LUND UNIVERSITY
Campus Helsingborg

# A piecewise regression model for identifying the strategic inflection point for organizational change

**Debora D'Avino, Pasquale Erto**
**PhD Student, Professor**
**Department of Aeronautical Engineering, University of Naples Federico II, Italy**

**d.davino@unina.it, ertopa@unina.it**

**Keywords:** *Organizational change, Piecewise regression, S-curve, Strategic inflection point*

**Category:** *Research paper*

## 1 INTRODUCTION

Nowadays, organizational change has become a mandatory condition to survive in the marketplace for all companies. In fact, a business needs to be able to adapt to changes of its external environment in order to remain competitive (Black and Crumley, 1997). In particular, since the late 1970s a number of change drivers, *i.e.* new technologies, new knowledge, new customer preferences, the deregulation of several industries and the increased globalization of trade, have caused an acceleration of environmental change (Dervitsiotis, 2003). Consequently, it was found that in 1991, between 60-70 per cent of the *Fortune* 100 largest global companies did not exist at all or in any form similar to what they were like in 1970 (Stockport, 2000). Therefore, many business school academics tried to identify the reasons of these failures. A typical example of the examined case studies is the history of the disk drive industry, where changes in technology, market structure, global scope and vertical integration have been very pervasive, rapid and unrelenting (Christensen, 1997). For this case, some scholars have attributed the high mortality rate to the unfathomable pace of technological change. On the contrary, a deeper study of the history of the disk drive industry revealed that the different impact of technological change were at the root of the leading firms' failures (Christensen, 1997). Two types of technological change were identified: the technological changes that *sustain* or reinforce established trajectories of product performance (*sustaining technologies*) and the technological changes that *disrupt* or redefine performance trajectories (*disrupting technologies*). The last ones result in the failure of the industry's leading firms.
In the same way, we can characterize two different environmental changes: *sustaining changes* and *disrupting changes*. Therefore, when a *sustaining change* occurs, an organization can survive in the marketplace with minor adjustments or adaptation.

Instead, when a *disrupting change* occurs, an organization has to face with a larger strategic transformation.

## 2 THE STRATEGIC INFLECTION POINT

If we assume a human organization as a living system, its lifecycle will follow similar patterns to that of the biological cycle for which the S-curve models were used to study populations growth and diffusion phenomena over time (Stockport, 2000; Dervitsiotis, 2005). Therefore, the growth of the organization quality performance can be conformed to a S-curve pattern as a function of time which starts when a business model is adopted. The curve's shape makes it easy to see that organization productivity will begin to decrease after the strategic inflection point ("point I"), that is the point on the curve where the arc changes from concave to convex (Fig. 1). At that point, management should begin a strategic transformation because further efforts in the old business model will result in diminishing returns (Asthana, 1995). Therefore, a key management task is to monitor the position of the company on its S-curve and identify the strategic inflection point for jumping from the present curve to a new performance curve, based on a more suitable new business model for the emerging conditions (Fig. 2). In this way, the organization can strategically transform itself before it starts to wither. In fact, "jumping the curve" must always be made while the organization has the ability and "slack" to transform itself. At "point I" there is still a momentum from continuing good results of past successful products and markets, which provides the "slack" of required resources to make an effective transition. On the contrary, if the leadership of a successful company is too slow to recognize a strategic inflection point and reacts too late, a much needed internal transformation may be too difficult or impossible to make, and decline follows (Dervitsiotis, 2003).

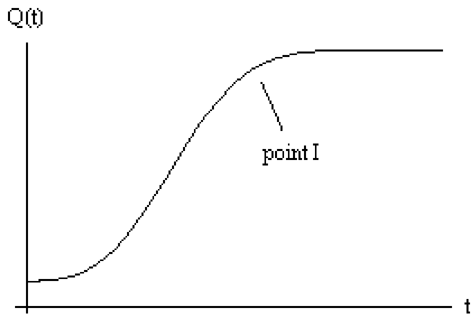


Figure 1: The strategic inflection point


Figure 2: Jumping the S-curve

## 3 A PIECEWISE REGRESSION MODEL

It is standard practice to approximate a regression curve by a single model over the entire range of time, $t$, which is relevant to the problem. However, in order to identify the strategic inflection point, is more useful to approximate the regression curve by a sequence of sub-models, each joined to its neighbours at the end points of the relevant parts of the range of $t$ (*piecewise regression model*). It is relatively simple to fit such a piecewise model if it is known in advance where the join points are. On the

