Passenger Flow Control with Multi-station Coordination on an Oversaturated Urban Rail Transit Line: A Multi-objective Integer Linear Programming Approach

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Abstract
With the booming travel demands in megacities, the limited transportation capacity hasn't satisfied them in urban rail transit. Passenger congestion problem become increasingly serious, which causes the potential accident risks on platforms. To further efficiently improve the conditions, this paper proposes an effective collaborative optimization method for the accurate passenger flow control strategies on an oversaturated urban rail transit line by simultaneously adjusting the number of inbound passengers entering multiple stations on the line. Through considering the space-time dynamic characteristics of passenger flow, a multi-objective integer linear programming model is formulated to minimize the number of passengers who are limited to enter stations, minimize the total passenger waiting time on platforms at all of involved stations where the optimal passenger flow control is imposed and maximize the passenger person-kilometres. And it is solved by CPLEX solver efficiently. Moreover, the passenger flow demands are time-variant, so it's very necessary for the accurate and easy-to-implement passenger flow control strategies to determinate the control time intervals. Hence, this paper develops a method based on Fisher optimal division to get an optimal determination of the control time intervals before solving the model. Finally, two sets of numerical experiments, including a small-scale case and a real-world instance with operation data of Chengdu metro system, are implemented to demonstrate the performance and effectiveness of the proposed approach.

Keywords
Urban rail transport, Passenger flow control, Multi-station coordination, Fisher optimal division method

1 Introduction

1.1 Background

With the acceleration of urbanization process and the drastic increase of urban population in China, urban rail transit (URT) transportation capacity has been unable to satisfy the booming travel demands in some cities (i.e. Beijing, Shanghai, etc.), especially in peak hours. Passenger congestion problem is becoming more and more serious, and it gradually affects the operation safety and reliability of URT. At the same time, due to the limitation of infrastructures, transportation capacity cannot be improved in the short term. Therefore, under the condition of the limited transportation capacity, it is urgent to develop a management strategy to relieve congestion and further improve the operation efficiency.

So passenger flow control becomes a better choice with the limited transportation capacity at present. In fact, the measures for passenger flow control have been taken widely in some cities (i.e. Beijing, Shanghai, et al.) in China. For example, passenger flow controls will be imposed when the ratio of the number of the passengers entering certain station to the maximum number that the station holds gets to 70% (Xu et al., 2016). In the daily operation, passenger flow control works by setting railings outside metro station, reducing the number of the used gates and slowing down the speed of the escalators to limit the number of passengers entering the platforms and relieve pressure on platforms. However, the control strategies are implemented at each station without coordination, respectively, and mainly depend on the staffs' subjective work experience currently, which is lack of mathematical programming and scientific method (Jiang et al., 2018).

1.2 Literature Review

On the level of passenger flow control, many researches have ever investigated it from different perspectives in recent years, including pedestrian boarding/alighting management, station capacity and station pedestrian management.

In pedestrian boarding/alighting management, Baee et al. (2012) proposed different boarding/alighting strategies for Tehran subway system to increase satisfaction level and service success rate while reducing travel time by simulation; Fernández et al. (2015) demonstrated the existence of pedestrian saturation flows in public transport doors and showed various capacities of train doors under different conditions by real-scale experiments.

In station capacity, Chen et al. (2012) proposed a M/G/c/c-based capacity model of staircases and corridors for passenger evacuation in consideration of space facility in metro stations through analysis of passenger movements; Xu et al. (2014) developed a SSC optimization model of station capacity according to the gathering and scattering process and the analytical queuing network to identify bottleneck facilities to improve capacity; Xu et al. (2016) proposed an approach to measure a transfer metro station capacity for different ratio of inbound, outbound and transfer passenger volumes to the total passenger volume, according to the passenger routes in the station.

In station pedestrian management, Hoogendoorn and Bovy (2004) put forward a new theory of pedestrian behaviors under uncertainty based on the concept of utility maximization to simultaneously optimize route choice, activity area choice, and activity scheduling using dynamic programming for different traffic conditions and uncertainty levels based on this normative theory; Davidich et al. (2013) evaluated the impact of waiting pedestrians and proposed a cellular automata model for waiting pedestrians to analysis and prediction waiting zone capacity in critical situations.

The above studies are in-depth in theory, and provide a theoretical basis and reference on the practical application. However, they focus on passenger flow control at a single station or several stations, the complex characteristics of passenger flow Origin and Destination (OD) and the passenger flow at other stations in the network aren't taken into consideration, which tends to result in the section capacity not being fully utilized and is bad to improve the service quality and economic benefits in the entire network.

In view of the above deficiencies, some researchers turn their attentions to the passenger flow control with multi-station coordination on an oversaturated line (Wang et al., 2015; Li et al., 2017; Shi et al., 2018; Jiang el at., 2018). For example, Wang et al. (2015) took average passenger delay as the objective to develop an integer programming model based on the analysis of passenger delay and the processes by which passengers alight and board, which aims to disperse the pressure of oversaturated stations into others and achieve the optimal state for the entire line, and model is verified by a case study; Jiang et al. (2018) proposed a method based on reinforcement learning to optimize the inflow volume during a certain period of time at each station with the aim of minimizing the safety risks imposed on passengers at the metro stations, and the performance of the approach was tested by the simulation experiment carried out on a real-world metro line in Shanghai.

