Optimal Layout for Wind Turbine Farms

Koby Attias[1,*], Shaul P. Ladany[1]

[1]*Ben- Gurion University, Beer-Sheva, Israel*
**Corresponding author. Tel: 972-52-5250815, Fax:972-77-2101037, E-mail:yattias@elta.co.il*

Abstract: A general discrete model was formulated for the expected Net Present Value (NPV) of the profit and for the expected yield of the investment (Internal Rate of Return - IRR) to be derived from rectangular grid shaped wind-turbine farms. The model considers the wind shade in the downwind direction and the effect of the wake behind the turbine, the joint wind-direction wind-velocity probability distribution, as well as the various relevant cost and revenue factors. It was assumed that the wind-turbines are identical and of equal heights, and are spaced equally along the axes of the rectangle, but not necessarily at the same equal distance at both axes. Using the model, the optimal layout that maximizes the expected NPV and/or IRR was derived numerically for a given data set, in stages, determining the optimal number of turbines in a row and the associated optimal distance in-between them, and also the optimal number of turbines in a column and the optimal distance in-between them. Sensitivity analysis has shown that minor changes in the parameters do not affect the selection of the optimal layout.

***Keywords**: wind energy, optimal layout, wind farms*

1. Introduction

The harvesting of wind energy is centuries old as manifested by the middle-age wind mills in Europe. Heier [1] describes how the use of wind energy to generate electricity started in 19th century, but only the oil crisis of October 1973 provided the strong impact. The understanding and use of wind energy has been investigated by Lindley et al.[2], and are summarized by Manwell et al. [3]and Burton et al. [4]. Plans for actions to increase the use of wind energy were introduced by Milborrow et al. [5] , and the success is evidenced by travelers in Denmark and North Western parts of Germany where thousands of wind-turbines have been installed in recent years. The performance of wind farms were evaluated by Haack [6], while the integration of wind power into general power systems is discussed by Ackermann [7]and Heier [1].

Obviously, the erection of multiple wind-turbines in windfarms necessitates the determination of their layout. Bossanyi et al.[8], has dealt with the issue of investigating the efficiency of different layouts and even for designing aerodynamically optimal layouts. However, they did not integrate the economic and financial optimization of windfarms with the aerodynamic aspects. Yet the methods of operational research and operations management determine such optimizations using the maximization of the expected profit as the objective function. For example, see Ladany [9] and [10] in which the optimal layout of urban gasoline-stations was determined.

Hence, the aim of this paper is to develop a model to determine economically optimal layouts for windfarms (i.e. the number of turbines and their setting), which include the aerodynamic interactions between the turbines, the various cost factors and the particular wind regime. Section 2 presents the model; Section 3 considers the aerodynamic interaction between the turbines; Section 4 describes the optimization procedure; Section 5 shows a numerical example; Section 6 offers the conclusions.

Searching through Tables 2 & 3, it is possible to detect solutions that provide the best combination of N_{PV}'s and I_r's, although each is less than its maximum value. For example, for a layout of I=4, J=6 (24 turbines), with the turbines separated with x=100 m and y=300 m, provides an N_{PV}=$21.5 million, and an I_r=19.7%, which is the authors recommend "optimal solution."

1.1. Aim of this paper

The aim of this paper is to explore and searching a model. By Using the model, we got the optimal layout that maximizes the expected NPV and/or IRR was derived numerically for a given data set, in stages,

2. The model

Consider a rectangular grid layout (see Figure 1) of I x J wind-turbines of equal size and height, 2 adjacent turbines separated by the distance x in one direction and y in the other direction (obviously the minimum of x & y is more than the diameter of the turbine's rotor).

