

Modal Models in Ptolemy*

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Abstract

Ptolemy is an open-source and extensible modeling and simulation framework. It offers heterogeneous modeling capabilities by allowing different models of computation to be composed hierarchically in an arbitrary fashion. This paper describes modal models, which allow to hierarchically compose finite-state machines with other models of computation, both untimed and timed. The semantics of modal models in Ptolemy are defined in a modular manner.

Keywords Hierarchy, State machines, Modes, Heterogeneity, Modularity, Modeling, Semantics, Simulation, Cyber-physical systems.

1. Introduction

Cyber-physical systems (CPS) consist of digital computers interacting among themselves and with physical processes. CPS applications are emerging at high rates today in many domains, including energy, environment, health-care, transportation, etc.

Designing CPS is a non-trivial task, as these systems manifest non-trivial dynamics, complex interactions, dynamic behavior, and a large number of components. The

design complexity is increased by the inherent *heterogeneity* in modeling such systems: parts of the system are digital, others are analog; parts are timed, others untimed; parts are discrete-time, others are continuous-time; parts are synchronous, others are asynchronous; and so on. This inherent heterogeneity implies a need for *heterogeneous modeling*. By the latter we mean a method and associated tools, that provide designers with a way of combining different *models of computation*, in an unambiguous way, in a single model of a system. A model of computation (MoC) here refers to a language or class of languages with a common syntax and semantics. Different MoCs realize different modeling paradigms, each being more or less suitable for capturing different parts of the system.¹

A number of modeling languages exist today, realizing different MoCs. Many of these languages are gaining acceptance in the industry, in so-called *model-based design* methodologies. Examples are UML/SysML, Matlab/Simulink/Stateflow, AADL, Modelica, LabVIEW, and others. These types of languages are raising the level of abstraction in designing CPS, by offering mechanisms to capture concurrency, interaction, and time behavior, all of which are essential concepts in CPS. Moreover, verification and code generation tools exist for many of these languages, allowing to go beyond simple modeling and simulation, and facilitating the process of going from high-level models to low-level implementations.

Despite these advances, however, no universally accepted solution exists for heterogeneous modeling. In fact, integration of modeling languages and tools is still a common theme in many research or industrial projects, as well as products (e.g., co-simulation environments) despite the fact that such solutions are often cumbersome to use and unsatisfactory, at best.

One of the longest efforts attacking the heterogeneous modeling problem is the Ptolemy project [15, 25]. Ptolemy follows the *actor-oriented* paradigm, where a system consists of a set of *actors*, which can be seen as processes executing concurrently and communicating using some mechanism. In Ptolemy, the exact manner in which actors execute (e.g., by interleaving, in lock-step, or in some other or-

*Ptolemy in this document refers to Ptolemy II, see <http://ptolemy.eecs.berkeley.edu>. **NOTE:** If you are reading this document on screen (vs. on paper) and you have a network connection, then you can click on the figures showing Ptolemy models to execute and experiment with those models on line. There is no need to pre-install Ptolemy or any other software. The models that are provided online are summarized at <http://ptolemy.eecs.berkeley.edu/ptolemyII/ptII8.0/jnlp-books/doc/books/design/modal/index.htm>.

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¹ We should emphasize the importance of syntax, in addition to semantics, in choosing a MoC. State machines, for example, can be given a discrete-time semantics, and so can a synchronous language such as Lustre [19]. Even though the two have the same semantics, their syntax (in the broad sense) is very different, which makes them suitable for different classes of applications.

der) and the exact manner in which they communicate (e.g., through message passing or shared variables) are not fixed once and for all: they are defined by an MoC. The implementation of an MoC in Ptolemy is called a *domain* and is realized by a *director*, which semantically is a composition operator. Currently, Ptolemy includes a number of different domains and corresponding directors, including, *finite-state machines* (FSM), *synchronous data flow* (SDF) [26], *synchronous reactive* (SR) [11, 19, 14], and *discrete event* (DE) [7, 33, 9, 37, 23, 30, 8].

Among the important characteristics of Ptolemy is that (a) it is open-source (and free); and (b) it is architected to be easily extensible. Thanks to these features, new domains and new actors are being added to the tool by different groups, depending on their specific interests.

Another essential feature of Ptolemy is that domains can be combined hierarchically, in an arbitrary fashion. For example, the model in Figure 1 combines SDF with hierarchical FSMs; the model in Figure 3 combines DE with FSMs. This is the fundamental mechanism that Ptolemy provides to deal with the heterogeneous modeling problem. It allows designers to build models where different parts are described in different MoCs, in a well-structured manner [15].

In this paper, we are particularly interested in one aspect of heterogeneous modeling in Ptolemy, namely, *modal models*. Modal models are hierarchical models where the top level model consists of an FSM, the states of which are *refined* into other models (possibly from different domains). Modal models are suitable for a number of applications. They are especially useful in describing event-driven and modal behavior, where the system’s operation changes dynamically by switching among a finite set of modes. Such changes may be triggered by user inputs, sensor data, hardware failures, or other types of events, and are essential in fault management, adaptivity, and reconfigurability (see, for instance, [36, 35]). A modal model is an explicit representation of this type of behaviors and the rules that govern transitions between behaviors.

The main contribution of this paper is to provide a formal semantics of Ptolemy modal models. In the process, we also give a modular and formal framework for Ptolemy in general, which is an additional contribution. We do not formalize all the domains of Ptolemy, however, as this is beyond the scope of this work.

The paper is organized as follows. Section 2 briefly reviews the visual syntax of Ptolemy through an example. In Section 3 we provide a formalization of the abstract semantics of Ptolemy. In Section 4 we provide the formal semantics of Ptolemy modal models. In Section 5 we discuss possible alternatives and justify our choices. Section 6 discusses related work. Section 7 concludes the paper.

2. Visual Syntax

Ptolemy models are hierarchical. They can be built using a *visual syntax*, an example of which is given in Figure 1. This example contains, at the top level of the hierarchy, a model with five actors, `Temperature Model`,

`Bernoulli`, `ModalModel`, `SequencePlotter` and `SequencePlotter2`. The SDF domain is used at this level, as indicated by the use of `SDF Director`, explained below. `Temperature Model` and `ModalModel` are *composite actors*: they are *refined* into other models, at a lower level of the hierarchy.