contrary, we deal with the more difficult case where the join points themselves have to be estimated from the data.
Therefore, we propose the following piecewise regression model of the quality performance level $Q(t)$ as a function of the time $t$. The model is very adaptive having six *shape* parameters, $Q_0$, $Q_{\lim}$, $a_1$, $a_2$, $a_3$ and $a_4$, four *time* parameters, $T_0$, $T_1$, $T_2$ and $T_3$, and only one explanatory variable, $t$:

$$\begin{cases} Q_1(t) = Q_0 e^{a_1(t-T_0)} & \text{for} \quad T_0 \le t \le T_1 \\ Q_2(t) = a_2 t + a_3 & \text{for} \quad T_1 \le t \le T_2 \\ Q_3(t) = Q_{\lim} - \dfrac{a_4}{t} & \text{for} \quad T_2 \le t \le T_3 \end{cases}$$

with $a_1 > 0$, $a_2 > 0$, $a_4 > 0$. $Q_0$, $Q_{\lim}$, $T_0$ and $T_3$ are given constants, while $T_1$ and $T_2$ are unknown and are to be estimated as well as the $\{a_i\}_{i=1,\dots,4}$.
The above model is composed of three sub-models: an exponential curve ($Q_1(t)$), that approximates the emergence or birth phase of the new business model; a straight line ($Q_2(t)$), that approximates its growth phase, and a branch of hyperbola ($Q_3(t)$), that approximates its maturity phase (Fig. 3). $T_0$ and $Q_0$ are the initial time and the corresponding quality performance level; $Q_{\lim}$ is the asymptotic value of the quality performance level or the maximum benefit reachable through the adopted business model; $T_1$ and $T_2$ are the join points, respectively between the exponential model and the straight line and between the straight line and the branch of hyperbola, or the times when the next phase starts; $T_3$ is the last sampled time. The most important parameter is $T_2$, that is the strategic inflection point. In addition, it is straightforward to show that:

$$\lim_{t \to T_0} Q_1(t) = Q_0 \qquad \lim_{t \to +\infty} Q_2(t) = Q_{\lim}$$

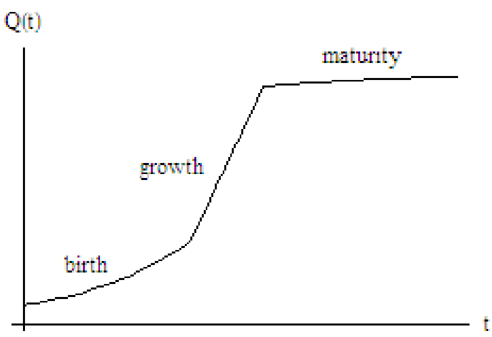

Figure 3: Organizational life cycle

## 4 LEAST SQUARE ESTIMATORS FOR PIECEWISE REGRESSION

The method of least squares (LS) is used throughout this paper. As anticipated, the number of the parameters to be estimated is six: two time parameters ($T_1$ and $T_2$) and four shape parameters ($\{a_i\}_{i=1,\dots,4}$). An important constraint is that the overall model is continuous at each join point, that is:

$$\begin{cases} Q_1(T_1) = Q_2(T_1) \\ Q_2(T_2) = Q_3(T_2) \end{cases} \quad \Rightarrow \quad \begin{cases} Q_0 e^{a_1(T_1 - T_0)} = a_2 T_1 + a_3 \\ a_2 T_2 + a_3 = Q_{\lim} - \dfrac{a_4}{T_2} \end{cases}$$

Therefore, the parameters $T_i$ and $a_i$ are not independent. Moreover, if the join points, $T_i$, are known, the equations will simply be linear constraints on the unknown shape parameters, $a_i$; while when $T_i$ are unknown, the equations will be nonlinear constraints on $T_i$ and $a_i$ (Hudson, 1966). On the contrary, no restriction is placed on the slopes of the adjacent curves.