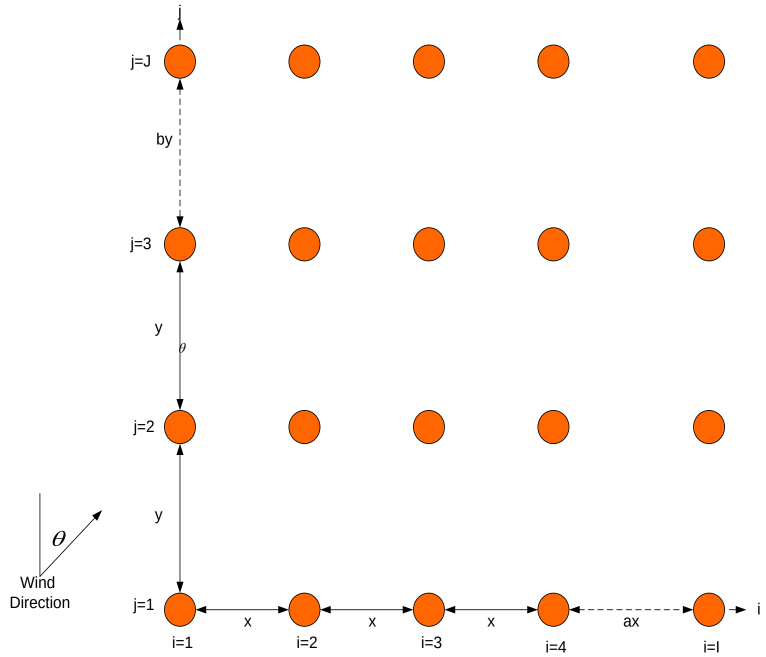


Figure 1: Location of turbine i, j in a rectangular grid layout of I x J turbines

When the wind blows in direction θ with a nominal velocity of V(θ), the effective wind velocity in front of the turbine at position i, j (which takes into account the aerodynamic interaction between the turbines, the wake affect – see the discussion in section 4) is Vij(θ, x, y).

Vij(θ, x, y) is obviously a function of V(θ), and it is developed in section 4. The effective wind velocity incident on 'turbine i, j' is Vij(θ, x, y), so the electrical power generated by the 'turbine i, j' is: eij(θ, x, y) [kwh]:

$$e_{ij}(\theta, x, y) = 0.5\rho V^3_{ij}(\theta, x, y) \cdot BC_p N_m \qquad (1)$$

where ρ is the air density [kg/m3], B is the swept rotor area [m2], Cp is the rotor efficiency coefficient (capacity factor) [%/100], Nm is the efficiency for converting the rotor mechanical power to electricity [%/100].

When the joint probability of the nominal wind velocity and its angle of incidence θ is defined as p (Vθ, θ), the expected annual energy to be generated by turbine i, j is

$$E(e \mid x, y)_{ij} = \sum_{v_\theta,\theta} e_{ij}(\theta, x, y) \cdot p(V_\theta, \theta) \qquad (2)$$

while the total expected energy to be generated by the whole farm, T(x, y), is

$$T(x, y) = \sum_{i=1}^{I}\sum_{j=1}^{J} E(e \mid x, y)_{ij} \qquad (3)$$

If the lifetime of a turbine is L, then K is the total investment in the windfarm (including the cost of turbines, installations and land cost), F is the net revenue from the selling electricity from the windfarm, r is the appropriate financial interest rate, H is the total operating time per period(for example operating time is: 24 h/d X 328.5 d/y =7884 h/y , and maintenance days are= 365-328.5=36.5d/y), P is the unit sale price of electricity , M is the cost of operation and maintenance of the windfarm per period, the Net Present Value, NPV, of the profit to be derived from the farm is

$$N_{pv}(x, y) = -K + \sum_{k=1}^{L} \frac{H \cdot T(x, y) \cdot P - M}{(1 + r)^{k-1}} = -K + \sum_{k=1}^{L} \frac{F}{(1 + r)^{k-1}} \qquad (4)$$

Where $H \cdot T(x, y) \cdot P - M$ =F
The Internal Rate of Return (IRR) on the investment, Ir (x, y) is the value of the interest rate, r, that results in
NPV (x, y) =0.

3. The aerodynamic interaction between the turbines

Since a wind-turbine generates electricity from the energy in the wind, the wind leaving the turbine has less energy content than the wind arriving in front of the turbine. Therefore a wind-turbine will always cast a wind shadow in the downwind direction. This is described as the wake behind the turbine, which is quite turbulent and has an average down-wind speed slower than the wind arriving in front of the turbine.

With effective yawing, we assume the direction of the wind is always perpendicular to the front of the turbine. Hence, the diameter of the turbine is considered always perpendicular to the wind's direction. Nybore [11] has shown that behind the turbine the wind creates a cone-sloped wake which extends 4.5º to the sides, as shown in Figure 2, (i.e. creating a truncated cone with a 9º head-angle.)