The refinement of `ModalModel` is an FSM with two *locations*,² `normal` and `faulty`. This FSM is hierarchical: each of its locations is refined into a new FSM, as shown in the figure. In Ptolemy, FSMs use implicitly the FSM domain. This is why no director is shown in the FSM models. The FSM director is implied. Note that, although in this example the location refinements are FSMs, this need not be the case: they can be models using any of the Ptolemy domains (e.g., see example of Figure 3).

The refinement of `Temperature Model` is shown in Figure 2. This refinement does not specify a domain (it contains no director). In such a case, the refinement uses implicitly the same domain as its parent, that is, in this case, the SDF domain. Since this model mixes SDF and FSM, it is an example of a heterogeneous model.

The visual syntax of Ptolemy contains other elements, which we briefly describe next. For details, the reader is referred to [27, 24] and the Ptolemy documentation [1]. Each actor contains a set of *ports*, used for communication with other actors. Ports are explicitly shown in the internal model of a composite actor: for instance, `fault` is an input port and `heat` is an output port of `ModalModel`. A port may be an input, an output, both, or neither. *Parameters* can also be defined: for instance, `heatingRate` is a parameter of the top-level model of Figure 1, set initially to 0.1 (the value of parameters can be modified dynamically during execution).

FSMs in Ptolemy consist of a finite set of locations, one of which is the initial location, and some of which may be labeled as final locations. Initial locations are indicated by a bold outline; the initial locations of the FSMs in Figure 1 are `normal` and `heating`. A transition links a source location to a destination location. A transition is annotated with a *guard*, a number of *output actions* and a number of *set actions*. Guards are expressions written in the Ptolemy expression language. Actions are written in the Ptolemy action language. Guards of two or more outgoing transitions of the same location need not be disjoint, in which case the FSM is non-deterministic. The user can indicate this, in which case transitions are visually rendered in red. *Default* transitions, indicated with dashed lines, are to be taken when no other transitions are enabled, i.e., their guard is the negation of the disjunction of the guards of all other transitions outgoing from the same source location. *Reset* transitions, indicated with open arrowheads, result in the refinement of the destination state being reset to its initial condition. *Preemptive* transitions, indicated by a red circle at the start of the transition, may prevent the execution of the current state refinement, when the guard evaluates to true.

² For the visual syntax, we use the term *location* instead of *state*, in order to distinguish it from the semantical concept of state (Section 3).

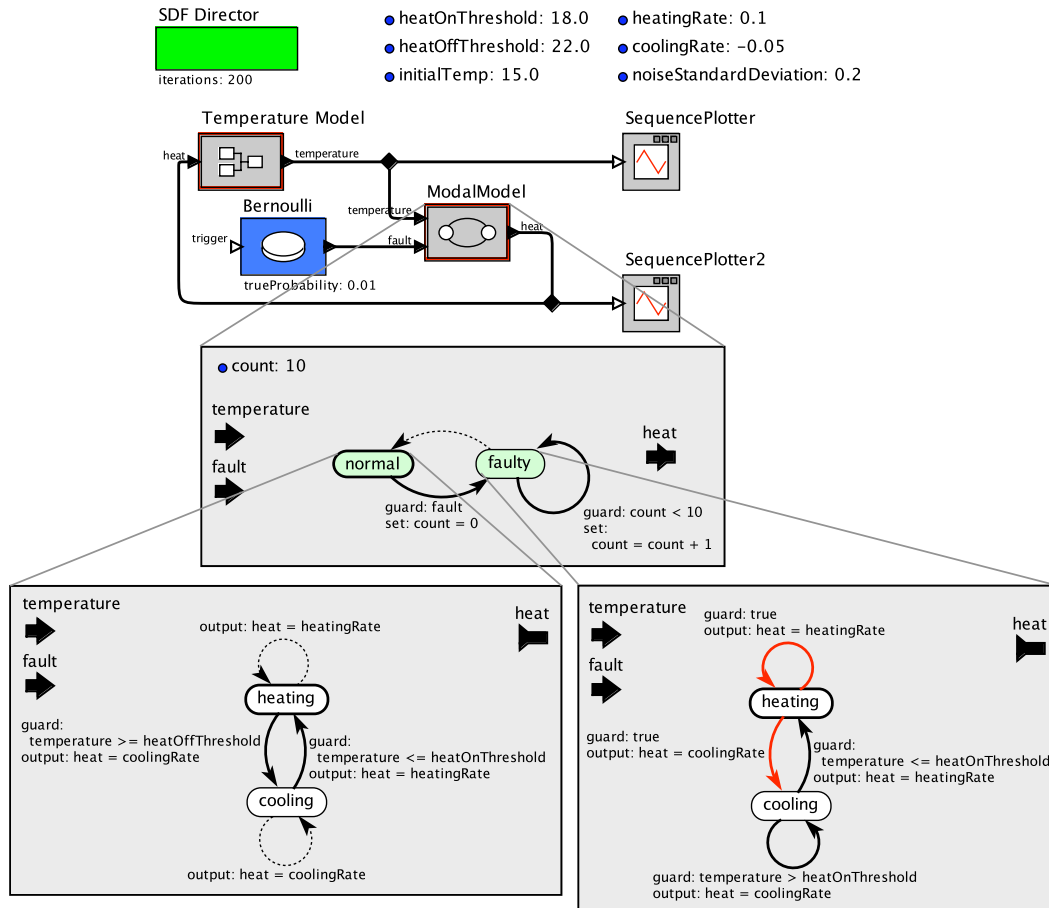


Figure 1. A hierarchical Ptolemy model.

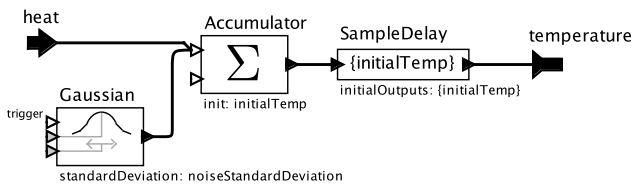


Figure 2. The Temperature Model composite actor of Figure 1.