In order to estimate the unknown parameters, we have to minimize the following residual sum of squares (RSS):

$$RSS(a_i, T_j, S, V) = \sum_{k=1}^{S} \left(Q_k - Q_1(t_k; a_1)\right)^2 + \sum_{k=S+1}^{V} \left(Q_k - Q_2(t_k; a_2, a_3)\right)^2 + \\ + \sum_{k=V+1}^{T_3} \left(Q_k - Q_3(t_k; a_4)\right)^2$$

with $i = 1, \dots, 4$; $j = 1, 2$; $k = 1, \dots, n$ ($n$ is the sample size) and $Q_k$ being the quality performance level corresponding to time $t_k$. The function is subject to the following constraints in addition to the constraints among parameters:

$$t_S \le T_1 \le t_{S+1} \qquad t_V \le T_2 \le t_{V+1}$$

where $S$ and $V$ are integer numbers which count the points that fall in each of the three phases.

For the complexity of the function , the solution requires an iterative technique. In fact, the proposed analytic solution led to intractable algebra (Hudson, 1966). Therefore, the recommended procedure is to search solutions for each relevant value of $S$ and $V$ (that is, for each relevant interval of values for $T_1$ and $T_2$) and, then, to choose the critical value respectively of $T_1$ and $T_2$ for which RSS is minimized (Hudson, 1966).

## 4.1 Obtaining the relevant intervals of values for $T_1$ and $T_2$

At first glance, achieving the convergence of the proposed iterative technique seems long and difficult. Instead, a simple procedure for obtaining the relevant intervals of values for $T_1$ and $T_2$, for which the solution has to be searched, makes it very fast. The procedure is to obtain a set of the parameters estimates by a simplified model composed of straight lines only. Then, the bootstrap confidence intervals (Efron and Tibshirani, 1994) of the obtained estimates are able to work as effective "initial" intervals to start the iterative procedure successfully.

Aiming to test the above procedure, we simulated a dataset with sample size 25 from the following S-curve regression model (Erto, 1997):

$$Q(t) = Q_0 + (1 - e^{-k(t-T_0)^s})(Q_{\lim} - Q_0) + \varepsilon$$

where $\varepsilon$ is a random variable generated to be stochastically independent and identically normally distributed with zero mean and constant variance $\sigma^2$ $((0.25)^2)$.

Subsequently, we estimated $B = 100$ bootstrap replications of the parameters by a model composed of three straight lines (this replications number was considered sufficient for our current aim). So, we obtained the empirical sampling distribution of $\hat{T}_1$ and $\hat{T}_2$ and constructed the confidence intervals based on bootstrap percentiles for both $\hat{T}_1$ and $\hat{T}_2$ (Fig. 4).

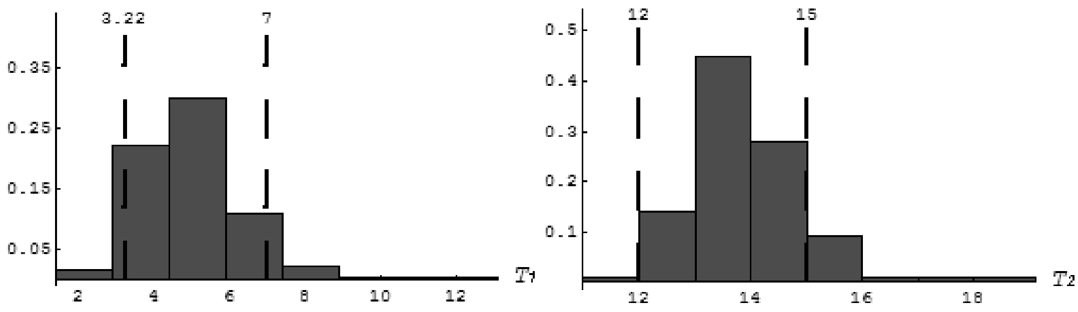


Figure 4: Confidence intervals based on bootstrap percentiles for both $\hat{T}_1$ and $\hat{T}_2$