In addition, some studies mainly proposed the stop-skipping strategy to enhance the operation efficiency and indirectly relieve the pressure caused by the huge passenger volume (Wang et al., 2014; Niu et al., 2015). Nevertheless, trains always run with relatively fixed all-stop patterns from the start station to the terminal station in URT.

To our knowledge, the majority of existing studies focus on passenger flow control at a single station or several stations while the researches on the passenger flow control at multi-station coordination on the line is relatively few. Also, there are still deficiencies in the existing studies on the passenger flow collaborative control. For example, there is no scientific method to determine the control time intervals while the proposed approach in this paper addresses precisely these gaps.

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1.3 Contributions

The proposed approach contributes to the state-of-the-art related passenger flow control research in three ways.

(1) In order to obtain accurate and easy-to-implement passenger flow control strategies, the method , Fisher optimal division ,to determine the control time intervals is proposed according to the historical passenger flow data.

(2)To take the interests of both passengers and operators into consideration simultaneously, based on characteristic of passenger flow OD and dynamic passenger demands, a multi-objective integer linear programming model is proposed to minimize the number of passengers limited to enter stations, minimize the total passenger waiting time on platforms at all of involved stations and maximize the passenger person-kilometres.

(3) Train dwelling time has an important influence on the boarding/alighting behaviors at stations, so it is taken into consideration and is taken as an important constraint in the proposed model.

The rest of the paper is structured as follows. Section 2 provides the concrete definition of passenger inflow control with multi-station coordination problem. Section 3 describes the integer linear programming models taking the characters of passenger flow into account. In Section 4, solution approaches, including the methods to determinate the control time intervals and solve the model, are described. Two sets of numerical experiments, including a small-scale case and a real-world instance with operation data of Chengdu metro system, are carried out to verify the effectiveness of these models in Section 5. Finally, conclusions and further studies are presented in Section 6.

2 Problem Description

2.1 Descriptions of the Passenger Flow Control System

This study considers a single-direction oversaturated urban rail transit line with n stations,

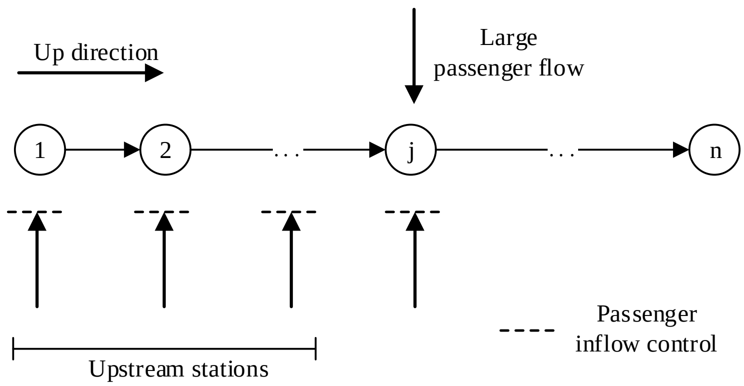

Figure 1: The illustration of urban rail transit line

high-frequency services and lack of train capacity in peak hours. The illustration of the line is shown in Figure 1. The line consists of stations and sections between these adjacent stations. Along certain operational direction (such as up direction.), the stations are numbered as 1, 2, ..., n consecutively, in which stations 1 and n represent the start and terminal stations respectively of in-service trains. For clarity, we use a set $N = \{1,2, \dots, n\}$ to identify stations in this direction. And trains run along the direction on the line according to fixed schedules.

Passenger flow collaborative control is related to the remaining loading capacity, which is dependent on the boarding and alighting passengers at upstream stations. As shown in Figure 1, it is assumed that there is large passenger flow at station j in peak hours. In order to rapidly evacuate passengers at this station, and guarantee operational safety and high service quality, the enough remaining loading capacity is necessary for this station. Therefore, the upstream stations, besides this station, start to limit the number of the inbound passengers according to the passenger demands at station j and other stations. For each station where passenger flow control is imposed, the inbound passengers are limited to queue at station entrances or gates, which reduce the congestion on platforms.

It is worth pointing out that the decrease of total demands will not be obvious after passenger flow control strategy is implemented. The ultimate objective for passenger flow control is to balance or reassign the limited transportation capacity among different stations (or sections) on the line, to achieve greater operation efficiency.

2.2 Definition of the Passenger Flow Control Problem

From the perspective of supply and demands, the participants are mainly operators and passengers in URT. When the passenger flow control strategies are formulated, the interests of both operators and passengers should be considered. For the passengers, they expect to quickly get to the destination from the departure, which can be reflected indirectly by the service quality of the operators. For the operators, increasing their revenues is an important goal. Since the revenues of URT are correlated with the travel distance and volumes of passengers, the passenger person-kilometres is the best indicator to measure their economic benefits.

As mentioned above, the passenger flow coordinated control is defined as a control

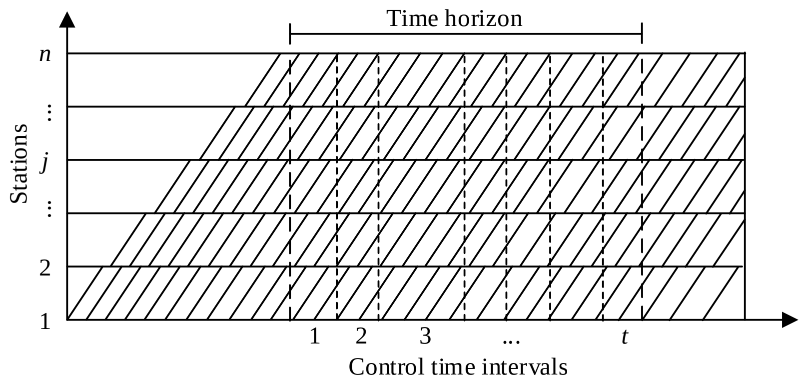

Figure 2: The illustration of the determination of control time intervals

congestion problem on the URT line to minimize the number of the passengers limited to enter the stations, minimize the total waiting time of stranded passengers and maximize the passenger person-kilometres in this paper. Note that the stranded passengers refer to passengers who cannot board the previous trains due to the limited train loading capacity and stay on platforms to wait the following trains.