3. Ptolemy Abstract Semantics

The semantics and execution of a Ptolemy model is defined by means of so-called *abstract semantics*. The same mechanism is used to ensure compositionality of Ptolemy domains. Mathematically, a Ptolemy model can be viewed as an abstract state machine, with a set of states, inputs and outputs. The abstract semantics defines the transitions of this machine, that is, how its state and outputs evolve according to the inputs.

Evolution can be seen as being *untimed*, that is, a sequence of transitions, or *timed*, that is, a sequence of transitions annotated by some timing information (e.g., a time delay since the previous transition). It is interesting to note that time is mostly external to the definition of the abstract state machine, that is, the times in which transitions are taken are primarily decided by the environment of the machine. However, *timed*, or *proactive*, machines can also be

defined, by providing means to impose constraints on these times. We do this using *timers* (see Section 3.1.2). In the absence of any such constraints, the machine is *untimed*, or *reactive*.

From an implementation point of view, the abstract semantics is essentially a set of methods (in the object-oriented programming sense) that every actor in a model implements. Composite actors also implement these methods, through their director. By implementing these methods in different ways, the various types of Ptolemy directors realize different models of computation. In the Java implementation of Ptolemy the abstract semantic methods form a Java interface that actor and director classes implement. This interface includes the following methods:³

- `initialize`: it defines the initial state of the machine.
- `fire`: it computes the outputs at a given point in time, based on the inputs and state at that point in time.
- `postfire`: it updates the state.

A formalization of the abstract semantics of Ptolemy is provided next.

³ For the purposes of this discussion, we omit some methods, e.g., `prefire`, etc. We also ignore implementation of communication between actors. See [1] for details.

3.1 A formalization of abstract semantics of actors

3.1.1 Untimed actors

An untimed actor is formalized as a tuple

$$(S, s_0, I, O, F, P).$$

The actor has a set of states S , a set of input values I , and a set of output values O . The `initialize` method of the actor is formalized as an initial state $s_0 \in S$. The `fire` and `postfire` methods of the actor are formalized as two functions F and P , respectively, of the following type:

$$\begin{aligned} F &: S \times I \times \mathbb{N} \rightarrow O \\ P &: S \times I \times \mathbb{N} \rightarrow S \end{aligned}$$

That is, F returns an output value $y \in O$, given a state $s \in S$, an input value $x \in I$, and an *index* $j \in \mathbb{N}$; P returns a new state, given a state, an input value and an index. The index is used to model non-determinism, and can be seen as a “dummy” input.⁴

We require that F and P be total in S and I . F and P may be partial in \mathbb{N} (i.e., in their index argument), however, we require that for any $s \in S$ and $x \in I$, there exists at least one $j \in \mathbb{N}$ such that both $F(s, x, j)$ and $P(s, x, j)$ are defined. If there is a unique such j for all $s \in S$ and $x \in I$ then the actor is *deterministic*. In that case we omit index j and write simply $F(s, x)$ and $P(s, x)$.

The semantics of an untimed actor can be then defined as a set of sequences of *transitions*, of the form:

$$s_0 \xrightarrow{x_0, y_0} s_1 \xrightarrow{x_1, y_1} s_2 \xrightarrow{x_2, y_2} \dots$$

such that for all $i = 0, 1, \dots$, we have $s_i \in S$, $x_i \in I$, $y_i \in O$, and there exists $j \in \mathbb{N}$ such that

$$y_i = F(s_i, x_i, j) \quad (1)$$

$$s_{i+1} = P(s_i, x_i, j) \quad (2)$$

Note that, exactly what the sets S, I and O are, and exactly how the functions F and P are defined, is a property of a given actor: it is in this sense that this semantics is *abstract*. Different actors will have different instantiations of this abstract semantics. Also note that the above elements essentially define a non-deterministic Mealy machine, where F is the *output function* of the machine, and P the *state update function*. This machine is not necessarily finite-state. The input and output domains may also be infinite.

Examples of untimed actors A simple untimed actor is the `Gain` actor which produces at its output a value $k \cdot x$ for every value x appearing at its input. k is a parameter of the actor. This actor is deterministic. It has a single state, and therefore a trivial (constant) update function P . Its F function is defined simply by: $F(x) = k \cdot x$.

⁴ Since $j \in \mathbb{N}$, we can model unbounded, but enumerable, non-determinism. The reader may wonder why we do not model the pair F, P simply as a single function with type $S \times I \rightarrow 2^{O \times S}$, that is, taking a state and an input and returning a set of state, output pairs. The reason is that we want to decouple output and update functions, which allows to give semantics of modal models in a modular manner: see Section 4.2. Once the F and P functions have been decoupled, it is necessary for some book-keeping in order to keep track of non-deterministic choices, that must be consistent among the two functions. This role is played by the index $j \in \mathbb{N}$.

3.1.2 Timed actors

The semantics of timed actors extend those of untimed actors with time. In particular, timed actors have special state variables, called *timers*, that measure time. Our timers are *dense-time* variables (they take values in the set of non-negative reals, $\mathbb{R}_{\geq 0}$) inspired by the model of [13]. A difference with [13] is that in our case timers can be created or destroyed dynamically, and the set of timers that are active at any given time is not necessarily bounded. Also, our timers can be *suspended* and *resumed*, which is not the case with the timers of [13]. Timers are *set* to some initial value when they are created, and then run downwards (i.e., decrease as time elapses) until they reach zero, at which point they *expire*. A timer can be *suspended* which means it is “frozen” and ceases to decrease with time. It can then be *resumed*. Suspended timers are also called *inactive*, otherwise they are *active*.

In the case of timed actors, the sets I and O often contain the special value ε denoting *absence* of a signal at the corresponding point in time. We will use this value in the examples that follow.

