Studies on the validity of the fixed-speed approximation for the real time Railway Traffic Management Problem

Pierre Hosteins [a,1], Paola Pellegrini [b], Joaquin Rodriguez [a]
[a] Univ Lille Nord de France, IFSTTAR, COSYS, ESTAS
F-59650 Villeneuve dAscq, France [b] Univ Lille Nord de France, IFSTTAR, COSYS, LEOST
F-59650 Villeneuve dAscq, France
[1] E-mail: pierre.hosteins@ifsttar.fr, Phone: +33 (0) 320438358

Abstract
In railway traffic management problems, a frequent approximation is the one of fixed-speed, *i.e.* the trains either run at their cruise speed or are stopped immediately without considering the acceleration and deceleration phases due to arising conflicts on the infrastructure. We assess the validity of the fixed-speed approximation for train speed dynamics in the real time Railway Traffic Management Problem. This is done through a statistical analysis on a number of perturbed scenarios on different railway infrastructures, for different objective functions commonly used in the literature. For each scenario, we analyze the ranking of the generated solutions both in the fixed-speed approximation, obtained by solving the optimization model, and with the variable-speed dynamics, obtained through micro simulation with the OpenTrack software. Our results indicate that some objective functions can be considered reliable when used in conjunction with the fixed-speed approximation, while others require more detailed studies. We also propose a modified fixed-speed approximation to better reflect the behaviour of trains speed dynamics and study its efficiency.

Keywords
Traffic Management, speed profile, fixed-speed approximation, Mathematical Programming, optimization

1 Introduction

At peak times, in critical parts of the railway network of many European countries, traffic is planned to occupy the infrastructure almost without interruption. When this happens, a delay of one train, even of a few seconds, may propagate to several other trains in a snowball effect: if one or more trains, running at the planned speed, would require the same piece of infrastructure concurrently, all but one of them must slow down or even stop to ensure safety. In this case, a *conflict* is said to emerge. Conflicts are particularly critical at *junctions*, where multiple lines cross. Here, the precedence between the involved trains must be specified and may have a strong impact on delay propagation. Moreover, it is often possible to route trains in different ways to go through a junction, also impacting delay propagation. This translates into a difficult combinatorial optimization problem.

Today, conflict management is performed mostly manually by dispatchers. Several algorithms have been proposed to solve the described routing and scheduling problem (Cacchiani et al., 2014), which is often named real-time Railway Traffic Management Problem (rtRTMP). Many variants exist to tackle such a problem, be it on the modelling side or on the methodology for recovering efficient solutions. For example, such models can advocate a macroscopic infrastructure representation, ignoring some details such as the locking and releasing of track sections. Conversely they can advocate a microscopic modelling of the infrastructure, taking into account all the necessary operational details. Different models focus on different aspects of the problem, such as, e.g., trains or passengers, details of the train speed variation dynamics when brakes and accelerations are necessary due to conflicts, etc... The choice of these aspects results in different objective functions to be optimized given the operational constraints of the problem. There also exists a wide range of algorithms to solve the rtRTMP. Exact resolutions usually make use of commercial branch-and-cut solvers to solve a Mixed Integer Linear Program which models the rtRTMP, as in, e.g., Caimi et al. (2011); Corman et al. (2012); D'Ariano et al. (2007a); Lamorgese and Mannino (2015); Meng and Zhou (2014); Törnquist and Persson (2007). Heuristic and meta-heuristic approaches have also been devised to tackle the problem. Prominent examples are the works of Khosravi et al. (2012); Dündar and Şahin (2013); Sama et al. (2017).

The impact of the modelling choices on the actual performance of the algorithms due to the validity of the underlying assumptions has not been deeply studied yet. In this paper, we try to assess the validity of the assumption underlying one of these choices, specifically the so-called fixed-speed model for the unplanned brakes and accelerations. According to this model, the speed profile of trains traveling according to the planned timetable is precisely computed, when the trains are free to reach their desired speed on each track section without encountering any conflict. However, if a train needs to slow down or stop due to traffic perturbations, the fixed-speed approximation considers that it passes from its planned speed to a halt in no space and time. When the track is free for the train to go, it reaches its planned speed in, again, no space and time. This means that there is inifinite acceleration and braking rate, which of course is not realistic. We illustrate in Figure 1 the difference in the speed profiles of a given train when it crosses an infrastructure without conflict, as well as when a conflict arises, using the exact speed dynamic and the fixed-speed approximation. The red part in the speed profile of train A shows where the train speed diverts from its basic conflictless profile due to the conflict with train B. While this part displays a smooth change in the speed value due to acceleration and deceleration, we can see that the speed jumps directly to 0 in the fixed-speed approximation and remains null until the train is free to move again, after which it jumps back to its maximum value over the track section.

Many optimization models regarding railway traffic management have been proposed, based on the fixed-speed approximation, as e.g. Corman et al. (2010); Pellegrini et al. (2014). The assumption behind the validity of this modelling choice is the preservation of the relative quality of solutions. In particular, if one takes two solutions A and B, then if the routing and scheduling decisions in A are better than the one in B according to the fixed-speed model, they will be better also in reality. Although the intuition suggests that this is likely to be true most often, no deep analysis has been performed so far to support this intuition. To the best of our knowledge, the only attempt to study quantitatively the fixed-speed approximation is the work of Sobieraj et al. (2011), which proposes experiments to characterize relations between some specific traffic conditions and its quality. In the following, we will try to assess on general grounds the quality of the fixed-speed approximation on a few different railways infrastructures and for different objective functions. We stress that our study aims at quantifying the quality of the fixed-speed approximation and therefore its