Meanwhile, in order to guarantee the feasibility of the coordinative control strategies, some systematic constraints should be formulated, such as train capacity, train dwelling time, station design passing capacity, platform capacity and passenger demands, etc.

2.3 Descriptions of Determination of the Control Time Intervals

Since the passenger demands are time-variant, in order to obtain the accurate and easy-to-implement passenger flow control strategies, it is necessary to discretize the considered time horizon into several control time intervals, so that the control time intervals and the time-variant characteristics of the passenger demands are as close as possible. Hence, we disperse the continues time horizon into a finite number of control time intervals. And these intervals will be denoted by a set $T = \{1,2, \dots, t\}$. The illustration of the determination of control time intervals is shown in Figure 2.

For the determination of control time intervals, it is a great ideal to take the train departure interval as control time intervals. However, in the daily operation, it is more difficult to formulate and implement the passenger flow control strategies in such a short interval, in which there is also no statistical laws for passenger flow. In addition, if each control time interval is too long, the passenger flow control strategy cannot be flexibly adjusted as the passenger demands change, which leads to losing the meaning of passenger flow control. Therefore, it is necessary to develop a scientific method to scientifically determine the number of control time intervals and the length of each interval. And the specific method to determinate the control time intervals is described in Section 4.

In summary, the problem of the passenger flow coordinative control on an urban rail transit line can be summarized as:

On an over-saturated urban rail transit line, according to the space-time distribution dynamic characteristics of passenger flow, all stations on the line are associated in the space and time dimensions; meanwhile, according to the historical data of passenger flow, the considered time horizon is dispersed into several control time intervals; Under the constraints of train capacity, train dwelling time, station design passing capacity, platform capacity and passenger demands, each station adjusts the number of the inbound passengers by the collaborative control strategies in each control time interval on the line, to make urban rail transit systems achieve greater transportation efficiency.

Table 1: Sets, subscripts, input parameters and decision variables

Symbols	Descriptions
Sets and subscripts	
N	Set of stations.
T	Set of control time intervals.
i, j, k	Index of stations, $\forall i, j, k \in N$.
t	Index of control time intervals, $\forall t \in T$.
Input parameters	
$\alpha_{i,t}$	The ratio of the passenger volumes choosing certain operating direction to the total passenger volumes entering station i in interval t.
$Q_{i,t}$	The maximum number of passengers loaded by trains at station i in interval t.
L	The maximum loading capacity per train.
Z	The maximum number of passengers who can enter stations within an hour.
$\beta_{i,j,t}$	The ratio of the number of passengers from station i to j to the number of boarding passengers at station i in interval t.
P	The maximum number of passengers that the platform can hold.
$A_{i,t}$	The maximum number of passengers who can be accumulated on the platform at station i in interval t.
t^{s}	The time spent boarding the trains per person.
t^{x}	The time spent alighting from the trains per person.
$t_{i,t}^{\mathrm{dwell}}$	The average dwelling time of train at station i in interval t.
t_{o}	The time it takes for the trains to open and close the doors.
B	The number of carriages per train.
m	The number of doors of each train carriage.
$n_{i,t}$	The number of trains passed station i in interval t.
$C_{i,t}$	The maximum number of passengers allowed to enter station i in interval t.
$D_{i,t}$	The passenger demands at station i in interval t.
d_i	The length of section i.
θ	The minimum ratio of the number of the inbound passengers to the total passenger demands at each station in any interval.
ΔT_t	The length of control time interval t.
Decision variables	
$x_{i,t}$	The total number of passengers entering station i in interval t, including up and down direction.
$x_{i,t}^{\mathrm{s}}$	The number of boarding passengers at station i in interval t.
$x_{i,t}^{\mathrm{x}}$	The number of passengers alighting from trains at station i in interval t.
$S_{i,t}$	The number of passengers stranded on the platform at station i in interval t.
$l_{i,t}$	The number of passengers in trains when leaving the station i in interval t.
$\omega_{i,t}$	The rate of passenger flow control at station i in interval t.

3 Mathematic Model

3.1 Assumptions and Notations

The following assumptions are made in this paper and notations of the model and their descriptions are shown in Table 1.

(1) Trains run well on the line with fixed schedules.

(2) Travel demands and the characteristics of passenger flow OD are known in each control time interval, and passenger flow OD can be obtained from historical data.

(3) Passengers follow the principle of "alighting first, boarding later" in the whole boarding/alighting activities.

(4) There is no passengers stranded on platforms at all stations before the first control time interval.

(5) All passengers board trains without the passengers stranded on the platform when the remaining train capacity is more than the number of passengers accumulated on the platform at each station in each control time interval during train operation.

(6) This paper studies the passenger flow control on an urban rail transit line rather than a single station, so the influences of the capacity of station gates, staircases, escalators and elevators are weakened. And the indicator, the maximum number of passengers who can enter the station within an hour, is adopted to replace them.