Consider a timed actor with fire and postfire functions F and P . The semantics of this actor can be defined as a set of sequences of *timed transitions* of the form:

$$s_0 \xrightarrow{x_0, y_0, d_0} s_1 \xrightarrow{x_1, y_1, d_1} s_2 \xrightarrow{x_2, y_2, d_2} \dots$$

such that there exists a sequence of indices $j_0, j_1, \dots \in \mathbb{N}$, and for all $i = 0, 1, \dots$, we have $s_i \in S$, $x_i \in I$, $y_i \in O$, $d_i \in T$, and

$$y_i = F(s_i, x_i, j) \quad (3)$$

$$s_{i+1} = P(s_i \ominus d_i, x_i, j_i) \quad (4)$$

$$d_i \leq \min\{c \mid c \text{ an active timer in } s_i\} \quad (5)$$

$d_i \in \mathbb{R}_{\geq 0}$ denotes the time elapsed at state s_i . $s_i \ominus d_i$ denotes the operation which consists in decrementing all active timers in s_i by d_i . Condition (5) ensures that this operation will not result in some timers becoming negative, i.e., that no timer expiration is “missed”. This condition therefore “forces” the environment of the actor to fire the actor at least at those instants when its timers are set to expire. Note that the actor could also be fired at other instants as well, for example, whenever an external event is received. The actor itself does not, and cannot, specify those other instants, because they are generally context-dependent.

Notice that, even though the above semantics does not explicitly mention suspensions and resumptions of timers, these actions can be easily modeled as part of the inputs x_i . In Ptolemy, these inputs are not accessible to the user, however, only to the director. This is particularly the case for hierarchical modal models, as described in Section 4.2.

Superdense time: It is worth noting that the delays d_i can be zero. This implies in particular that multiple output events can occur at the same real-time instant. It is convenient to model such cases using so-called *superdense* time, i.e., the set $\mathbb{R}_{\geq 0} \times \mathbb{N}$ [32, 28, 31]. Then, an output y can be seen as a signal with a superdense time axis, that is, as

a partial function from $\mathbb{R}_{\geq 0} \times \mathbb{N}$ to a set of values. For instance, in a run of the form

$$s_0 \xrightarrow{x_0, y_0, d_0} s_1 \xrightarrow{x_1, y_1, d_1} s_2 \xrightarrow{x_2, y_2, d_2} \dots$$

where $d_0 = d_1 = 0$ and $d_2 > 0$, the output signal y can be seen as a function on superdense time, such that $y(0, 0) = y_0$, $y(0, 1) = y_1$, $y(d_2, 0) = y_2$, and so on.

Examples of timed actors

First, let us consider a `DiscreteClock` actor that periodically emits some value v at its output, with period $\pi \in \mathbb{R}_{>0}$. Both v and π are parameters of the actor. The `DiscreteClock` actor has no inputs. It has a single state variable which is a timer c . Initially, $c = 0$ (as an option, the user can also set initially $c = \pi$, in which case the actor will not produce an event until after one period). The F and P functions of `DiscreteClock` are defined below (the actor is deterministic, so we omit reference to index j):

$$\begin{aligned} F(c) &= v \text{ if } c = 0, \text{ and } \epsilon \text{ if } c > 0 \\ P(c) &= \pi \text{ if } c = 0, \text{ and } c \text{ if } c > 0 \end{aligned}$$

The definition of F states that an output with value v is produced when the timer c reaches zero, otherwise, the output is absent. The definition of P states that the timer is reset to π when it reaches zero and is left unchanged otherwise.

Next, let us consider the `ConstantDelay` actor, which, for every input with value x that it receives at time t , it produces an output with value x at time $t + \Delta$, where $\Delta \in \mathbb{R}_{>0}$, is a parameter of the actor. As state variables, `ConstantDelay` maintains a set of *active* timers C plus, for each $c \in C$, a variable v_c to memorize the value that must be produced when c expires. Initially C is empty. A new timer c is added to C whenever an input is received: at that point, c is set to Δ and v_c is set to x , the value of the input. When a timer c expires it is removed from C and output with value v_c is produced. Formally, a state s of `ConstantDelay` is a set of triples of the form (c, δ_c, v_c) , where c is a timer, $\delta_c \in \mathbb{R}_{\geq 0}$ is the current value of c , and v_c is as explained above. The initial state is $s_0 = \emptyset$. The F and P functions of `ConstantDelay` can be defined as follows (again we omit j because of determinism):

$$\begin{aligned} F(s, x) &= \begin{cases} v_c & \text{if } \exists(c, 0, v_c) \in s \\ \epsilon & \text{otherwise} \end{cases} \\ P(s, x) &= \begin{cases} (s \setminus \{(c, 0, v_c)\}) \cup Q & \text{if } \exists(c, 0, v_c) \in s \\ s \cup Q & \text{otherwise} \end{cases} \\ Q &= \begin{cases} \{(c', \Delta, x)\} & \text{if } x \neq \epsilon \text{ and } \nexists(c', \delta_{c'}, v_{c'}) \in s \\ \emptyset & \text{otherwise} \end{cases} \end{aligned}$$

Note that a “fresh” timer c' is created only when the input x is not absent, as defined by Q .

3.1.3 Untimed actors as a special case of timed actors

As expected, an untimed actor can be seen as a special case of a timed actor, with no timers. Because of this, untimed actors can also be given semantics in terms of sequences of timed transitions. In this case, Condition (4) reduces to

Condition (2), and Condition (5) is trivially satisfied with the convention that the minimum of an empty set is infinity. This means that the time instants when untimed actors are fired are entirely determined by the context in which these actors are embedded.

3.2 Composite actors

As illustrated in Section 2, Ptolemy allows to build hierarchical models, by encapsulating a set of actors, plus a director, within a composite actor. The latter is itself an actor, thus can be further encapsulated to create new composite actors. Models of arbitrary hierarchy depths can be built this way.

A composite actor C has an abstract semantics just like any actor. How this abstract semantics is instantiated depends on: (a) the instantiation of the abstract semantics of the internal actors of C ; and (b) the director that C uses.

Directors can be viewed formally as *composition operators*: they define functions F and P of a composite actor C , given defined such functions for all internal actors of C .

A large number of directors are included in Ptolemy, implementing various models of computation. It is beyond the scope of this document to formalize all these directors. We informally describe two of them, namely, *SR (synchronous reactive)* and *DE (discrete event)*. More information can be found in [23, 14, 29, 30, 8]. In the next section, we formalize the semantics of the *FSM Director*. The latter implements modal models, which is the main topic of this paper.

