Modelica *simulation problem* in the general case is a Modelica *model* that can be reduced to a hybrid DAE in the form of equations (1), (2), (3) and (4), together with additional constraints on variables and their derivatives called *initial conditions*.

The initial conditions prescribe initial start values of variables and/or their derivatives at simulation time=0 (e.g. expressed by the Modelica `start` attribute value of variables, with the attribute `fixed = true`), or default estimates of start values (the `start` attribute value with `fixed = false`).

The simulation problem is *well defined* provided that the following conditions hold:

- The total model system of equations is consistent and neither underdetermined nor overdetermined.
- The initial conditions are consistent and determine initial values for all variables.

- The model is specific enough to define a unique solution from the start simulation time  $t_0$  to some end simulation time  $t_1$ .

The initial conditions of the simulation problem are often specified interactively by the user in the simulation tool, e.g. through menus and forms, or alternatively as default `start` attribute values in the simulation code. More complex initial conditions can be specified through `initial equation` sections in Modelica.

## 4.2 Simulation Techniques

There are three different kinds of equation systems resulting from the translation of a Modelica model to a flat set of equations, from the simplest to the most complicated and powerful:

- ODEs – Ordinary differential equations for continuous-time problems.
- DAEs – Differential algebraic equations for continuous-time problems
- Hybrid DAEs – Hybrid differential algebraic equations for mixed continuous-discrete problems.

In the following we present a short overview of methods to solve these kinds of equation systems. However, remember that these representations are strongly inter-related: an ODE is a special case of DAE without algebraic dependencies between states, whereas a DAE is a special case of hybrid DAEs without discrete or conditional equations. We should also point out that in certain cases a Modelica model results in one of the following two forms of purely algebraic equation systems, which can be viewed as DAEs without a differential equation part:

- Linear algebraic equation systems
- Nonlinear algebraic equation systems

However, rather than representing a whole Modelica model, such algebraic equation systems are usually subsystems of the total equation system.

## 4.3 The Notion of DAE Index

The DAE index is an important property of DAE systems. Consider once more a DAE system on the general form (neglecting the hybrid part, parameters and constants):

$$F(x(t), \dot{x}(t), y(t), u(t)) = 0 \quad (5)$$

We assume that this system is solvable with a continuous solution, given an appropriate initial solution. There are several definitions of DAE *index* in the literature, of which the following, also called *differential index*, is informally defined as follows:

- The index of a DAE system (5) is the minimum number of times certain equations in the DAE must be differentiated in order to solve  $\dot{x}(t)$  as a function of  $x(t)$ ,  $y(t)$ , and  $u(t)$ , i.e. to transform the problem into ODE explicit state space form.

The index gives a classification of DAEs with respect to their numerical properties and can be seen as a measure of the distance between the DAE and the corresponding ODE

An ODE system on explicit state space form is of index 0 since it is already in the desired form:

$$\dot{x}(t) = f(t, x(t)) \quad (6)$$

The following *semi-explicit* form of DAE system is of index 1 under certain conditions:

$$\begin{aligned} \dot{x}(t) &= f(t, x(t), y(t)) & (a) \\ 0 &= g(t, x(t), y(t)) & (b) \end{aligned} \quad (7)$$

The condition is that the Jacobian of  $g$  with respect to  $y$ ,  $(\partial g / \partial y)$  – usually a matrix – is *non-singular* and therefore has a well-defined inverse. This means that in principle  $y(t)$  can be solved as a function of  $x(t)$  and substituted into (7a) to get state-space form. A DAE system in the general form (5) may have higher index than one. Mechanical models often lead to index 3 DAE systems. We conclude:

- There is no need for symbolic differentiation of equations in a DAE system if it is possible to determine the *highest order derivatives* as continuous functions of time and lower derivatives using stable numerical methods. In this case the index is at most 1.
- The index is zero for such a DAE system if there are no algebraic variables.

#### 4.4 Mixed Symbolic and Numerical Solution of higher-index DAEs

A mixed symbolic and numerical approach to solution of DAEs avoids the problems of numeric differentiation. The DAE is transformed to a lower index problem by using index reduction. The standard mixed symbolic and numeric approach contains the following steps:

1. Use Pantelides algorithm to determine how many times each equation has to be differentiated to reduce the *index* to one or zero.
2. Perform *index reduction* of the DAE by analytic symbolic differentiation of certain equations and by applying the method of dummy derivatives.
3. Select the core state variables to be used for solving the reduced problem. These can either be selected statically during compilation, or in some cases selected dynamically during simulation.
4. Use a numeric ODE solver to solve the reduced problem.

In the following we will discuss the notions of index and index reduction in some more detail.

#### 4.5 Higher Index Problems are Natural in Component-Based Models

The index of a DAE system is not a property of the modeled system but the *property* of a *particular model representation*, and therefore a function of the modeling methodology. A natural object-oriented component-based methodology with reuse and connections between physical objects leads to high index in the general case. The reason is the constraint equations resulting from setting variables equal across connections between separate objects.