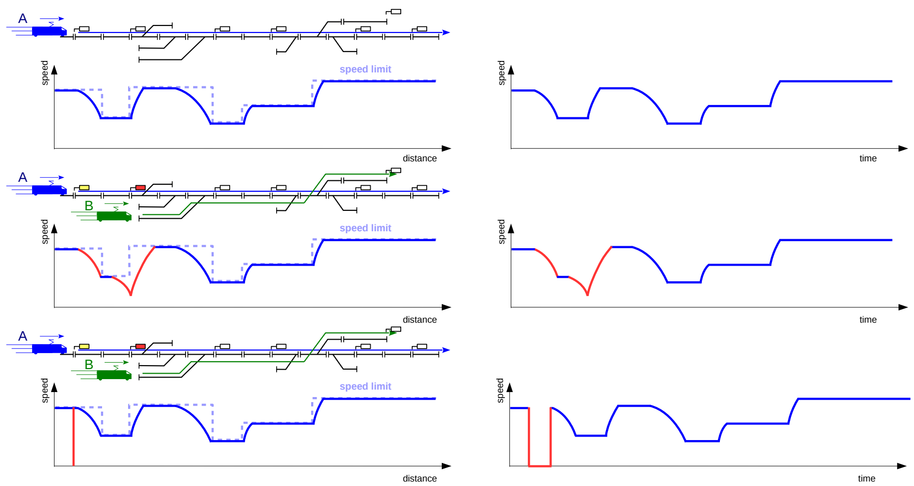

Figure 1: Speed profiles of train A (blue) in three cases (from top to bottom): no conflict, conflict with train B (green) with accurate speed dynamics (simulation) and conflict with the same train using the fixed-speed approximation. The left curves represent the value of speed as a function of distance while the right curves represente the same speed as a function of time. Dashed lines represent the speed limit on a given track section.

validity in terms of the ranking of the best solution returned by an optimization model when using such an approximation. Indeed, all solutions found are feasible from the point of view of the model constraints but the approximation used for the trains dynamics will change the value of the trains delay and therefore the value of the objective function. Let us note, however, that not all the proposed models in the literature make use of the fixed-speed approximations. Some examples exist which propose an iterative loop which solves the model with approximated speed profiles and then improve such speed profiles when they are found to be infeasible or suboptimal in the current solution D'Ariano et al. (2007b); Mazzarello et al. (2007); Lüthi (2009). This trend of work is sometimes preoccupied with the reduction of power consumption on the railway network, which is directly linked to the trains speed and accelerations. Examples of such energy consumption oriented works can be found in D'Ariano and Albrecht (2006); Albrecht (2009).

In the next section we will recall the MILP formulation already used in previous works to solve the rtRTMP and also propose a refined model that tries to better model the effects of unanticipated braking on the running time of trains. In section 2, we recall the formulation of the rtRTMP and introduce a refined model to try to take into account the effect of deceleration on the trains running time. In section 3, we will detail our methodology to assess the validity of the fixed-speed approximation by comparing its results to the variable-speed dynamic obtained by a simulation tool. In Section 4 we will present the different infrastructures that we use for our numerical experiments and then provide a comparison between the approximated and simulated speed dynamics. Finally we conclude in Section 5.

2 Integer Linear formulation for the real-time Railway Trafic Management Problem

2.1 Formulation for the classic rtRTMP with fixed-speed approximation

We will use a MILP to solve the rtRTMP, similar to the previous works of Pellegrini et al. (2014, 2015), called RECIFE-MILP. It models the infrastructure at the microscopic level and implements the route-lock sectional-release interlocking system (Pachl, 2008). The tracks are divided into *track-circuits*, i.e., track segments on which the presence of a train is automatically detected. *Block sections* represent groups of track-circuits whose access is controlled by a signal. Moreover, before a train can *occupy* a sequence of block sections, all their track-circuits must be reserved for the train itself. RECIFE-MILP uses the following sets:

- T: the set of trains;
- Θ: set of train types;
- R_t: the set of routes available to train $t \in T$, with $R = \cup_{t \in T} R_t$ the total set of routes;
- TC_t: the set of track-circuits which can be used by train $t \in T$;
- TC^r: the set of track-circuits belonging to route $r \in R$;
- $OTC_{ty,r,tc}$: set of track-circuits occupied by a train $t \in T$ of type $ty \in \Theta$ along $r \in R_t$ if t's head is at the end of $tc \in TC^r$ ($\emptyset$ if t shorter than tc);
- $TC(tc, tc', r)$: set of track-circuits between tc and $tc' \in RT^r$ along $r \in R$;
- S_t, $TCS_{t,s}$: set of stations where $t \in T$ has a scheduled stop and set of track-circuits that can be used by t for stopping at $s \in S_t$;

and parameters:

- tc_0 and tc_∞: entry and the exit locations of the infrastructure considered;
- $sched_t$: scheduled arrival time of traint $t \in T$ at destination;
- ty_t: type corresponding to train t (train characteristics);
- $init_t, exit_t$: earliest time at which train $t \in T$ can be operated and earliest time at which it can reach its destination given $init_t$, the route assigned in the timetable and the intermediate stops;
- $i(t', t)$: indicator function equal to 1 if t' and t use the same rolling stock and t results from the turnaround, join or split of t', 0 otherwise; $ms \equiv$ minimum separation between the arrival and the departure of two trains using the same rolling stock;
- $rt_{ty,r,tc}, ct_{ty,r,tc}$: running and clearing time of $tc \in RT^r$ along $r \in R$ for a train of type $ty \in \Theta$;
- $ref_{r,tc}$: reference track-circuit for the reservation of $tc \in TC^r$ along $r \in R$, depending on block-sections structure;

- $e(tc, r)$: indicator function equal to 1 if track-circuit $tc \in TC^r$ belongs to either the first or the last block section of $r \in R$, 0 otherwise;
- $bs_{r,tc}$: block section including track-circuit $tc \in TC^r$ along route $r \in R$;
- for_{bs}, rel_{bs}: formation and release time for block section bs;
- S_t, $TCS_{t,s}$: set of stations where $t \in T$ has a scheduled stop and set of track-circuits that can be used by t for stopping at $s \in S_t$;
- $dw_{t,s}, a_{t,s}, d_{t,s}$: minimum dwell time, scheduled arrival and scheduled departure times for train $t \in T$ at station $s \in S_t$;
- $p_{r,tc}, s_{r,tc}$: set of track-circuits preceding and following $tc \in RT^r$ along $r \in R$;
- M: a large constant.