Synchronous Reactive (SR): Every time a composite actor C with an SR director is fired, the SR director repeatedly fires all actors within C until a *fixpoint* is reached. This fixpoint assigns values to all ports of actors of C . Note that, because of interconnections between actors, some output ports are typically connected to input ports of other actors of C , and therefore obtain equal values in the fixpoint. The fixpoint is defined with respect to a *flat CPO*, namely, the one that has a bottom element \perp representing an “undefined” or “unknown” value, and all other, “true” values, greater than \perp in the CPO order (see [14]). The fixpoint is computed by assigning initially \perp to all outputs, and then iterating in a given order the F functions of all actors of C . Any execution order can be used and is guaranteed to reach the fixpoint, although some execution orders may be more efficient (i.e., may converge faster). When the fixpoint is reached, the `fire()` method of the SR director (and consequently, of C) returns.⁵ The `postfire()` method P of C is implemented by invoking the P methods of all internal actors of C .

Discrete Event (DE): DE leverages the SR semantics, but extends it with time. (see [23, 8]). As is typical with DE simulators, the DE director maintains an *event queue* that stores events in timestamp order. Initially, the event queue is empty. When actors are initialized, some of them may post initial events to the event queue. Whenever the composite actor is fired, the earliest events are extracted from

⁵ The fixpoint may contain \perp values, which means the model contains feedback loops with *causality cycles*. In this case, the Ptolemy implementation returns a Java exception.

the event queue and presented to the actors that receive them. In contrast to standard DE simulators, Ptolemy incorporates the SR semantics for processing simultaneous events. In particular, a fixpoint is computed, starting with the extracted events at the specified ports, and \perp values for all other, unknown, ports. Fire() returns when the fixpoint is found, as in the SR case. Postfire() consists in calling postfire() of internal actors, as in the SR case. During postfire(), actors may post new events to the queue.

4. Modal Model Semantics

A modal model M is a special kind of composite actor. In the visual syntax, M is defined by a finite-state machine M_c whose locations can be *refined* into sub-models, as illustrated in Figure 1. In Ptolemy terminology, M_c is called the *controller* of M . Each of these sub-models is itself a composite actor. Therefore, the internal actors of M are the composite actors that refine the locations of M_c , plus M_c itself. Note that a special case of modal model is an FSM actor: this is a modal model whose controller has no refinements. Another special case of modal model is a hierarchical state machine: this is a modal model whose refinements are FSM actors or are themselves hierarchical state machines.

In this section, we describe the semantics of modal models, starting with the simple case of FSM actors, and extending to the general case of modal models.

4.1 Semantics of FSM actors

FSM actors are untimed actors. For a given FSM actor M , its set of states is the set of all possible *valuations* of the *state variables* of M . The set of state variables includes all parameters of M (which in Ptolemy can be changed dynamically) as well as a state variable to record the current location of M . A valuation is a function assigning a value to every state variable. The initial state assigns to each parameter its default value (specified by the user) and to the location variable the initial location (also specified by the user). M may also have inputs and outputs, defined in Ptolemy's visual syntax by input and output ports that the user specifies, as in Figure 1.

A state s and an input x of M , together with the transitions of M , define a finite set s_1, s_2, \dots, s_k of *successor states*, as follows. Let l be the location of M at s and consider an outgoing transition from l . If the guard of this transition is satisfied by s and x we say that the transition is *enabled*. Suppose there are $k \geq 1$ enabled transitions. Enabled transition j defines successor state s_j . In particular, if the destination location of the transition is l_j , then the location at s_j is set to l_j . Moreover, any parameters that are set in the set action of the transition are assigned a corresponding new value at s_j , and the rest of the parameters remain unchanged (i.e., have the same value at s_j as at s). Therefore, this defines function P on s and x , as follows: $P(s, x, j) = s_j$, for $j = 1, \dots, k$. If there are no enabled transitions at s and x , then there is a unique successor state, namely s itself, and we define $P(s, x, j) = s$, only for $j = 1$.

The output actions of the enabled transitions define a set y_1, y_2, \dots, y_k of output values, therefore, they define function F on s and x , as follows: $F(s, x, j) = y_j$, for $j = 1, \dots, k$. The output values are the values that the output action assigns to output ports of the actor. If an output port is not mentioned in the output action, or if no transitions are enabled (and therefore no output actions are executed) then the value of this port is ϵ , i.e., “absent”.

4.2 Semantics of general modal models

A general modal model M consists of its controller M_c , which is an FSM actor with n locations, l_1, \dots, l_n , plus a set of composite actors M_1, \dots, M_n , where M_i is the refinement of location l_i . Some locations may have no refinement: this is handled as explained below. Without loss of generality, we assume that the *initial location* of M_c is l_1 (a controller has a single initial location). We also denote by S^c the set of locations: $S^c = \{l_1, \dots, l_n\}$.

We denote by S^i, s_0^i, F^i, P^i , respectively, the set of states, initial state, fire and postfire functions of M_i . As explained in Section 3.1.3, untimed actors are special cases of timed actors, therefore, without loss of generality, we can assume that all composite actors M_i are timed. Then, denote by C^i the set of timers of M_i .

In addition, without loss of generality, we can assume that every location has a refinement. Indeed, if location l_i has no refinement, then the above elements can be defined trivially: S^i as a singleton set (i.e., containing a single state which is also the initial state), F^i as the identity function from inputs to outputs, and P^i as the constant function, since the state is unique.

In a modal model M , the sets I and O of input and output values are the same for all internal actors of M , namely, M_c, M_1, \dots, M_n , and the same for M as well.

The set of states S of M is the cartesian product of the sets of states of all internal actors of M , and similarly for the sets of initial states, i.e.:

$$\begin{aligned} S &= S^c \times S^1 \times \dots \times S^n \\ s_0 &= (l_1, s_0^1, \dots, s_0^n) \end{aligned}$$

Although timers are just a special kind of state variables, it is convenient to be able to refer to them specifically. Therefore, we define C to be the set of timers of M , as

$$C = \bigcup_{i=1, \dots, n} C^i$$

In s_0 , all timers except those in C^1 are set to their suspended state. Those in C^1 are set to their active state.