Since the index is not a property of the modeled system it is possible to reduce the index by symbolic manipulations. High index indicates that the model has algebraic relations between differentiated state variables implied by algebraic relations between those state variables. By using knowledge about the particular modeling domain it is often possible to manually eliminate a number of differentiated variables, and thus reduce the index. However, this violates the object-oriented component-based modeling methodology for physical modeling that is intended to be supported by the Modica language.

We conclude that high index models are natural, and that automatic index reduction is necessary to support a general object-oriented component-based modeling methodology with a high degree of reuse.

### 5 Finding Consistent Initial Values at Start or Restart

As we have stated briefly above, at the start of the simulation, or at restart after handling an event, it is required to find a consistent set of initial values or restart values of the variables of the hybrid DAE equation system before starting continuous DAE solution process.

At the *start* of the simulation these conditions are given by the initial conditions of the problems (including `start` attribute equations, equations in `initial equation` sections, etc., together with the system of equations defined by (1), (2), and (3). The user specifies the initial time of the simulation,  $t_0$ , and initial values or guesses of initial values of some of the continuous variables, derivatives, and discrete-time variables so that the algebraic part of the equation system can be solved at the initial time  $t=t_0$  for all the remaining unknown initial values. In some application examples it is even necessary to calculate initial values of parameters (`fixed = false`), that afterwards be kept constant during simulation.

At *restart* after an event, the conditions are given by the *new values* of variables that have changed at the event, together with the current values of the remaining variables, and the system of equations (5), (6), and (7). The goal is the same as in the initial case, to solve for the new values of the remaining variables. In the initial case, however, the causality can be different since initial equations are included to calculate start values for the state variables, whereas at restart the state variables are always known.

## 6 Robust Initialization of Higher-Index DAEs

Initializing DAEs using the Modelica language has been quite cumbersome in the past, since initial equations have to be provided on the system level, where the user needs to know details on the underlying transformation and index-reduction algorithms, that are in general applied to simulate a Modelica model. Especially, when higher-index DAEs are involved the number of locally defined state variables no longer coincide with the number of state variables of the overall system. Although, one can influence the index-reduction algorithm by setting some attribute values (`stateSelect=always,prefer,...`), cases can be constructed which don't allow the straight forward prediction of the number of state variables left after transformation.

In order to make the initialization procedure more convenient a new concept is necessary, which allows to define the initial equations locally in each relevant component where the corresponding states appear, even if these states are eliminated during index-reduction. Naturally, this leads to an overdetermined system of equations, which has to be solved during the initialization process. In this context, we call a higher-index problem “well-posed” if enough equations of the system are redundant so that initial values can be determined which fulfill the whole set of initial equations. The main idea of the new approach is to reformulate the problem of finding roots of the set of non-linear equations to an equivalent optimization problem.

Considering the general mathematical description of the initialization problem:

$$\begin{aligned} f_1(z_1, \dots, z_n) &= 0 \\ &\vdots \\ f_m(z_1, \dots, z_n) &= 0 \end{aligned} \tag{8}$$

Cases where  $m \geq n$  means that more equations ( $m$ ) than variables ( $n$ ) are given. Every solution to (8) minimizes the problem:

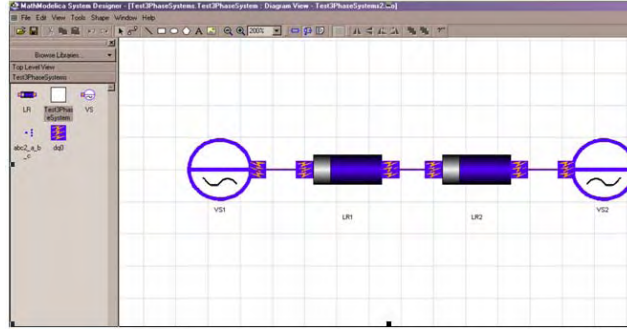
$$F(z_1, \dots, z_n) = \sum_{i=1}^m f_i(z_1, \dots, z_n)^2 \rightarrow \min \tag{9}$$

On the other hand, every global minimum of (9) is a solution to (8). In order to solve (9) a number of different algorithms have been developed during the past. The algorithm can be categorized depending on the order of derivatives needed during the solution process. In the OpenModelica environment the Simplex-method of Nelder and Mead as well as the Brent's method are currently implemented, only working with the minimization function  $F$ . The OpenModelica prototype already shows reliable results for the evaluated examples.

Further improvements can be achieved as soon as the Jacobian of  $F$  with regards to the unknown is available. In that case, more advanced algorithms like the method of Fletcher-Reeves, Quasi-Newton, and/or Levenberg-Marquardt methods can be applied which would provide a speed-up in convergence. We regard this as a quality of implementation, since the described approach is working in principle already.