(7)The process from the entrance facilities to the platform will not be considered. That is, passengers allowed to enter the station can arrival at the platform immediately. This similar assumption can also be found in by Shi et al. (2018).

(8)The passengers are evenly distributed on platforms with the staffs' guidance before trains arrive at stations in peak hours.

3.2 Decision Variables and the Related Expressions

The decision variables in this paper are defined in Table 1, and the related expressions are shown in formula (1)-(5). Note that all the decisions variables are non-negative integers.

(1) The number of passengers alighting from trains

The number of alighting passengers at the station i equals to the sum of the products of the number of boarding passengers at each upstream station and the alighting rate from it to i during interval t. And it can be expressed as:

$$x_{i,t}^{\mathrm{x}} = \sum_{k=1}^{i-1} x_{k,t}^{\mathrm{s}} \cdot \beta_{k,i,t}. \tag{1}$$

Where $\beta_{i,j,t}$ is known and given according to the historical data from Automatic Fare Collection (AFC).

(2) The number of passengers stranded on platforms

According to the principle of flow conservation, the number of passengers stranded on the platform at station i during interval t can be expressed as:

$$S_{i,t} = \begin{cases} x_{i,t} \cdot \alpha_{i,t} - x_{i,t}^{\mathrm{s}}, & t = 1 \\ S_{i,t-1} + x_{i,t} \cdot \alpha_{i,t} - x_{i,t}^{\mathrm{s}}, & t > 1 \end{cases}. \tag{2}$$

Where $\alpha_{i,t}$ is known and given according to the historical data from AFC.

(3) The number of passengers in trains

According to the principle of flow conservation, the number of passengers in trains when leaving station i during interval t can be expressed as:

$$l_{i,t} = \begin{cases} x_{i,t}^{\mathrm{s}}, & i = 1 \\ l_{i-1,t} + x_{i,t}^{\mathrm{s}} - x_{i,t}^{\mathrm{x}}, & i > 1 \end{cases}. \tag{3}$$

(4) The number of boarding passengers

The number of boarding passengers is the minimum of both the remaining loading capacity and the number of passengers accumulated on the platform at station i during interval t. For the start station, there is no alighting passengers. And there is no passengers in trains before trains arrival at the start station. So it can be given as:

$$x_{i,t}^{\mathrm{s}} = \min\{Q_t - l_{i-1,t} + x_{i,t}^{\mathrm{x}}, S_{i,t-1} + x_{i,t} \cdot \alpha_{i,t}\}. \tag{4}$$

(5) The rate of passenger flow control at stations

During the control time interval t, the rate of passenger flow control at station i is defined as the ratio of the number of passengers limited to enter this station to total passenger demands at this station during this control time interval.

$$\omega_{i,t} = \frac{D_{i,t} - x_{i,t}}{D_{i,t}}, i \neq n. \tag{5}$$

Where $D_{i,t}$ is known and given according to the historical data from AFC.

3.3 Constraints

The descriptions and the specific expressions the systematic constraints, including train capacity, platform capacity, train dwelling time, station design passing capacity, passenger demands, etc., are given in this subsection.

(1) Train capacity constraint

The number of passengers in trains should not exceed their maximum capacity in any control time interval. Note that the maximum capacity in any control time interval equals to the product of the number of trains passing station i in the interval and the maximum loading capacity of each train.

$$\begin{cases} l_{i,t} \leq Q_{i,t} \\ Q_{i,t} = n_{i,t} \cdot L \end{cases}. \tag{6}$$

(2) Platform capacity constraint

The number of passengers stranded on the platform should not exceed the platform capacity under the safe level at the end of the interval t. Meanwhile, the number of passengers accumulated on the platform will reach the maximum after passengers alight from trains. To ensure the safety, all the number of the inbound passengers, the stranded passengers and the alighting passengers combined should not exceed the maximum number of passengers who can be accumulated on the platform at station i in interval t.

$$\begin{cases} S_{i,t} \le P \\ x_{i,t} \cdot \alpha_{i,t} + S_{i,t-1} + x_{i,t}^{\mathrm{x}} \le A_{i,t} \\ A_{i,t} = n_{i,t} \cdot P \end{cases}. \tag{7}$$

(3) Train dwelling time constraint

When studying the influence of the large passenger flow on stations in peak hours, the passengers' boarding/alighting activities cannot be ignored. The trains will stay at each station for a period of time in order to complete the passengers' boarding/alighting service during their operation. According to the actual operation, passengers' boarding and alighting service is completed from the time when the doors are fully open to the time when the doors start to be closed after the train arrives at the station.

$$\frac{x_{i,t}^{\mathrm{s}} \cdot t^{\mathrm{s}} + x_{i,t}^{\mathrm{x}} \cdot t^{\mathrm{x}}}{n_{i,t} \cdot m \cdot B} \le t_{i,t}^{\mathrm{dwell}} - t_{\mathrm{o}}. \tag{8}$$

Where $t^{\mathrm{s}} = 0.76\mathrm{s/p}$, $t^{\mathrm{x}} = 0.55\mathrm{s/p}$, according to Cao (2009); Li (2011) pointed out that at the stations with screen doors, it takes $t_{\mathrm{o}} = 15s$ for the trains to open and close the doors; $t_{i,t}^{\mathrm{dwell}}$ can be determined according to the fixed schedules.