We also make use of the following variables:

- $sU_{t,tc}, eU_{t,tc}$: continuous positive variable representing the time at which $tc \in TC_t$ starts and ends being utilized by $t \in T$;
- $x_{t,r}$: binary variable equal to 1 if train $t \in T$ uses route $r \in R_t$, 0 otherwise;
- $y_{t,t',tc}$: binary variable equal to 1 if train $t \in T$ utilizes track-circuit tc before train t', such that the index t is smaller than the index t' ($t \prec t'$), with $tc \in TC_t \cap TC_{t'}$, and 0 otherwise;
- $o_{t,r,tc}$: time in which $t \in T$ starts the occupation of $tc \in TC^r$ along $r \in R_t$;
- $l_{t,r,tc}$: longer stay of $t \in T$'s head on $tc \in TC^r$ along $r \in R_t$, due to dwell time and scheduling decisions (delay).

In addition to the existing track-circuits, we introduce the dummy ones: tc_0 and tc_∞ which represent the entry and the exit locations of the infrastructure considered. Depending on the objective function used, we also have to define the following variables:

- D_t: delay suffered by train t when exiting the infrastructure;
- Δ: maximum secondary delay among all trains;
- δ_t: binary variable equal to 1 if train $t \in T$ suffers some delay compared to its original timetable.

The model has to respect the following sets of constraints:

$$o_{t,r,tc} \geq init_t \, x_{t,r} \qquad \forall t \in T, r \in R_t, tc \in TC^r, \tag{1}$$

$$o_{t,r,tc} \leq M x_{t,r} \qquad \forall t \in T, r \in R_t, tc \in TC^r, \tag{2}$$

$$o_{t,r,tc} = o_{t,r,p_{r,tc}} + l_{t,r,p_{r,tc}} + rt_{r,ty_t,p_{r,tc}} x_{t,r} \qquad \forall t \in T, r \in R_t, tc \in TC^r, \tag{3}$$

$$o_{t,r,s_{r,tc}} \geq \sum_{\substack{s \in S_t: \\ tc \in TCS_{t,s} \cap TC^r}} d_{t,s} x_{t,r} \qquad \forall t \in T, r \in R_t, tc \in \bigcup_{s \in S_t} TCS_{t,s}, \tag{4}$$

$$l_{t,r,s_{r,tc}} \geq \sum_{\substack{s \in S_t: \\ tc \in TCS_{t,s} \cap TC^r}} dw_{t,s} x_{t,r} \qquad \forall t \in T, r \in R_t, tc \in \bigcup_{s \in S_t} TCS_{t,s}, \tag{5}$$

$$\sum_{r\in R_t} x_{t,r} = 1 \qquad \forall t \in T, \tag{6}$$

$$\sum_{\substack{r\in R_t, tc\in TC^r:\\ p_{r,tc}=tc_0}} o_{t,r,tc} \geq \sum_{\substack{r\in R_{t'}, tc\in TC^r:\\ s_{r,tc}=tc_\infty}} o_{t',r,tc} + (ms + rt_{r,ty_{t'},tc})x_{t',r}$$
$$\forall t, t' \in T : i(t', t) = 1, \tag{7}$$

$$\sum_{\substack{tc\in TC_t:\\ \exists r\in R_t, p_{r,tc}=tc_0}} sU_{t,tc} \leq \sum_{\substack{tc\in TC_{t'}:\\ \exists r\in R_{t'}, s_{r,tc}=tc_\infty}} eU_{t',tc} \qquad \forall t, t' \in T : i(t', t) = 1, \tag{8}$$

$$sU_{t,tc} = \sum_{\substack{r\in R_t:\\ tc\in TC^r}} \Big(o_{t,r,ref_{r,tc}} - for_{bs_{r,tc}}\, x_{t,r}\Big) \qquad \forall t \in T,\ tc \in TC_t :$$
$$(\nexists t' \in T : i(t', t) = 1) \vee (\forall r \in R_t : ref_{r,tc} \neq s_{r,tc_0}), \tag{9}$$

$$eU_{t,tc} = \sum_{\substack{r\in R_t:\\ tc\in TC^r}} o_{t,r,ref_{r,tc}} + (for_{bs_{r,tc}} + rel_{bs_{r,tc}})\, x_{t,r} + ul_{t,r,tc} \qquad \forall t \in T, tc \in TC_t, \tag{10}$$

$$eU_{t,tc} - M(1 - y_{t,t',tc}) \leq sU_{t',tc} \qquad \forall t, t' \in T, \text{index } t < \text{index } t',\ tc \in TC_t \cap TC_{t'} :$$
$$i(t, t') \sum_{r\in R_t} e(tc, r) = 0 \ \wedge\ i(t', t) \sum_{r\in R_{t'}} e(tc, r) = 0, \tag{11}$$

$$eU_{t',tc} - My_{t,t',tc} \leq sU_{t,tc} \qquad \forall t, t' \in T, \text{index } t < \text{index } t',\ tc \in TC_t \cap TC_{t'} :$$
$$i(t, t') \sum_{r\in R_t} e(tc, r) = 0 \ \wedge\ i(t', t) \sum_{r\in R_{t'}} e(tc, r) = 0. \tag{12}$$