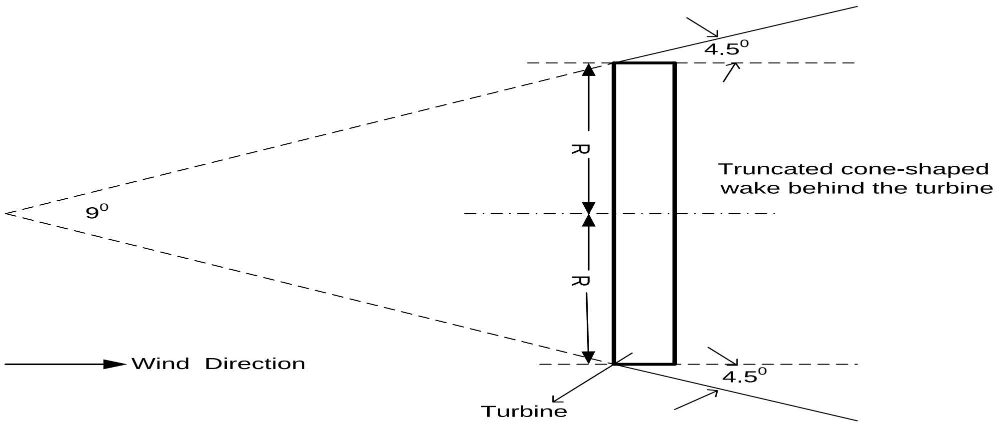


Figure 2: A 2-dimensional representation (top view) of the cone-shaped wake created by the wind behind the turbine

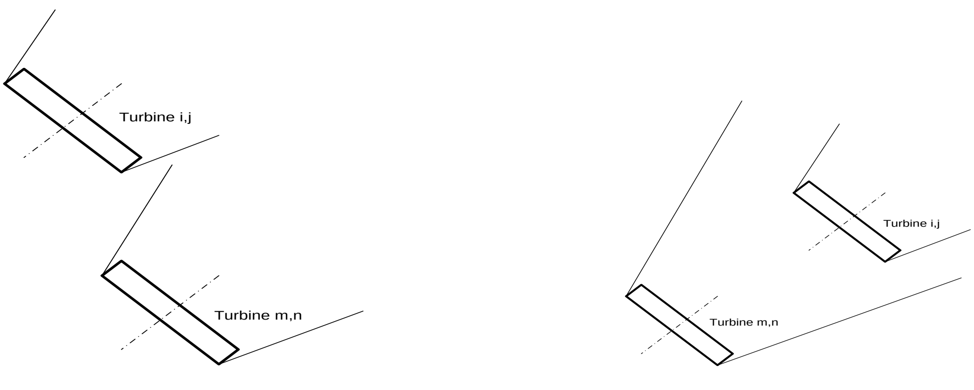
In a windfarm, 'turbine i, j' might or might not be affected by the wake created by another turbine positioned in front of it (in relation to the direction of the wind.) Moreover, the effect might be partial or complete. As a result, we distinguish 4 different states for the wind velocity hitting 'turbine i, j', as shown in Figure 3.

Adapting the findings of Nybore [11] to the notation required for dealing with a grid shaped farm, the wind velocity onto 'turbine i, j', when the general wind velocity in direction θ is V(θ), and the grids are separated by the distances x and y, is:

For state (a) when 'turbine i, j', is not affected by the wake of another turbine m, n ,

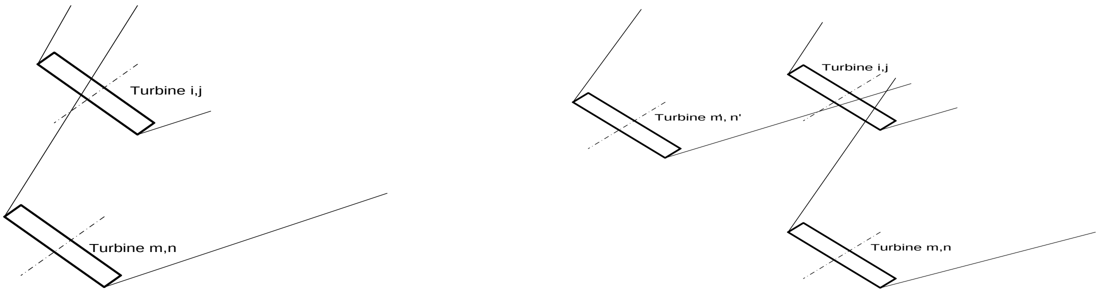
$$V_{ij}(\theta, x, y) = V(\theta) \tag{5}$$

Where m, n are the coordinates on the grid of another turbine in front of 'turbine i, j'. m might be less, equal or more than i, and likewise n might be less, equal or more than j, but (m, n) ≠ (i, j).