(4) Station design passing capacity constraint

The number of passengers entering the station at station i during interval t should not exceed the maximum number of passengers allowed to enter this station during this interval.

$$x_{i,t} \le C_{i,t}. \tag{9}$$

(5) Passenger demands constraint

The number of inbound passengers at station i during interval t should not be more than the realistic passenger demands in this interval. At the same time, there is still certain service ability at each station.

$$\begin{cases} x_{i,t} \ge \theta \cdot D_{i,t} \\ x_{i,t} \le D_{i,t} \end{cases}. \tag{10}$$

(6) Additional constraint

In order to reduce the waiting time of the passengers stranded on platforms as much as possible, it is necessary to ensure that the passengers stranded on platforms during interval t are served during interval $t + 1$.

$$S_{i,t} \le x_{i,t+1}^{\mathrm{s}}, t < T. \tag{11}$$

3.4 Objective function

In this paper, the objective functions for the problem of the passenger flow collaborative control on an urban rail transit line are as follows:

$$\begin{cases} \min z_1 = \sum_{i=1}^{n} \sum_{t=1}^{T} \left(D_{i,t} - x_{i,t} \right) \\ \min z_2 = \sum_{i=1}^{n} \sum_{t=1}^{T} S_{i,t} \cdot \Delta T_t \\ \max z_3 = \sum_{i=1}^{n} \sum_{t=1}^{T} l_{i,t} \cdot d_i \end{cases}. \tag{12}$$

Where z_1 is defined to minimize the number of passengers limited to enter the stations; z_2 is defined to minimize the waiting time of the passengers stranded on the platform involved all stations; z_3 is defined to maximize the passenger person-kilometres.

The multi-objective integer linear programming model of the passenger flow collaborative control with multi-station on an URT line in this paper is formulated by combining (1)-(12).

4 Solution Approaches

Note that the control time intervals must be determinated before solving the model to obtain all the input parameters of the model. Therefore, the method to determinate the control time intervals is described firstly,and finally the method to solve the model is depicted.

4.1 Algorithm for Determining Control Time Intervals

In the determination of the control time intervals, the existing studies disperse the time horizon into equal intervals (10min or 15min) as the control time intervals, without giving a scientific method. Therefore, in order to determine the control time intervals that is consistent with the time-variant characteristics of passenger demands and develop the accurate and easy-to-implement passenger flow control strategies, based on Fisher optimal division method (Xiao et al., 2014), the optimal control time intervals can be obtained by cluster analysis on the historical inbound passenger flow time series during the time horizon in this paper. The specific steps are as follows.

Step 1 Constructing inbound passenger flow time series and matrix.

Firstly, the historical data of passenger flow during time horizon is counted at intervals of Δt, then the inbound passenger flow time series is obtained. Let $\boldsymbol{H}_t$ be the inbound passenger flow matrix for the interval t, then the time series of the inbound passenger flow during the time horizon can be expressed as:

$$\boldsymbol{H} = \{\boldsymbol{H}_1, \boldsymbol{H}_2, \dots, \boldsymbol{H}_T\}. \tag{13}$$

Where $\boldsymbol{H}_t$ is given as:

$$\boldsymbol{H}_t = (p_1^t \quad p_2^t \quad \dots \quad p_n^t)^{\mathrm{T}}. \tag{14}$$

Where p_i^t is defined as the number of the inbound passengers at station i in control time interval t.

Step 2 Calculating class diameter.

Let the class G contain samples $\{\boldsymbol{H}_{(i)}, \boldsymbol{H}_{(i+1)}, \dots, \boldsymbol{H}_{(j)}\}$, denoted as $\mathrm{G} = \{\boldsymbol{H}_{(i)}, \boldsymbol{H}_{(i+1)}, \dots, \boldsymbol{H}_{(j)}\}$, where $j > i$, and the mean vector of the class is given as:

$$\boldsymbol{H}_{\mathrm{G}} = \frac{1}{j-i+1}\sum_{t=i}^{j} \boldsymbol{H}_{(t)} . \tag{15}$$

Then the diameter of the class G is:

$$D(i,j) = \sum_{t=i}^{j}\left(\boldsymbol{H}_{(t)} - \boldsymbol{H}_{\mathrm{G}}\right)^{\mathrm{T}}\left(\boldsymbol{H}_{(t)} - \boldsymbol{H}_{\mathrm{G}}\right). \tag{16}$$

The $D(i,j)$ is obtained by Step 2.

Step 3 Calculating the classification loss function.

The classification loss function is calculated by the following recursive formula. When n and k are fixed, the smaller $L[b(n,k)]$ is, the smaller sum of deviation square of all classes is, and the more reasonable the classification is. Therefore, it is necessary to find a classification method to minimize $L[b(n,k)]$. $P(n,k)$ is denoted as the classification method that $L[b(n,k)]$ takes the minimum value.

$$\begin{cases} L[b(n,2)] = \min\{D(1,j-1) + D(j,n)\}, 2 \le j \le n \\ L[b(n,k)] = \min\{L[P(j-1,k-1)] + D(j,n)\}, k \le j \le n \end{cases}. \tag{17}$$

The calculation steps are as follows:

- Calculate the optimal two-partition for $\boldsymbol{H} = \{\boldsymbol{H}_1, \boldsymbol{H}_2, \dots, \boldsymbol{H}_T\}$ according to (17);
- Calculate the optimal k-partition for $\boldsymbol{H} = \{\boldsymbol{H}_1, \boldsymbol{H}_2, \dots, \boldsymbol{H}_T\}$ according to (17).