Eqs. (1) and (2) force train t to be operated no earlier than $init_t$ on its chosen route. In Eq. (3), a train starts occupying a given track-circuit after spending its effective running time in the preceding one (if the route is used). Eq. (5) ensures that train t which stops at station s along route r does not leave track-circuit $tc \in TCS_{t,s}$ before the scheduled departure time from s. In (6), a single route is chosen for train t. Eqs. (7) and (8) are used to guarantee consistency for trains using the same rolling stock, i.e. minimum time between arrival and departure of such trains and consistent *utilization* time of the track-circuits. The utilization time is defined as the sum of the *reservation* and the *occupation* time. In Eq. (9), a train's utilization of a track-circuit starts as soon as the train starts occupying the track-circuit $ref_{r,tc}$ along one of the routes including it, minus the formation time. Furthermore, in Eq. (10), the utilization of a track-circuit lasts till the train utilizes it along any route, plus the formation and the release time. Here $ul_{t,r,tc}$ is the total utilization time which includes: the running time of all track-circuits between $ref_{r,tc}$ and tc, the longer stay of the train's head on each of these track-circuits and the clearing time of tc. Finally, Eqs. (11) and (12) ensure that the track-circuit utilizations by two trains must not overlap. We refer the reader to Pellegrini et al. (2015) for additional details about the above formulation.

In our subsequent analysis, we consider the four following objective functions commonly used in the literature:

- the total delays (i.e. the sum of delays for each train with respect to its timetable):

$$\min \quad \sum_{t\in T} w_t D_t, \tag{13}$$

$$D_t \geq \sum_{r\in R_t} o_{t,r,tc_\infty} - sched_t \qquad \forall t \in T; \tag{14}$$

- the maximum secondary delay (i.e. the maximum propagation delay between all trains):

$$\min \quad \Delta, \tag{15}$$
$$\Delta \geq D_t \qquad \forall t \in T, \tag{16}$$
$$D_t \geq \sum_{r \in R_t} o_{t,r,tc_\infty} - exit_t \qquad \forall t \in T; \tag{17}$$

- the number of delayed trains:

$$\min \quad \sum_{t \in T} \delta_t, \tag{18}$$
$$M\delta_t \geq \sum_{r \in R_t} o_{t,r,tc_\infty} - exit_t \qquad \forall t \in T; \tag{19}$$

- the total travel times of all trains in the infrastructure:

$$\min \quad \sum_{t \in T} \sum_{r \in R_t} (o_{t,r,tc_\infty} - o_{t,r,tc_0}). \tag{20}$$

2.2 Modified formulation for the Min-fixed-speed approximation

In order to try to include the effects of deceleration on the trains dynamics more accurately, we introduce a new approximation based on a slight modification of the fixed-speed based model described above. The basic idea is that when a train is forced to decelerate because of a conflict, it loses a minimum amount of time, even if in the fixed-speed approximation it would only have to stop for a couple of seconds. We therefore introduce a minimum *forfait* f for the stopping time of a train: when a train has to stop due to a conflict, we impose that its running time increases of at least f seconds. The value of f is a parameter that might depend on the infrastructure considered and the type of train considered.

In order to generalize the fixed-speed model we introduce the new binary variables:

- $\sigma_{t,r,tc}$: variable equal to 1 if train $t \in T$ needs to stop on track circuit $tc \in TC_t$ on route $r \in R_t$,

and we add the following constraints to the fixed-speed model described above:

$$l_{t,r,tc} \leq M\sigma_{t,r,tc}, \qquad t \in T, r \in R_t, tc \in TC_t, \tag{21}$$
$$l_{t,r,tc} \geq f\sigma_{t,r,tc}, \qquad t \in T, r \in R_t, tc \in TC_t. \tag{22}$$

Hence, when variable l must be greater than 0 on a certain track-circuit due to some conflict, the corresponding binary variable σ is equal to 1, thus forcing l to be at least equal to the forfait value f.

3 Methodology

We now present the statistical methodology chosen for our numerical experiments and analysis. In order to perform a relevant statistical analysis, we need to generate a large set of different solutions for a given perturbation scenario. For a given scenario on a given infrastructure, we generate five hundred solutions[1]. For each solution, we randomly fix the route of the trains among all available routes. We also randomly set 5% of scheduling decisions (y variables in the MILP described in Section 2). The remaining scheduling decisions are decided by solving RECIFE-MILP with CPLEX 12.8 with three minutes of running time and extracting the best found solution[2]. These solutions are then executed using the OpenTrack microscopic railway simulator for obtaining the exact speed profile results and we use CPLEX 12.8 to compute the objective functions in the (min-)fixed-speed approximation using RECIFE-MILP with fixed binary variables. Then, for each scenario, we identify the ranking of the solutions in terms of objective function value according to either the (min-)fixed-speed model or the simulation results.

We focus on the relative ranking of the solutions since our aim is to identify the optimal (or best possible) solution regarding trains routing and scheduling. Therefore, it is not crucial that the approximate objective is the same as the objective obtained in the micro-simulation, as long as the optimization algorithm provides the best possible solution for the network operator. In other words, the relative ranking of a set of solutions according to the fixed-speed approximation and the simulated speed dynamics (also called *variable-speed dynamics* in contrast to the fixed-speed approximation) should be the same. Therefore, when we choose to represent the solutions in a plane where the x and y-axis are the relative ranking of each solution in both (min-)fixed-speed approximation and with variable-speed dynamics, the points should be located on the diagonal. We will quantify the departure from the diagonal by performing a linear regression and extract the linear coefficient and the correlation factor. We perform the same statistical analysis for the four different objective functions described in Section 2, as well as for the min-fixed-speed approximation with different forfait values.

We also decided to perform an aggregation of the solutions which have very close objective values, in order to avoid the artificial discrimination of somewhat equivalent solutions. For each objective function we choose a threshold value θ which fixes the precision with which we intend to round the value of the objective function for each solution. In practice, we divide the value of the objective by θ, round the obtained value to the nearest integer and multiply the result by θ. This operation creates more solution with the same objective value, which won't be discriminated by the ranking procedure. We fix the value of θ to 100 seconds for the total delay and total travel time objective functions, 10 seconds for the maximum consecutive delay and 1 for the number of late trains. In practice, this means that we do not perform any aggregation for the number of late trains as we feel that the difference is already clear enough between two different solutions.