(a) Turbine i, j not affected by the wake of another turbine m, n

(b) Turbine i, j fully affected by the wake of another turbine m, n



(c) Portion of turbine i, j is affected by the wake of another turbine m, n

(d) Portion of turbine i, j is affected simultaneously by the wake of turbine m, n and of turbine m', n'

Figure 3: Top view of the 4 states in which turbine i, j can be affected by the wake

For state (b) when turbine i, j is fully affected by the wake of turbine m, n,

$$V_{ij}(\theta, x, y) = V(\theta) \cdot \{1 - [1 - \frac{V_{mn}(\theta, x, y)}{3V(\theta)}] \cdot [\frac{R}{R + 0.078d}]^2\} \tag{6}$$

where
R is the radius of the turbine's rotor, and
d is the distance between the centers of turbine i, j and turbine m, n (See fig. 4)
Note: This formula was developed by engineer Niels Otto Jensen from Risoe.

The wind in the wake $V_{ij}(\theta, x, y)$ is related to the surrounding free wind $V(\theta)$, the downwind distance (d meters), the rotor radius (R meters) and the spreading angle of the wind (about 4.5^0) . The factor 0.078 is called the constant of spreading, its corresponds to a spreading angle of about 4.5^0

For state (c) (when a portion (either a major or a minor portion) of turbine i, j is affected by the wake of another turbine m, n), Nybore's [11] equation is adjusted to the prevailing conditions:

$$V_{ij}(\theta, x, y) = V(\theta)\frac{A_{ij}}{\Pi R^2} \cdot \{1 - [1 - \frac{V_{mn}(\theta, x, y)}{3V(\theta)}] \cdot [\frac{R}{R + 0.078d}]^2\} + V(\theta)[\frac{\Pi R^2 - A_{ij}}{\Pi R^2}], \tag{7}$$

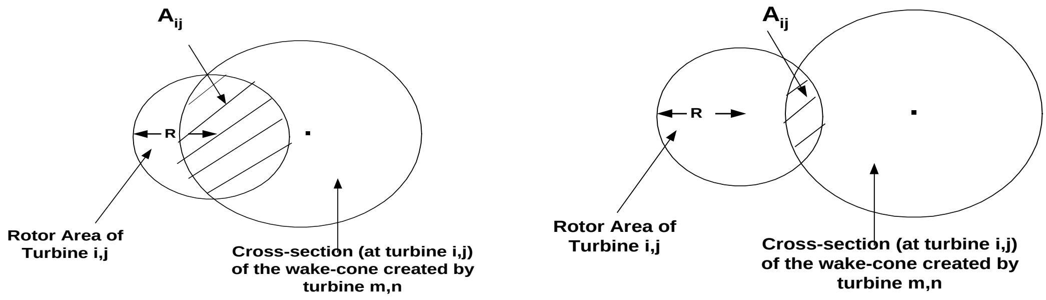
where
Aij is the area of intersection between the area of the rotor of turbine i, j and the cross-section (at turbine i, j) of the wake cone affected by turbine m, n.

For state (d) when a portion of turbine i, j is simultaneously affected by the wakes of turbine m, n and also of turbine m', n', Nybore's[11] equation is further adjusted:

$$\begin{aligned} V_{ij}(\theta, x, y) = {} & V(\theta)\frac{A_{ij}}{\Pi R^2} \cdot \{1 - [1 - \frac{V_{mn}(\theta, x, y)}{3V(\theta)}] \cdot [\frac{R}{R + 0.078d}]^2\} \\ & + V(\theta)\frac{A_{i'j'}}{\Pi R^2} \cdot \{1 - [1 - \frac{V_{m'n'}(\theta, x, y)}{3V(\theta)}] \cdot [\frac{R}{R + 0.078d'}]^2\} + V(\theta)[\frac{\Pi R^2 - A_{ij} - A_{i'j'}}{\Pi R^2}], \end{aligned} \tag{8}$$

where
$A_{i'j'}$ is the area of intersection between the area of the rotor of turbine i, j and the cross-section (at turbine i, j) of the wake cone affected by turbine m', n', and
d' is the distance between the centers of turbine i, j and turbine m', n'.
d is the distance between the centers of turbine i, j and turbine m, n.