It remains to define functions F and P of M . Consider a state $s \in S$ and an input $x \in I$. Let $s = (s_c, s_1, \dots, s_n)$ be the vector of component states of M_c, M_1, \dots, M_n , respectively. Suppose the location of M_c at s_c is l_i . Let $J \subseteq \mathbb{N}$ be the set of indices j for which $F^i(s_i, x, j)$ and $P^i(s_i, x, j)$ are defined. We distinguish cases:

1. There are no outgoing transitions of M_c from location l_i that are enabled at s and x . Then, for $j \in J$, we define $F(s, x, j) = F^i(s_i, x, j)$, $P(s, x, j) = (s_c, s'_1, \dots, s'_n)$, where:

- (a) $s'_i = P^i(s_i, x, j)$;
- (b) for all $m = 1, \dots, n$ with $m \neq i$, we have $s'_m = s_m$.
2. There exist $k \geq 1$ preemptive outgoing transitions from l_i that are enabled at s and x . Suppose, without loss of generality, that the j -th such transition goes from location l_i to location l_j , for $j = 1, \dots, k$, and denote its output action and set action by α_j and β_j , respectively. Then, for $j = 1, \dots, k$, we define $F(s, x, j) = y_j$ and $P(s, x, j) = (s'_c, s'_1, \dots, s'_n)$, where:
- (a) y_j is obtained from α_j , as in the FSM actor semantics;
- (b) s'_c is obtained from l_j and β_j as in the FSM actor semantics;
- (c) if the j -th transition is not a reset transition then s'_j is identical to s_j , except that all suspended timers in C^j are resumed; if the j -th transition is a reset transition then s'_j is the initial state of M_j : $s'_j = s_0^j$; (note that the timers of M_j , if any, are also re-initialized in the case of a reset transition);
- (d) s'_i is identical to s_i , except that all timers in C^i are suspended;
- (e) for all $m = 1, \dots, n$ with $m \neq j$ and $m \neq i$, we have $s'_m = s_m$.
3. There are no preemptive outgoing transitions from l_i that are enabled at s and x , but there exist $k \geq 1$ non-preemptive outgoing transitions from l_i that are enabled at s and x . Let $j_1 = 1, \dots, k$ and suppose that the j_1 -th such transition goes from l_i to l_{j_1} and has output and set actions α_{j_1} and β_{j_1} . Let j_2 range in J . Then, for $j = j_1 \cdot j_2$, we define $F(s, x, j) = y_j$ and $P(s, x, j) = (s'_c, s'_1, \dots, s'_n)$, where:
- (a) y_j is obtained by applying the output action α_{j_1} to $F^i(s_i, x, j_2)$, that is, to the output produced by M_i for non-determinism index j_2 ;
- (b) s'_c is obtained as in Case 2b.
- (c) s'_j is obtained as in Case 2c.
- (d) s'_i is obtained by applying the set action β_{j_1} to $P^i(s_i, x, j_2)$ and suspending all timers in C^i ;
- (e) for all $m = 1, \dots, n$ with $m \neq j$ and $m \neq i$, we have $s'_m = s_m$.

Item 1 treats the case where no transition of the controller is enabled: in this case, the modal model M behaves (i.e., fires and postfires) like its current refinement M_i . Item 2 treats the case where preemptive transitions of the controller are enabled, possibly in addition to non-preemptive transitions. In this case the preemptive transitions preempt the firing and postfiring of M_i , and only the outputs produced by the transition of the controller can be emitted. Item 3 treats the case where only non-preemptive transitions of the controller are enabled. In this case, before choosing and taking such a transition non-deterministically, we must fire (again, non-deterministically in general) the current refinement M_i .

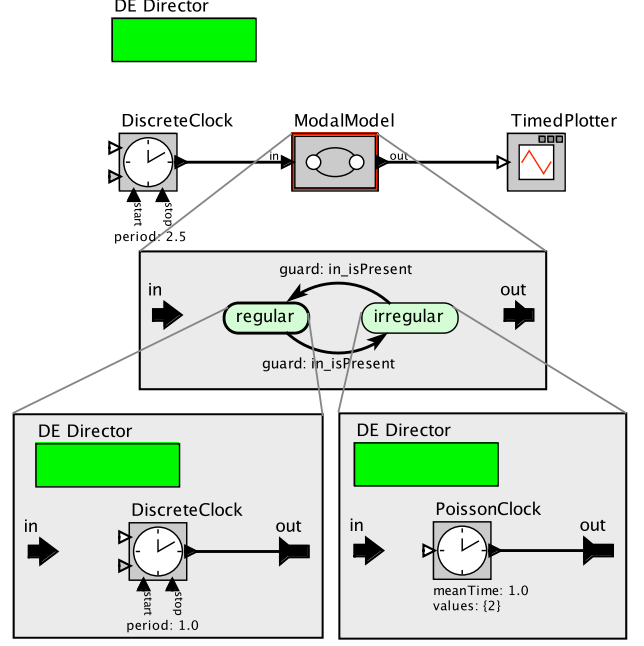


Figure 3. A Ptolemy model with a timed modal model.

Examples As a first example, consider the `ModalModel` actor of Figure 1. The controller of `ModalModel` is the automaton with locations labeled `normal` and `faulty`. The refinements of both these locations are FSM actors. As all refinements are untimed, `ModalModel` is also untimed. The refinement of `faulty` is a non-deterministic FSM actor, as the outgoing transitions of its `heating` location have both guard `true`. The state variables of `ModalModel` are the location variables of all FSM actors, plus the `count` parameter (the other parameters, such as `heatingRate`, etc., should in principle also be included in the state; however, they can be omitted since they remain invariant). A sample of the values that the F and P functions of `ModalModel` take is given below (because of determinism, the index parameter j is omitted):

$$\begin{aligned}
 F((normal, heating, cooling, 10), (22, \overline{fault})) &= -0.05 \\
 P((normal, heating, cooling, 10), (22, \overline{fault})) &= \\
 &\quad (normal, cooling, cooling, 10) \\
 F((normal, heating, cooling, 10), (22, fault)) &= -0.05 \\
 P((normal, heating, cooling, 10), (22, fault)) &= \\
 &\quad (faulty, cooling, heating, 0)
 \end{aligned}$$

The first two equations correspond to Case 1 whereas the last two equations correspond to Case 3. No preemptive or reset transitions exist in this model.