4 Statistical analysis

4.1 Railway infrastructures and scenarios

We propose an experimental analysis based on two French control areas: the Pierrefitte Gonesse junction and the Parisian St. Lazare station. These two control areas have very different characteristics. The former is a complex junction about 18 km long where freight,

[1] Note that equivalent solutions are discarded, so that some scenarios have less than 500 solutions in practice.

[2] This method aims at generating sufficiently different solutions while avoiding very bad ones which are not likely to be returned by optimization algorithms.

conventional and high speed lines cross. A weekday timetable includes 336 trains. The latter is a terminal station area of slightly more than 7 km, with 27 platforms. A weekday timetable includes 459 trains, most of which linked by rolling-stock re-utilization constraints. A simplified map of the Pierrefitte Gonesse junction is shown in Figure 2 while a map of the St. Lazare station is shown in Figure 3.

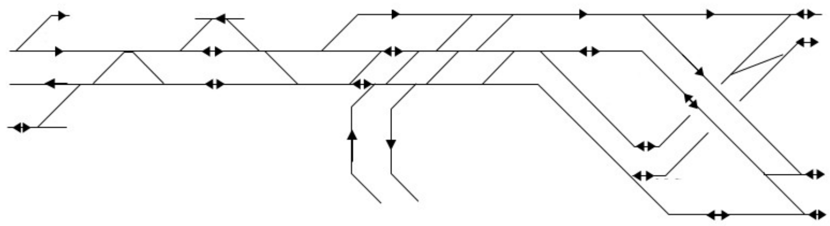

Figure 2: Simplified map of the Pierrefitte Gonesse junction.

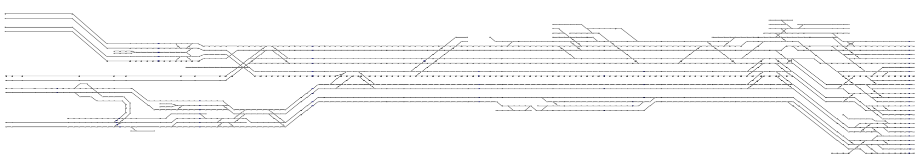

Figure 3: Simplified map of the St. Lazare station.

For each control area, we consider four different daily perturbation scenarios. These scenarios are obtained by randomly assigning an entrance delay between 5 and 15 minutes to 20% of the trains. From each of these daily scenarios we extract three peak-hour scenarios, starting at 6, 7 and 8 a.m. and lasting one hour. Therefore we have a total of twelve different perturbation scenarios for each railway infrastructure.

4.2 Numerical Results

We first present the results for the Pierrefitte Gonesse junction. We start by providing an example of the objective function values for both fixed-speed approximation and variable-speed dynamics for each solution in Figure 4 (left panels) on a representative perturbation scenario. We also plot on the same figure (right panels) the variable-speed ranking of the same solutions as a function of the fixed-speed rankings. As advocated in the Section 3, we perform a linear regression on the obtained cloud of points. We display on Figure 4 the straight line obtained, together with the diagonal. As can be seen, even though the linear regression and the diagonal are very close, there is a substantial dispersion of the solutions around it. In order to quantify the dispersion, we compute the average correlation

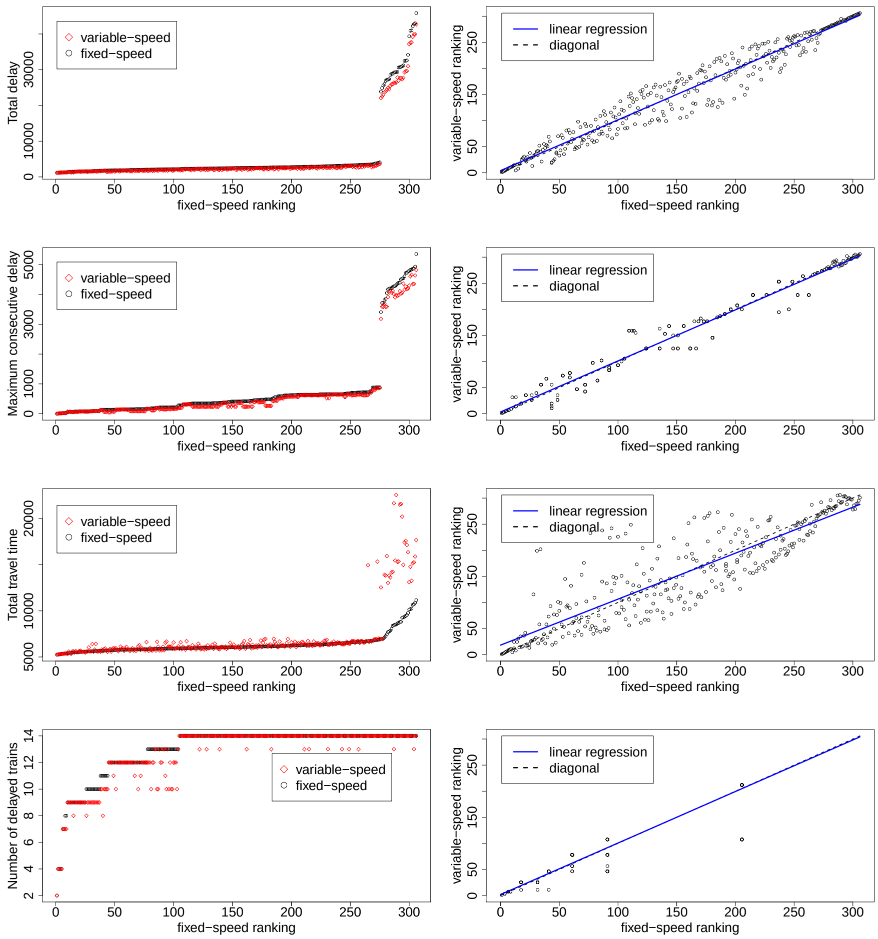

Figure 4: Value of the objective function (left panels) as a function of the ranking in the fixed-speed approximation (black circle) and variable-speed model (red diamond) and ranking with the variable-speed dynamics (right panels) as a function of the ranking in the fixed-speed approximation for a perturbation scenario on the Pierrefitte Gonesse junction. The different objective functions from top to bottom are: total delay, maximum consecutive delay, total travel time and number of delayed trains. The first three objective functions are expressed in seconds.