The $L[b(n,k)]$ is obtained by the above steps.

Step 4 Determine the number of classifications-k .

Since the number of classifications- k cannot be predetermined, generally speaking, the inflection points in the k -changing trend diagram of $L[b(n,k)]$, can be used as the basis for determining the number of classifications- k. However, the inflection points maybe not unique. Therefore, In order to further determinate the number of classifications- k, this paper also uses an indicator, slope difference, to help find better k.

This paper calculates the slope difference between adjacent line segments in the graph according to the following formula. Note that $\gamma(k)$ is the slope difference between adjacent line segments at k.

$$\gamma(k) = \left|\frac{L[b(n,k-1)] - L[b(n,k)]}{(k-1)-k}\right| - \left|\frac{L[b(n,k)] - L[b(n,k+1)]}{k-(k+1)}\right|, 3 \le k \le n-1. \tag{18}$$

From the geometric sense, when $\gamma(k)$ reaches its maximum, point corresponding to k in the graph of $L[b(n,k)] \sim k$ divides the line in the graph into two parts. It is steep before this point and it is relatively flat after this point in the graph of $L[b(n,k)] \sim k$, which means there are few differences among classifications after this point and it is meaningless if it continues to be divided. That is, k is the better number of classifications at this moment. And the better k is obtained. In addition, when $\gamma(k)$ is close to 0, it no longer continues to be divided.

4.2 Model Solution

This model is a multi-objective integer linear programming model, which can be effectively handled through linear weighted methods in this paper.

5 Numerical Experiments

In this section, a series of numeric experiments, involving a small-scale case study and a real-world case study in Chengdu Metro System, are implemented to illustrate the applications of the proposed model. And the model is solved by CPLEX solver.

5.1 A Small-scale Case Study

In this part, we consider a single-direction line with 3 stations and 4 control time intervals, to test the performance of the model.

(1)Experiment descriptions and Parameter settings

For the convenience of description, we particularly name the stations A, B, C, and name the control time intervals T1, T2, T3, T4. In the experiments, the data is shown in Table 2. The train capacity is 80 persons, the length of each section is 3 km and the platform capacity is 140 persons.

Table 2: The data of the small-scale case

Origin	**Interval number(Length)**	**Destination**	$\boldsymbol{\beta_{i,j,t}}$	$\boldsymbol{\alpha_{i,t}}$	**Demands**
A	T1（6）	B C	0.3 0.7	1	300
	T2（3）	B C	0.4 0.6	1	200
	T3（6）	B C	0.5 0.5	1	200
	T4（6）	B C	0.6 0.4	1	100
B	T1（6）	C	1	0.8	200
	T2（3）	C	1	0.8	100
	T3（6）	C	1	0.8	100
	T4（6）	C	1	0.8	50

(2)Computational results analysis

The experiment is solved by CPLEX solver, and the optimal solution is obtained: $z_1 = 304$ persons, $z_2 = 2004$min, $z_3 = 3345$person-kilometres. The number of the boarding passengers, the alighting passengers, the passengers stranded on platforms and the passengers limited to enter the stations from the case is shown in Table 3.

Table 3: The results of the numerical example

Stations	**Intervals**	$\boldsymbol{x^{s}_{i,t}}$	$\boldsymbol{x^{x}_{i,t}}$	**The number of the stranded passengers**		$\boldsymbol{\omega_{i,t}}$ **(%)**
				Platforms	**Out of stations**	
1	T1	155	0	0	145	48.3
	T2	80	0	60	60	30.0
	T3	160	0	100	0	0
	T4	160	0	40	0	0
2	T1	52	47	30	97	48.5
	T2	32	32	76	2	2.0
	T3	80	80	76	0	0
	T4	96	96	20	0	0
3	T1	0	160	0	0	0
	T2	0	80	0	0	0
	T3	0	160	0	0	0
	T4	0	160	0	0	0

The optimal solution is shown in Table 3. As seen in Table 3, the time intervals to implement the flow control measures at stations 1 and 2 is mainly in the first two control time intervals, and passengers entering the stations aren't limited in the last two intervals. During the first two control time intervals, passenger demands are large, and some passengers cannot enter the stations due to the limited train capacity and platform capacity. In the following intervals, passenger demands are decreased at each station, and these passengers can be gradually satisfied by the platform capacity. And the passenger flow control measures are gradually removed. Meanwhile, train capacity gradually satisfies the demands of the stranded passengers on the platform at each station, and the number of the stranded passengers are gradually decreased. During the last interval, there are still the passengers stranded on the platform at stations 1 and 2, and these passengers can be loaded by the following available trains. The experimental results shows that the model can accurately describes the behaviors of the passenger flow control for the problem of passenger flow coordination control on an urban rail transit line.

5.2 Numerical Experiments on Chengdu Metro Line 2

To further demonstrate the performance of the proposed model for large-scale problems, we next consider a real-world case study on the Chengdu metro line 2 with 32 stations. And we only consider the up direction. In the implementations, all the dynamic input parameters are obtained from AFC. In the following, we shall first give the detailed experimental descriptions and parameters.

(1)Experiment descriptions and parameter settings

In the experiments, the considered time horizon is set as 7:30-9:00, which is the morning peak-hours. And the stations are numbered as 1, 2, ..., 32 consecutively along up direction on the line, in which stations 1 and 32 represent the start and terminal stations respectively of in-service trains. Note that the transfer passenger flow is converted into the inbound passenger flow or the outbound passenger flow at the transfer station on the line. The parameters settings are as follows.