Another example, that illustrates timed modal models, is shown in Figure 3. This model switches between two modes every 2.5 time units. In the `regular` mode it generates a regularly-spaced clock signal with period 1.0 (and with value 1, the default output value for `DiscreteClock`). In the `irregular` mode, it generates pseudo-randomly spaced events using a `PoissonClock` actor with a mean time between events set to 1.0 and value set to 2. The result of a typical run is plotted in Figure 4, with a shaded

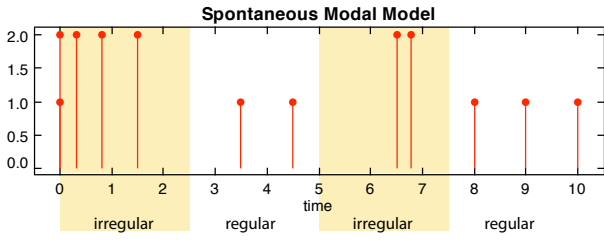


Figure 4. A plot of the output from one run of the model in Figure 3.

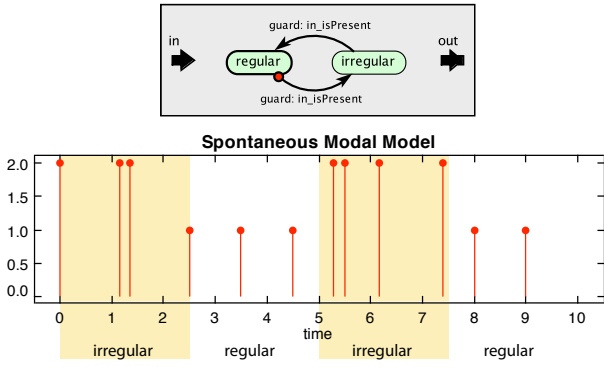


Figure 5. A variant of Figure 3 where a preemptive transition prevents the initial firing of the innermost `DiscreteClock` actor of that model.

background showing the times over which it is in the two modes. A number of observations worth making arise from this plot.

First, note that two events are generated at time 0, a first event with value 1, at superdense time $(0,0)$, and a second event with value 2, at superdense time $(0,1)$. The first event is produced by `DiscreteClock`, according to the semantic rules of Case 3a. If we had instead used a preemptive transition, as shown in Figure 5, then that first output event would not appear: this is according to the semantic rules of Case 2a and the fact that the action of the preemptive transition does not refer to the output port.

The second event is produced by `PoissonClock`, according to the semantic rules of Case 1. The reason for this second event is the following. When the model is initialized, a timer is set by `PoissonClock` to value zero: this means that this timer is to expire immediately, i.e., `PoissonClock` will produce an output immediately when it starts, and at random intervals thereafter.⁶ When the `irregular` state is entered, this timer is resumed and since it has value 0, is ready to expire. This forces a new firing of `ModalModel` and ultimately of `PoissonClock`, which produces the event at superdense time $(0,1)$.

Another interesting observation concerns the output events with value 1 occurring at times 3.5, 4.5, 8, and so on. These events occur at times during which the model is at the `regular` mode. Notice that the model begins in the `regular` mode but spends zero time there, since it

⁶ This is the default operation, which can be optionally modified by the user by setting the appropriate parameter of the `PoissonClock` actor.

immediately transitions to the `irregular` mode. Hence, at time 0, the `regular` mode becomes inactive and the timer of `DiscreteClock` is suspended. Since no time has elapsed yet, the timer is equal to 1, the value of the period, at this time. When `regular` is re-entered at time 2.5, this timer is resumed, and expires one time unit later, i.e., at time 3.5. This explains the event at that time. Moreover, the timer is reset to 1 during `postfire()`, according to Case 1a. It expires again 1 time unit later, which explains the event at time 4.5. Finally, it is reset to 1 at time 4.5, suspended at time 5, and resumed at time 7.5, which explains the event at time 8.

The above examples may appear rather artificial, however, they are given mainly for purposes of illustration of the semantics. More interesting and realistic examples can be found in the open-source distribution of Ptolemy available from <http://ptolemy.eecs.berkeley.edu/>. Detailed descriptions of some of these examples can be found in other publications of the Ptolemy project. For modal models in particular, we refer the reader to the case studies described in [8].

5. Alternative Modal Model Patterns

It is instructive to briefly discuss alternative definitions of modal model semantics and justify our choices.

First, consider our design choice to have the refinement M_i of location l_i in a modal model M “freeze” while the controller automaton is in a location different from l_i . “Freezing” here means that M_i is inactive, in terms of its state which does not evolve at all. This includes in particular the timers of M_i , which are suspended until l_i is re-entered. An alternative would be to consider all refinements “live”, but to feed the inputs of M only to the currently “active” refinement, say M_i , and to use the outputs of M_i as outputs of M . Let us term this alternative as the “non-freezing” semantics, for the purposes of this discussion.

One issue with the non-freezing semantics is that it is redundant from a modeling point of view. Indeed, as we show next, there exists a simple design pattern that allows the non-freezing semantics to be easily implemented in Ptolemy. Since this mechanism already exists, there would be no need to add modal models to get the same semantics. In fact, using different modeling patterns that result in the same semantics may be confusing.

This design pattern, which we call the *switch-select pattern*, is illustrated in Figure 6. There are five actors in this model, `M1`, `M2`, `Controller`, `BooleanSwitch` and `BooleanSelect`. `M1` and `M2` represent the refinements of the non-freezing modal model that the pattern captures, and `Controller` is its controller (which is assumed to have 2 locations in this example). The switch and select actors control the routing of the inputs/outputs to/from either `M1` or `M2`, depending on the state that the `Controller` is in. The latter may in turn generally depend on outputs of these actors, which is captured by the communication links between `Controller`, `M1` and `M2`.