coefficient over the 12 perturbation scenarios and display it in Table 1, for the four objective functions, both for the fixed-speed and min-fixed-speed approximation with a forfait $f = 30$ seconds. Since our aim is to minimise a given objective, it is reasonable to suppose that an optimization algorithm will tend to provide good quality solutions. Hence, in a sense, one may consider the correlation of the set of best solutions as more important than the one of just any solution. Therefore, we also indicate the same correlation coefficient computed on a fraction of the best solutions, i.e., on the 50% and 25% best solutions.

Table 1: Average correlation factor for the fixed-speed approximation on the Pierrefitte Gonesse junction, over the whole sample, the best 50% and the best 25% solutions, for the four objective functions. The best correlation between all four objective functions is displayed in bold font. Results are displayed both for the fixed-speed (left) and min-fixed-speed (right) approximations with a forfait of $f = 30$ seconds.

	fixed-speed			min-fixed-speed		
	whole sample	50% best	25% best	whole sample	50% best	25% best
totD	0.92	0.81	**0.74**	0.92	0.81	**0.74**
maxConsD	**0.93**	0.77	0.59	**0.93**	0.77	0.58
num	0.91	**0.89**	0.73	0.91	**0.88**	0.72
totT	0.63	0.42	0.38	0.62	0.40	0.35

What we can infer from this table is that on this particular infrastructure, the total travel time does not seem to be a very reliable objective function with respect to the fixed-speed approximation. The other three objectives provide good correlation factors on the whole sample, however the correlation decreases when we restrict ourselves to a fraction of the best solutions. This might be due to the lower statistics but also to the stabilizing effect of the worst solutions, who tend to have the same ranking in both models. In this case, the total delay and number of delayed trains seem to hold better. We observe that the results in the min-fixed-speed approximation are very similar to those of the fixed-speed approximation with a forfait of 30 seconds. We also tested the min-fixed-speed model with a larger forfait of 60 seconds and obtained no better results than the more basic fixed-speed approximation. This result seems to imply that differences between the exact speed profiles and the fix-speed approximation do not come from very short decelerations (akin to a short stop in the fixed-speed model) but from more complicated dynamics between the train conflicts. In general, the correlation coefficient shows that both models with all objective functions but the total travel time are able to properly distinguish very bad from very good solutions. However, the discrimination ability decreases when only good solutions are concerned. This says that an optimization algorithm implementing these models may possibly not return the very best solution, evaluating it slightly worse than other ones. However, these other solutions will not be much worse than the best one.

In order to provide a better idea of the statistical distribution of our results, we also display in Figure 5 histograms indicating the difference for each solution between the ranking in the fixed-speed approximation and the variable-speed model. We report one histogram per objective function. Ideally, one would expect a bell-shaped, narrow distribution centered on value 0 and with a short and thin tail. This would mean that the ranking of a solution in each model is very close, which would guarantee the validity of the approximation. What we can see in the figure is that the distribution for the total delay and maximum consecutive delay are roughly bell-shaped and centered around 0, though their tails are fatter than one would expect in an ideal situation. The distribution for the total number of delayed trains has a thinner width, though it is not centered around value 0. This may explain the good correlation factors for that particular objective function. Finally, the shape of the distribution for the total travel time sheds some light on the weak correlation factors displayed in Table 1, since it has a shape almost opposite to what one could hope for. Specifically, we

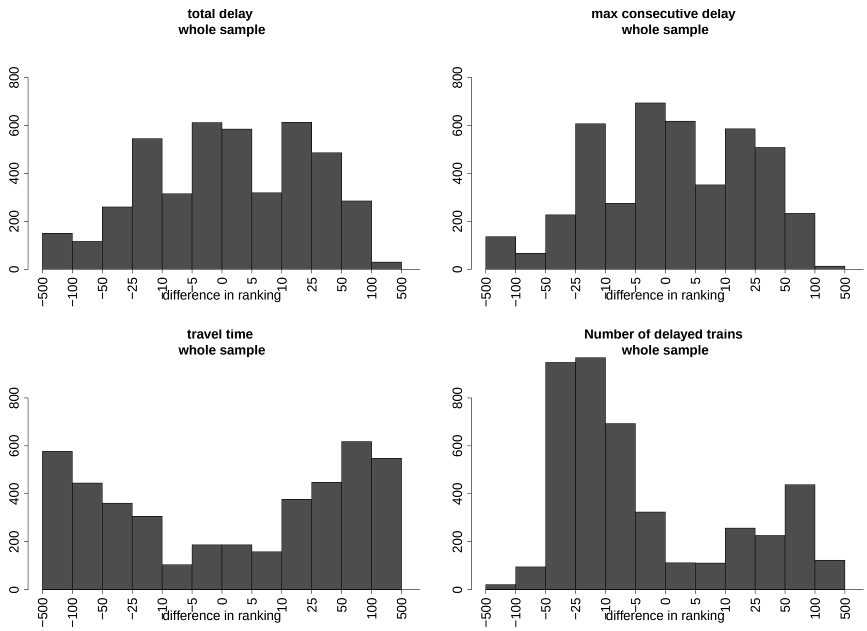

Figure 5: Histograms of the difference between the fixed-speed and variable-speed rankings for the four objective functions, on the Pierrefitte Gonesse junction (total over the twelve perturbation scenarios). Each rectangle represents the total number of solutions with a difference in ranking located in the range displayed on the x axis.

see that many solutions have an absolute rank difference larger than 50 or even 100.