For states (c) and (d), A_{ij} is depicted in Figure 4.



(a) Major portion of turbine i, j is affected (b) Minor part of turbine i, j is affected

Figure 4: Front view of the intersection (shaded) of the rotor area of turbine i, j, and of the cross-section (at turbine i, j) of the wake-cone affected by turbine m, n.

While the occurrence of state (d) in itself is very rare, theoretical situations exist in which in state (d) the cross-section (at turbine i, j) of the wake-cone created by turbine m, n and of turbine m', n' partially overlap while intersecting the rotor area of turbine i, j . Such other states can be dealt with by adjusting Nybore's [11] formula similarly to equation (8).

4. The optimization procedure

The optimization is performed in a sequential manner. An initial minimal sized layout with least possible number of turbines, (J=1, I=1) is selected and its NPV and Ir are calculated. At the second stage the number of turbines is increased, say to J=1, I=2, then numerically the value of x is searched that maximizes NPV, x1, and separately the value of x is searched that maximizes Ir, x2, retaining the resulting maximal NPV and Ir values. At the 3rd step, the number of turbines is further increased, say to J=2, I=2, the combination of values of x and y that maximize N_{PV}, (x_1, y_1), and that maximize IRR, (x_2, y_2), are evaluated, retaining the obtained maximal values of N_{PV} & I_r. The number of turbines is further increased, and the optimal outcomes of x & y for each layout are evaluated, and the corresponding maximal values of N_{PV} & I_r are retained.

5. Numerical example

To demonstrate the use of the model, a realistic set of data was assumed:
L= 20 y, R= 18 m, ρ = 1.225 kg/m^3, H= 7884 h/y (24 h/d X 328.5 d/y =7884 h/y),
P= 0.05 $/kWh, capacity of turbine=600kWh
C_p= 0.4, N_m= 0.95, and r=5% /y.
Total investment in the windfarm $K=I\cdot J\cdot\{C_T+C_I+C_L[(I-1)x\cdot(J-1)y]\}$, (9)
Where (cost in US dollar, $US)
C_T=cost per turbine=450,000$
C_I=cost per turbine installation=100,000$
C_L=cost of land per turbine=10$/m^2
Cost of operation&maintenance of the windfarm per turbine $M=0.015\cdot C_T\cdot I\cdot J$, $/y (10)

The results of the optimization procedure with the objective to maximize N_{pv} are presented in Table 2. The table provides for each combination of values of I x J the maximal achievable N_{pv}, and the values of x and y that generated this N_{pv}. In addition, the value of I_r generated by these values of x and y is also listed. T able 3 lists similar results, providing for each combination of I x J the maximal achievable I_r, and the values of x and y that generated this I_r. In addition, the value of N_{pv} generated by these values of x and y is also presented .

Table 2: Optimal results of x and y (for any given I x J) for maximal N_{PV}, and I_r

Parameter	**J- Turbines in Column**	**I-Number of Turbines in Row**				
		2	**3**	**4**	**5**	**6**
x (m)	**2**	300	300	300	300	300
y (m)		100	100	100	100	100
N_{PV} (M$)		4.6	6.5	8.5	10.4	12.4
I_r (%)		27.7	25.3	24.4	23.9	23.5
x (m)	**5**	100	100	100	100	100
y (m)		300	300	500	500	500
N_{PV} (M$)		10.6	14.3	18.8	21.9	25.6
I_r (%)		24.1	21.3	17.9	17.2	16.8
x (m)	**6**	100	100	100	100	100
y (m)		300	300	300	500	500
N_{PV} (M$)		12.6	17.1	21.5	25.9	*** 30.3**
I_r (%)		23.9	21.0	19.7	16.9	16.4

* Optimal results

Table 3: Optimal results of x and y (for any given I x J) for maximal I_r, and N_{PV}

Parameter	**J- Turbines in Column**	**I-Number of Turbines in Row**				
		2	**3**	**4**	**5**	**6**
x (m)	**2**	300	300	300	300	300
y (m)		100	100	100	100	100
N_{PV} (M$)		4.6	6.5	8.5	10.4	12.4
I_r (%)		*** 27.7**	25.3	24.4	23.9	23.5
x (m)	**5**	100	100	100	100	300
y (m)		300	300	300	300	100
N_{PV} (M$)		10.6	14.3	18.0	21.4	24.3
I_r (%)		24.1	21.3	19.9	19.0	18.1
x (m)	**6**	100	100	100	100	100
y (m)		300	300	300	300	300
N_{PV} (M$)		12.6	17.1	21.5	25.6	28.9
I_r (%)		23.9	21.0	19.7	18.8	17.9