## 7 Test and Evaluation with OpenModelica

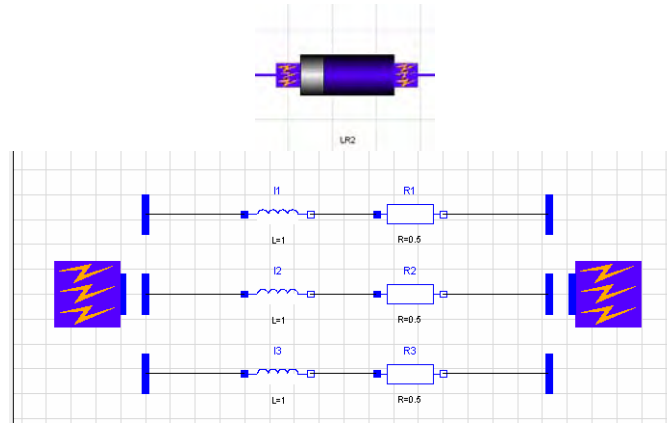
Consider the following electrical 3-phase power system, where two generating units  $vs1$  and  $vs2$  are connected via a transmission line modeled by components  $LR1$  and  $LR2$ .



**Fig. 1.** An electrical power system where two generating units  $vs1$  and  $vs2$  are connected via a transmission line.

The connectors are written in  $dq0$ -coordinates implementing the potential variable  $u_{dq0}$  and the flow variable  $i_{dq0}$ . These quantities are constant in case of a nondistributed steady state, which is generally assumed during the initialization process. Introducing the Park-Transformation  $P$  the 3-phase rotating system (voltages  $u_{abc}$  and currents  $i_{abc}$ ) can be calculated from the  $dq0$ -representation and vice versa.

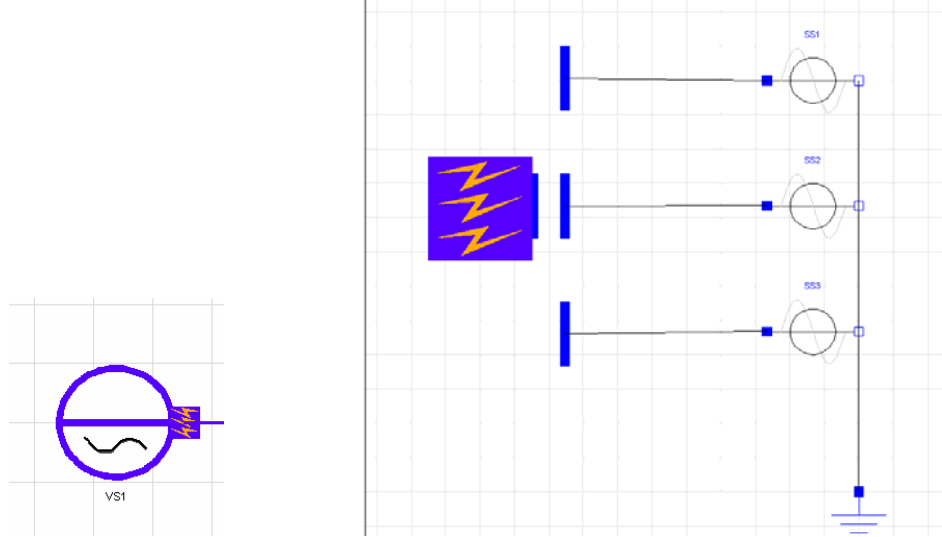
The transmission line ( $LR1$  and  $LR2$ ) is modeled by a purely inductive and resistive component, based on the Modelica Electrical Library. Since  $LR1$  and  $LR2$  are connected in series, giving a higher index system, index reduction has to be applied for simulation purposes.



**Fig. 2.**  $LR2$  component with  $dq0$  connectors.



The voltage source is described similarly using the Modelica Standard Library combined with the dq0-connectors.



**Fig. 3.** Voltage source.

In order to initialize the model correctly to steady state the following initial equations have been added to the local components LR1 and LR2.

```

model LR
  ...
equation
  ...
  initial equation
    der(dq0_1.i_dq0)={0,0,0};
  end LR;

```

Due to the higher-index of the overall system, index-reduction is applied. The system finally is determined by 3 state variables  $LR1.I1.i$ ,  $LR1.I2.i$ ,  $LR1.I3.i$ . The corresponding initial equation system has 3 equations more than number of unknowns, but these equations are redundant and could be eliminated. Due to the involvement of the Park-transformation, redundancy is not easy to detect. However, applying the concept described above correct initialization of the system is performed.

## 8 Conclusions and Future work

In this paper we have presented an overview of our implementation of initializing Modelica models in the OpenModelica compiler. A new concept has been developed to describe the initial equations locally in the relevant component where the corresponding states appear, that also works for arbitrary well-posed higher-index problems. Due to the necessary index reduction some of the states get changed to dummy states that means that they will be algebraic during the simulation of the model. The

corresponding initial equations are therefore redundant, but can be handled correctly by the new initialization process, if they are consistent. If not, an error/warning is issued to the user.

The described method has been implemented in the OpenModelica compiler. In the future we also wish to implement calculation of the Jacobian matrix of the equation system with regards to the state variables. This gives the possibility to implement more advanced and robust numerical algorithms in order to solve the corresponding optimization (minimization) problem during initialization of the DAE.

## 9 Acknowledgements

This work was supported by the University of Applied Sciences in Bielefeld, by MathCore Engineering AB, by the Swedish Research Council (VR), and by SSF in the VISIMOD project.

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