Table 2: Same results as Table 1 but with aggregated solutions.

	fixed-speed			min-fixed-speed		
	whole sample	50% best	25% best	whole sample	50% best	25% best
totD	**0.99**	0.77	0.78	**0.99**	0.77	0.78
maxConsD	0.98	0.71	0.61	0.98	0.71	0.61
num	0.93	**0.91**	**0.88**	0.93	**0.91**	**0.88**
totT	0.58	0.27	0.29	0.57	0.24	0.26

In accordance with the methodology described in Section 3, we perform the same statistical analysis after rounding the objective values with respect to some precision threshold. The effet of this rounding is to aggregate some solutions together, which means that they will not be differentiated by the ranking procedure. The results displayed in Table 2 are interesting, since the correlation factors evolve significantly for different objectives. Specifically, the correlation factor for the number of late trains generally improves, especially for the 25% best solutions, while the correlation for the total travel time is even further degraded.

We can now perform the same statistical analysis for the St.Lazare station, where trains

Table 3: Average correlation factor for the fixed-speed approximation on the St.Lazare station, over the whole sample, the best 50% and the best 25% solutions, for the four objective functions. The best correlation between all four objective functions is displayed in bold font. Results are displayed both for the fixed-speed (left) and min-fixed-speed (right) approximations with a forfait of $f = 30$ seconds.

	whole sample	50% best	25% best	whole sample	50% best	25% best
totD	0.94	0.83	**0.80**	0.94	0.83	**0.80**
maxConsD	**0.96**	**0.85**	0.77	**0.96**	**0.85**	0.77
num	0.82	0.67	0.57	0.82	0.67	0.57
totT	0.87	0.68	0.60	0.87	0.68	0.60

circulate at lower speed and can encounter more conflicts at junctions. The results, as displayed in Table 3, lead to slightly different conclusions than those of the Pierrefitte Gonesse junction. In particular, it is now the number of delayed trains whose correlation factor decreases more when restricted to the best 25% solutions while the maximum delay remains more reliable in the same conditions. The total delay, however, seems to be reliable on both infrastructures and imposes itself as the most reliable objective function so far. As before, the min-fixed-speed results do not improve on the simple fixed-speed model.

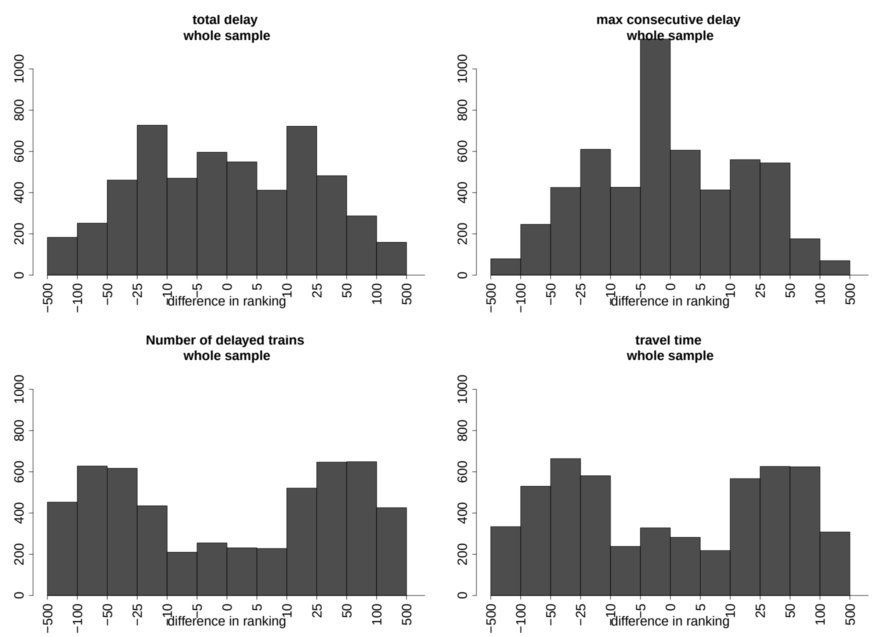

Figure 6: Same as Figure 5 with the St-Lazare station.

We complete the analysis of the second infrastructure with the histograms of the distribution of the difference in fixed and variable-speed rankings in Figure 6 and the average correlation factors with aggregated solutions in Table 4. The histograms show that the distri-

Table 4: Same results as Table 3 but with aggregated solutions.

	fixed-speed			min-fixed-speed		
	whole sample	50% best	25% best	whole sample	50% best	25% best
totD	0.95	0.88	**0.87**	0.95	0.88	**0.87**
maxConsD	**0.98**	**0.92**	**0.87**	**0.98**	**0.92**	**0.87**
num	0.85	0.75	0.72	0.85	0.75	0.72
totT	0.88	0.76	0.72	0.88	0.76	0.72

butions for the total delay and maximum consecutive delay are still roughly bell-shaped but now both the total travel time and the number of delayed trains look like double bell-shaped symmetric distributions, which once again explains the degredation of the correlation coefficient for the latter objective function. This time the correlation factors for aggregated solutions improve for all objective functions, in particular the correlations for the 25% best solutions improve between 0.07 and 0.15. This is a hint that the fixed-speed approximation cannot pretend to compute the objective functions reliably with a precision of the order of a second, or at least to discriminate between solutions which differ by a relatively small delay, in our case a handful of seconds.

5 Conclusion

We have provided a study of the validity of the so-called fixed-speed approximation for train dynamics in optimization models of the real time Railway Traffic Management Problem. We used two different railway infrastructures with various train behaviours, generated a dozen perturbation scenarios for each and hundreds of solutions for said scenarios. A statistical analysis was performed over the solutions for each scenario and four different objective functions used in the literature, to assess whether the ranking of solutions was the same with both the fixed-speed approximation and variable-speed dynamics. We also considered a slightly more refied model, in which the fixed-speed approximation is slomehow brought a step closer to the variable-speed dynamics. However, this model did not prove to behave differently from the fixed-speed one.