From Table 2, it is evident that maximal Net Present Value, N_{PV} of $ 30.3 million can be achieved for I=6 and J=6 (36 turbines), each one separated x=100 m on the x direction, and y=500 m on the y direction. However, this solution provides only an I_r=16.4%. On the other hand, Table 3 shows that a maximal rate of return, I_r of 27.7% is attainable with I=2 and J=2, the turbines being apart, in x=300 m on the x direction, and y=100 m on the y direction. The disadvantage of this high rate of return is that it generates only a Net Present Value of $4.6 million. If the decision would be done by an investment company, they should opt for the largest possible rate of return on their its investments in any given project. However, a power-generating company that is not diversifying its investments in many different types of projects with different level of risks, it would decide on a project that generates a large N_{PV} while satisfying at least a minimal level of I_r. Searching through Tables 2 & 3, it is possible to detect solutions that provide the best combination of N_{PV}'s and I_r's, although each is less than its maximum value. For example, for a layout of I=4, J=6 (24 turbines), with the turbines separated with x=100 m and y=300 m, provides an N_{PV}=$21.5 million, and an I_r=19.7%, which is the authors recommend "optimal solution."

6. Conclusions

The developed model optimizes windfarm layouts on flat ground or over water for both economic and aerodynamic criteria. Although there are other criteria (e.g. visual impact) to consider, the model performance is an advance for economic optimization. The model with its accompanying computer program, can handle different wind-regimes and all the different combinations of cost and technical parameters for rectangular layouts, of which single-line layouts are special cases. Even the seeming dependence of the solution on the initial selection of the directions of the perpendicular axes can be eliminated, by repeating the calculations for different directions of the axes, and by selecting the layout that has the axis given by the "recommended optimal solution" Furthermore, the computer program can be adjusted, for layouts that are not rectangles, but also parallelograms.

References

[1] Heier, Siegfried, Grid Integration of Wind Energy Conversion Systems, 2nd ed., John Wiley & Sons, Inc., New York, N.Y., 2004.

[2] Lindley, D. et al., "The Effect of Terrain and Construction Method on the Flow over Complex Terrain Models in a Simulated Atmospheric Boundary Layer," in Proceedings of the Third B.W.E.A. Wind Energy Conference, edited by Musgrove,P.J., Cranfield, U.K., April 1981, pp.198-199.

[3] Manwell, J.F., Mcgowan, J.G. and Rogers, A.L., Wind Energy Explained, John Wiley & Sons, Inc., New York, N.Y., 2002

[4] Burton, Tony, Sharpe, David, Jenkines, Nick and Bossanyi, Ervin, Wind Energy Handbook, John Wiley & Sons, Inc., New York, N.Y. 2001.

[5] Milborrow, David ,Garrad, Andrew and Madsen, Birger, A Plan for Action in Europe:Wind Energy –The Facts, European Wind Energy Association(EWEA) European Communities, 1999, pp. 133-134.

[6] Haac, Barry N. An Examination of Small Wind Electric Systems in Michigan, Department of Geography, University of Michigan,1977, pp.27.

[7] Ackermann, Tomas, Wind Power in Power Systems, John Wily& Sons, Inc., New York, N.Y.,2004.

[8] Bossani, E.A.et al., "The Efficiency of Wind Turbine Clusters," in Third International Symposium on Wind Energy Systems, edited by Stephens,H.S., and Stapleton, C.A. , Cranfield, U.K., August 1980, pp.403-406.

[9] Ladany, Shaul P., "Optimal Layout for Urban Gasoline Stations ," in Gilad, I.E.& M. '98, Haifa 1998, pp.45-49.

[10] Ladany, Shaul P ., and Li, Jingwen, "Layout Design for Urban Service Facilities," Communications in Dependability and Quality Management, Vol.5, No.2, 2002, pp.16-30.

[11] Nybore, Claus, The WindFarm-Planning Windphysics, The Danish Centre for Renewable Energy, Copenhagen, 1988, pp. 1-29.