• The determination of control time intervals

The considered time horizon (7:30-9:00) is dispersed into 18 statistical intervals at intervals of $\Delta t = 5min$, to construct an inbound passenger flow time series $\boldsymbol{H} = \{\boldsymbol{H}_1, \boldsymbol{H}_2, \dots, \boldsymbol{H}_{18}\}$. Calculate it according to the steps in Subsection 4.1, and the $L[b(n,k)] \sim k$ trend diagram is obtained by using C#, as shown in Figure 3.

According to Fisher optimal division method, the optimal number of classifications is obtained at the inflection point of the curve. However, the inflection points are not unique (such as 4,5,6), as seen in Figure 3. So the slope difference between adjacent line segments is calculated according to the formula (18). And the $\gamma(k) \sim k$ trend diagram is plotted and shown in Figure 4.

As seen Figure 4, when $k = 5$, $\gamma(k)$ reaches the maximum. According to step 4 in subsection 4.1, the inbound passenger flow time series is divided into 5 clusters, and the results of classification are shown in Table 4.

According to the divided control time intervals, combing the historical AFC data and the schedules, $\beta_{i,j,t}$, $\alpha_{i,t}$, $D_{i,t}$ and $n_{i,t}$ can be obtained.

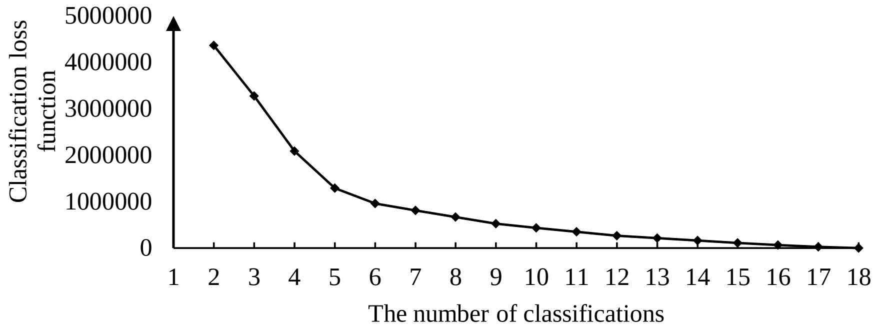

Figure 3: $L[b(n,k)]\sim k$ trend diagram

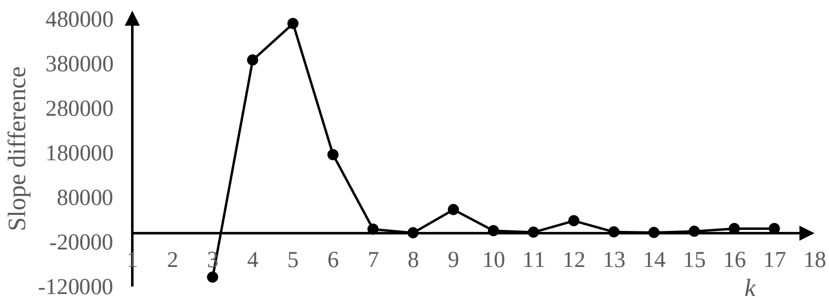

Figure 4: $\gamma(k)\sim k$ trend diagram

Table 4: The determination of the control time intervals

Classification number	Statistical time interval	Control time interval
1	T1-T4	7:30-7:50
2	T5-T7	7:50-8:05
3	T8-T11	8:05-8:25
4	T12-T15	8:25-8:45
5	T16-T18	8:45-9:00

- Model other parameters

The values of model other parameters are shown in Table 5.

Table 5: Model parameters

Parameters	Values
L	1888
Z	21000
P	1100
θ	0.5
Time horizon	7:30-9:00

(2) Computational results analysis between collaborative and non- collaborative control

The input parameters are obtained in (1), and the model is solved by CPLEX solver efficiently. The objective function values, the mean crowding degree on the platform at each station and the average number of passengers limited to enter stations at each station are selected as three indicators to make a comparative analysis between the passenger flow non-collaborative and collaborative control strategies.

The optimal solution, passenger flow collaborative control strategy, is shown in Table 6, and the objective function values are shown in Table 7.

Note that the passenger flow collaborative control strategy is obtained by solving the proposed model while the passenger flow non-collaborative control strategy is based on the idea that passengers at upstream stations board the trains with the priority and obtained by successive recursion method after giving the upstream stations greater weights under the constraints of train transportation capacity, train dwelling time and station design passing capacity constraints. And the objective function values of the passenger flow non-collaborative control strategy are shown in Table 7.

Table 6: Time-dependent and average inflow control rate of stations

Control time intervals	**Stations**							（%）
	8	9	10	11	12	13	15	16
7:30-7:50	49.9	—	34.9	47.6	—	—	20.5	—
7:50-8:05	49.9	48.8	49.3	49.3	7.6	1.2	10.3	—
8:05-8:25	49.9	48.2	48.6	50.0	36.1	0.7	24.1	—
8:25-8:45	—	—	50.0	—	33.7	—	37.6	9.4
8:45-9:00	—	—	—	—	18.0	—	30.5	28.9
The average control rate	29.9	19.4	36.6	29.4	19.1	0.4	24.6	7.7

As seen in Table 6, during the time horizon, the stations with the limited passenger flow are mainly from station 8 to 16, which is consistent with the practical situation. From the perspective of time, the control time interval is mainly 7:50-8:25. In the first four intervals, passenger flow control is imposed at the upstream stations, to ensure the rapid evacuation of the large passenger flow at stations 12, 15 and 16. In the last interval, passenger flow control is mainly implemented at stations 12, 15 and 16, to guarantee the rapid evacuation of the passengers stranded on platforms at the upstream stations. As time goes by, passenger demands are decreased. Some stations can remove passenger flow control measures or reduce passenger flow control intensity. During the last interval, some passengers at stations 12, 15 and 16 are still limited to enter the stations, and these passengers can be served by the following available trains. From the average rate of passenger flow control at the stations, passenger flow control intensity is large at stations 8, 10 and 11, which shows that these stations suffered from greater passenger flow organization pressure.