From Fig. 4, we can see that, if $B = 100$ and $\alpha = 0.05$, the 90% confidence intervals based on bootstrap percentiles are:

$$I_1 = \left(\hat{T}_1^{(0.05)}, \hat{T}_1^{(0.95)}\right) = \left(3.22, 7.00\right) \text{ and } I_2 = \left(\hat{T}_2^{(0.05)}, \hat{T}_2^{(0.95)}\right) = \left(12.0, 15.0\right)$$

We advise cautiously to round down the lower limit, $\hat{T}_{lower}\left(\hat{T}^{(\alpha)}\right)$, and to round up the upper limit, $\hat{T}_{upper}\left(\hat{T}^{(1-\alpha)}\right)$, obtaining the following intervals of values for $T_1$ and $T_2$:

$$t_3 \leq T_1 \leq t_7 \text{ and } t_{12} \leq T_2 \leq t_{15}$$

being, in our simulated dataset, $t_k = 1, \ldots, 25$.

Therefore, in order to minimize the function $RSS$, we could search solutions for the following values of $S$ and $V$:

$$S = 3, \ldots, 7 \text{ and } V = 12, \ldots, 15.$$

In this way, the critical values of $T_1$ and $T_2$ for which RSS is minimized, were obtained without computational difficulties:

$$\hat{T}_1 = 5.535 \text{ and } \hat{T}_2 = 13.46$$

being $\hat{T}_2$ the strategic inflection point estimate.

## 5 CONFIDENCE INTERVALS FOR JOIN POINTS

It is very useful to supplement the above point estimates of $T_1$ and $T_2$ by a confidence interval. Some authors studied the difficult problem of finding confidence limits for the above join points finding any suitable solution for our case (Robinson, 1964; Hudson, 1966). For this purpose, we examined the sampling properties of the join points LS estimators by a simulation study. Data were generated using a set of predetermined values of the parameters of the model . An error term, $\varepsilon$ , was added to each sub-model:

$$\begin{cases} Q_1(t) = Q_0 e^{a_1(t-T_0)} + \varepsilon & for \quad T_0 \le t \le T_1 \\ Q_2(t) = a_2 t + a_3 + \varepsilon & for \quad T_1 \le t \le T_2 \\ Q_3(t) = Q_{\lim} - \dfrac{a_4}{t} + \varepsilon & for \quad T_2 \le t \le T_3 \end{cases}$$

As before, $\varepsilon$ was generated to be stochastically independent and identically normally distributed with zero mean and constant variance $\sigma^2$ $((0.25)^2)$. By this means, 500 sets of simulated data were produced, each of which provided a set of the parameters LS estimates (Ratkowsky, 1983). Moreover, the predetermined values of the parameters were assumed as the "true" values of them. In particular, the assumed "true" values of $T_1$ and $T_2$ were respectively 5 and 14. Consecutively, we present the histograms of both 500 $\hat{T}_1$ and 500 $\hat{T}_2$ estimates (Fig. 5):

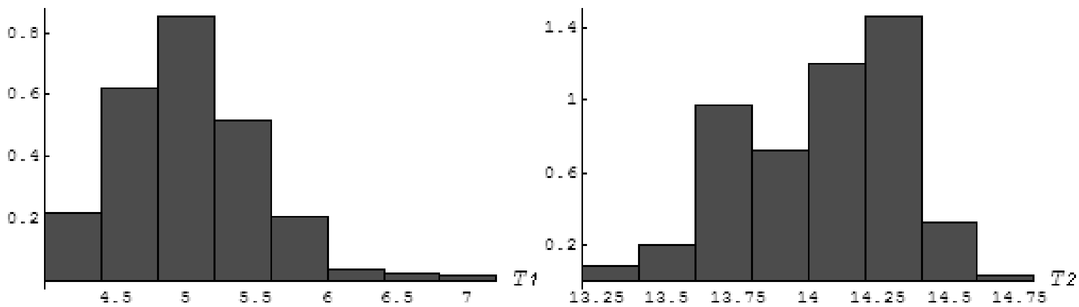


Figure 5: Histograms of simulated estimates of both $\hat{T}_1$ and $\hat{T}_2$

From Fig. 5, we can see that the observed sets of the parameters estimates are not normally distributed. In fact, the coefficients of skewness and kurtosis were compared with their expected values under sampling from a normal distribution (0 and 3 respectively) (Ratkowsky, 1983). From the results of the hypothesis tests, it was clear that the distribution of both $\hat{T}_1$ and $\hat{T}_2$ is not close to normal. Therefore, classical confidence intervals are not applicable. So, we used the bootstrap method again in order to construct a confidence interval of the obtained estimates for both $\hat{T}_1$ and $\hat{T}_2$. Unfortunately, for our current aim, we needed greater accuracy than that one provided by the percentile method used before. Then, we implemented an improved version of the percentile method called $BC_a$, the abbreviation standing for *bias-corrected and accelerated* (Efron and Tibshirani, 1994).