The average results on all perturbation scenarios show that the fixed-speed approximation seems to be reliable for most objective functions on the whole set of solutions. However, when we look specifically at the best solutions, the correlation factors between fixed and variable-speed rankings tend to decrease. Overall, the total delay objective function seems the most robust in terms of fixed-speed approximation while the total travel time provides questionable correlation factors with respect to the variable speed dynamics computed through micro-simulation. The results generally tend to improve when we aggregate the solutions with a larger granularity, which hints at the fact that the fixed-speed model cannot claim to reliably differentiate between solutions that differ by a handful of seconds. Instead, when solutions are significantly different, the fixed-speed model correctly capture the quality of the different routing and scheduling decisions. We now plan to extend the analysis to a third infrastructure in order to confirm our results.

References

Albrecth, T., 2009. “The influence of anticipating train driving on the dispatching process in railway conflict situations”. *Networks Spatial Economics*, vol. 9:1, pp. 85–101.

Cacchiani, V., Huisman, D., Kidd, M., Kroon, L., Toth, P., Veelenturf, L., and Wagenaar, J., 2014. “An overview of recovery models and algorithms for real-time railway rescheduling”. *Transportation Research Part B: Methodological*, vol. 63, pp. 15–37.

Caimi, G., Chudak, F., Fuchsberger, M., Laumanns, M., and Zenklusen, R., 2011. “A new resource-constrained multicommodity flow model for conflict-free train routing and scheduling”. *Transportation Science*, vol. 45:2, pp. 212–227.

Corman, F., D’Ariano, A., Pacciarelli, D., and Pranzo, M., 2010. “A tabu search algorithm for rerouting trains during rail operations”. *Transportation Research Part B*, vol. 44:1, pp. 175–192.

Corman, F., D’Ariano, A., Pacciarelli, D., and Pranzo, M., 2012. “Optimal inter-area coordination of train rescheduling decisions”. *Transportation Research Part E*, vol. 48, pp. 71–88.

D’Ariano, A., and Albrecht, T., 2006. “Running time re-optimization during real-time timetable perturbations”. In: *WIT Transactions on the Built Environment*, vol. 88. WIT Press, pp. 532–540.

D’Ariano, A., Pacciarelli, D., and Pranzo, M., 2007. “ A branch and bound algorithm for scheduling trains in a railway network”. *European Journal of Operational Research*, vol. 183, pp. 643–657.

D’Ariano, A., Pranzo, M., and Hansen, I.A., 2007. “Conflict Resolution and Train Speed Coordination for Solving Real-Time Timetable Perturbations”. *IEEE Transactions on Intelligent Transportation Systems*, vol. 8:2, pp. 208–222.

Dündar, S., and Şahin, I., 2013. “Train re-scheduling with genetic algorithms and artificial neural networks”. *Transportation Research Part C: Emerging Technologies*, vol. 27:0, pp. 1–15.

Khosravi, B., Bennell, J.A., and Potts, C.N., 2012. “Train scheduling and rescheduling in the UK with a modified shifting bottleneck procedure”. In: *Delling, D., Liberti L. (Eds), Proceedings of the 12th Workshop on Algorithmic Approaches for Transportation Modelling, Optimization and Systems*, vol. 25 of OpenAccess Series in Informatics (OASIs), Dagstuhl, Germany, Schloss Dagstuhl-Leibniz-Zentrum für Informatik, pp. 120–131.

Lamorgese, L. and Mannino, C., 2015. “An exact decomposition approach for the real-time train dispatching problem”. *Operations Research*, vol. 63:1, pp. 48–64.

Lüthi, M., 2009. “Improving the efficiency of heavily used railway networks through integrated real-time rescheduling”. *Diss., Egenössische Technische Hochschule Zürich*.

Mazzarello, M., and Ottaviani, E., 2007. “A traffic management system for real-time traffic optimization in railways”. *Transportation Research Part B*, vol. 41:2, pp. 246–274.

Meng, L., and Zhou, X., 2014. “Simultaneous train rerouting and rescheduling on an n-track network: A model reformulation with network-based cumulative flow variables”. *Transportation Research Part B: Methodological*, vol. 67, pp. 208–234.

Pachl, J. (2008). ‘Timetable design principles”. In Hansen, I. and Pachl, J., editors, *Railway Timetable & Traffic*, chapter 2, pages 9–42. Eurailpress — DVV Rail Media, Hambourg, Germany.

Pellegrini, P., Marlière, G., and Rodriguez, J., 2014. “Optimal train routing and scheduling for managing traffic perturbations in complex junctions”. *Transportation Research Part*

B: Methodological, vol. 59, pp. 58–80.
Pellegrini, P., Marliere, G., Pesenti, R., and Rodriguez, J., 2015. “RECIFE-MILP: An Effective MILP-Based Heuristic for the Real-Time Railway Traffic Management Problem”. *IEEE Transactions on Intelligent Transportation Systems*, vol. 16:5, pp. 2609–2619.
Samà, M., D’Ariano, A., Corman, F., and Pacciarelli, D., “A variable neighbourhood search for fast train scheduling and routing during disturbed railway traffic situations”. *Computers & Operations Research*, vol. 78, pp. 480–499.
Sobieraj, S., Marlière, G., and Rodriguez, J., 2011. “Simulation of solutions of a fixed-speed model for the real-time railway traffic optimization problem”. In: *Proceedings of The 4th International Seminar on Railway Operations Modelling and Analysis (RailRome2011)*, Rome, Italy.
Törnquist, J., and Persson, J., 2007. “N-tracked railway traffic re-scheduling during disturbances”. *Transportation Research Part B*, vol. 41, pp. 342–362.