Another issue with the non-freezing semantics is that it is less modular than the freezing semantics. In the freez-

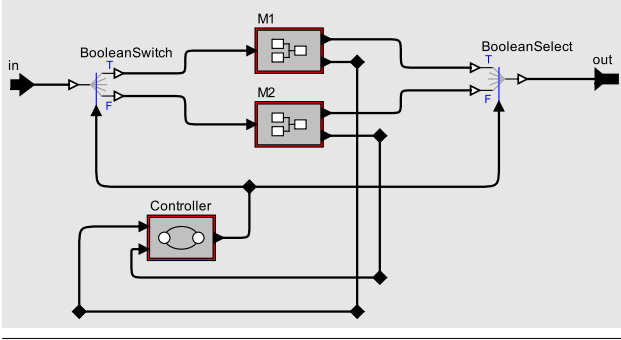


Figure 6. The switch-select pattern.

ing semantics, a subsystem (refinement of a certain location) is *completely unaffected* by being suspended. In the non-freezing semantics, behavior of a subsystem continues while the latter is inactive, only with absent inputs. Thus the evolution of the subsystem depends on how much time it remains inactive, for instance.

Another design choice could be to have time pass in inactive subsystems, i.e., to have their timers active, while having the rest of their state be frozen. The disadvantage of this approach is that for many components (e.g. `DiscreteClock`), the state is intrinsically bound to time. It is therefore hard to separate the two notions.

Finally, it is worth mentioning the approach taken in the Simulink/Stateflow tool from the Mathworks. Simulink is a hierarchical block diagram language. Some Simulink blocks can be Stateflow models, that is, hierarchical state machines similar to Statecharts [22]. Simulink blocks, however, cannot be embedded into Stateflow as state refinements. The way to get modal behavior in Simulink/Stateflow is by connecting Stateflow outputs to *enable* inputs of Simulink blocks. When a block is disabled, it is frozen, as in the Ptolemy semantics. Contrary to Ptolemy, however, the output of a disabled block can still be used as it still exists: it is simply held constant while time passes.

6. Related work

A number of formalisms based on hierarchical state machines (HSMS) have been studied in the literature, including Statecharts [22], SyncCharts [5], and commercial variants such as Stateflow from the Mathworks or Safe State Machines from Esterel Technologies [6] (SSMs are based on SyncCharts). Hierarchical state machines are also one of the diagrams of UML. The main difference of Ptolemy modal models with respect to the above is that in Ptolemy modal model refinements are not restricted to state machines or concurrent state machines (built with AND states). In Ptolemy, refinements can include other domains as well, for instance, as in Figure 3. Note that AND states can still be modeled in Ptolemy, using concurrent `ModalModel` actors. For instance, the `TemperatureModel` and `ModalModel` actors shown in Figure 1 are concurrent: the `TemperatureModel` could very well be another modal model. Note that in this case, a MoC such as SR or DE must be specified, in order to define the semantics of the composition of these actors.

Even when we restrict our attention to pure HSMs with no concurrency, there are differences between the Ptolemy version and the models above. A variety of different semantics has been proposed for Statecharts for instance, see [10, 16]. Operational and denotational semantics for Stateflow are presented in [21, 20]. Implicit formal semantics of Stateflow by translation to Lustre are given in [34].

Also, contrary to Statecharts, SyncCharts and Stateflow, Ptolemy modal models do not use broadcast events for communication.⁷ Guards may refer to input events, however, these events are transmitted using explicit ports and connections, and are evaluated when the `fire()` or `postfire()` methods are called (e.g., guard `in_isPresent` in Figure 3 is evaluated to true or false depending on whether the value of the input is present or absent when the `fire()` method is called).

Another difference with the above languages is that Ptolemy modal models include both untimed (*reactive*) and timed (*proactive*) models. Timed versions of Statecharts and UML (but not general modal models) have been proposed in [12, 17].

The semantics we present are somewhat operational in nature, given by functions that produce outputs and update the state. Our semantics is also abstract, as in Abstract State Machines [18]. Most importantly, our semantics is modular, in the sense that we show how the output and state update functions of composite actors are defined given output and state update functions of sub-actors.

Formal studies of HSMs can be found in [3, 4, 2].

7. Conclusions

We presented a modular and formal framework for Ptolemy, and described the semantics of modal models, as these are implemented in Ptolemy. Modal models allow hierarchical composition of state machines with other MoCs, therefore generalizing hierarchical state machines and enriching heterogeneous modeling with modal behavior.

Existing Ptolemy models emphasize actor semantics, by having an explicit notion of inputs and outputs. This is in contrast to languages such as Modelica, which are based on undirected equations. Note that feedback loops are allowed in Ptolemy, and can be used to capture some form of equational constraints. How these loops are handled depends on the domain used. In the SR and DE domains, for instance, the equations are solved by fixpoint computations, as mentioned above. In the future we intend to study equational constraints in more depth, borrowing ideas from languages such as Modelica. One direction would be to implement a Modelica domain in Ptolemy, which would work by translating Modelica models into, essentially, CT models, and

⁷ It is worth pointing out that, although it is common to refer to communication in Statecharts as being “broadcast”, this is slightly misleading, since it implies that all processes receive all signals, which is not the case. A more accurate description is “name matching” since, in fact, only those processes that refer to the signal by name receive it. Name matching is as static as ports in Ptolemy, but is less modular (changing the name in one part of the model requires changing it at other places as well). It also requires more effort to identify the communication links between processes (e.g., when determining causality loops in a diagram).

then handle the latter using numerical solvers. This translation could benefit from the code generation framework available in Ptolemy [38].

Although our discussion in this paper focused on discrete-time modal models, Ptolemy currently supports continuous-time models as well, via the *CT domain*, which allows a number of numerical solvers to be expressed, including those that use backtracking [28, 29]. A formalization of CT using the framework developed in this paper is a topic of future work.

The semantics developed in this paper are operational. It would be interesting to study also a denotational semantics of modal models. The work reported in [20] could be beneficial in that context.

Acknowledgments

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