We found the following 90% $BC_a$ confidence interval for the strategic inflection point estimates $\hat{T}_2$ (Fig. 6):

$$I_{BC_a} = \left(\hat{T}_2^{(\alpha_1)}, \hat{T}_2^{(\alpha_2)}\right) = (12.0, 15.2)$$

being $\hat{T}_2 = 13.46$ the point estimate of the strategic inflection point previously obtained.

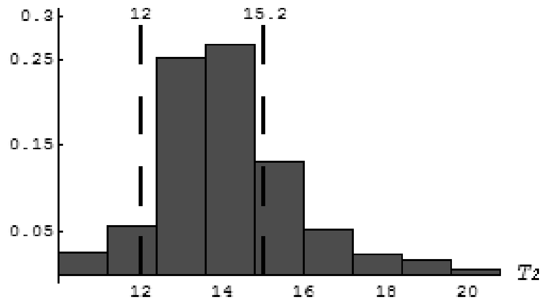


Fig. 6: $BC_a$ confidence intervals for $\hat{T}_2$

Moreover, we report the following estimates of the bias of the join points LS estimators, expressed as a percentage of the true value of the parameters:

$$\%Bias_1 = \frac{\sum_{i=1}^{n} \hat{T}_1 - 5}{5} = 0.0053 \quad \text{and} \quad \%Bias_2 = \frac{\sum_{i=1}^{n} \hat{T}_2 - 14}{14} = 0.00297$$

These bias values indicate that join points LS estimators tend to overestimate the parameters.

## 6 CONCLUSIONS

Organizational change has been the subject of many papers. The greater part of these consider case studies and highlight the important factors that affect a successful transformation. Differently from them, this paper intends to be operative by providing the managers with an effective statistical methodology to face the most important task: identifying the strategic inflection point. Besides, in this way, management will become aware timely of a critical environment change (disrupting change particularly) that may pose a threat to the organization's future success.

## REFERENCES

Asthana, P., (1995), "Jumping the technology S-curve", *IEEE Spectrum*, June, pp. 49-54

Black, S. A., Crumley, H. C., (1997), "Self-assessment: What's in it for us?", *Total Quality Management*, vol. 8, no. 2&3, pp. S90-S93.

Christensen, C. M., (1997), *The Innovator's Dilemma*, Harvard Business School Press

Dervitsiotis, K. N., (2003), "The pursuit of sustainable business excellence: guiding transformation for effective organizational change", *Total Quality Management*, vol. 14, no. 3, pp. 251-267

Dervitsiotis, K. N., (2005), "Creating conditions to nourish sustainable organizational excellence", *Total Quality Management*, vol. 16, no. 8, pp. 925-943

Efron, B., Tibshirani, R. J., (1994), *An introduction to the bootstrap*, FL: CRC Press, Boca Raton

Erto, P., (1997), "Modelling of the technological innovation process", *Conf. ISO 9000 Series 9000 Standards, Statistical Methods, Problems of Certification*, Nizhny Novgorod, Russia

Hudson, D. J., (1966), "Fitting segmented curves whose join points have to be estimated", *Journal of the American Statistical Association*, vol. 61, no. 316, pp. 1097, 1129

Ratwosky, D. A., (1983), *Nonlinear regression modeling. A unified practical approach*, Marcel Dekker, Inc., New York

Robinson, D. E., (1964), "Estimates for the points of intersection of two polynomial regressions", *Journal of the American Statistical Association*, vol. 59, no. 305, pp. 214-224

Stockport, G. J., (2000), "Developing skills in strategic transformation", *European Journal of Innovation Management*, vol. 3, no. 1, pp. 45-52