Table 7: The objective under the uncoordinated and coordinated conditions

Objective function	**Passenger flow non-collaborative control**	**Passenger flow collaborative control**
z1	25587	23190
z2	88650	0
z3	908175	909861

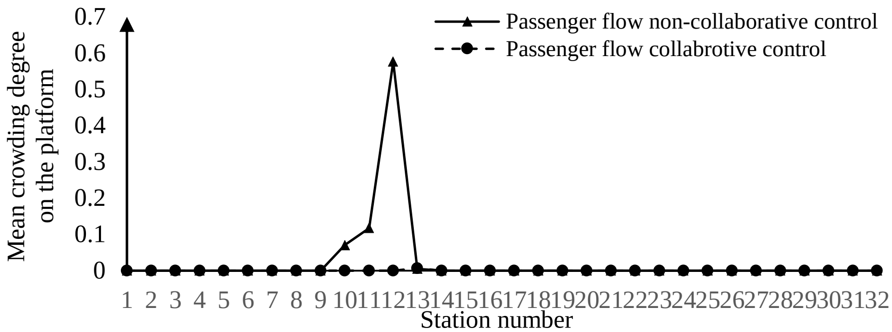

Figure 5: Mean crowding on the platform of stations

As seen in Table 7, the number of passengers limited to enter stations under the collaborative control is reduced by about 9.4% than that under the non-collaborative control. There are no passengers stranded on platforms under the collaborative control. And the passenger person-kilometres is increased by roughly 0.2% than that under the non-collaborative control. Overall, for the operation enterprises, the service quality can be intensely improved and effectively alleviate the contradiction between the limited transportation capacity and passenger demands under the collaborative control. Meanwhile, there is a small increase on the passenger person-kilometres. That is, the economic performance is also enhanced.

Mean crowding degree on the platform under passenger flow non-collaborative and collaborative control is shown in Figure 5. The mean crowding degree on the platform is defined as the ratio of the average number of the passengers stranded on the platform at each station in the five control time intervals to the capacity of the platform at each station in this paper, and this indicator can well reflect the passenger flow on the platform at each station in the whole time horizon.

As shown in Figure 5， the passengers are accumulated with an unbalanced situation at several stations under the non-collaborative strategies. For example, the mean crowding degree of platform at the station 12 is highest, which shows that the congestion situation is very serious on the platform. And stations 10 and 11 are less crowded, which represents that passenger flow pressure is relatively small on the platform at these stations; nevertheless, the mean crowding degree of platform at each station is 0 under passenger flow collaborative control, which shows that the proposed model can effectively reduce the number of the passengers stranded on the platform at each station and avoid potential accident risks caused by it.

The average number of the passengers limited to enter stations at each station under the passenger flow non-collaborative and collaborative control is shown in Figure 6. As seen in Figure 6, under the passenger flow non-collaborative control, the limited inbound passengers are mainly at stations 12, 15 and 16, and there is huge passenger volume at these stations, which shows these stations undertake the huge pressure caused by the large passenger flow. However, under the passenger flow collaborative control, the limited inbound passengers are mainly at stations from 8 to 16. And the number of the limited inbound passengers is reduced at stations 12 and 15 than that under the non-collaborative control, which shows that the proposed model can effectively balance the passenger flow pressure among all the stations and accelerate the evacuation of the passenger flow at the s-

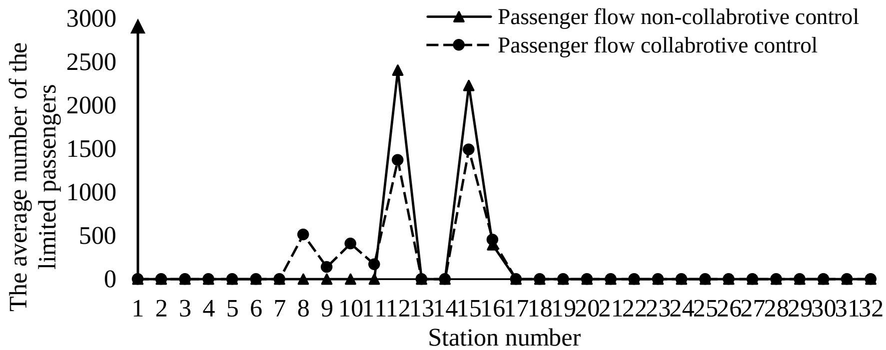

Figure 6: The average number of the passengers limited to enter stations

tations where there is the large passenger flow.

For the transfer passenger flow, since the cross-line passenger flow is converted into the passenger flow of the line in this paper, the transfer passenger flow and the inbound passenger flow are not distinguished at the transfer station during the calculation. Therefore, when the passenger flow control strategies are formulated, appropriate adjustments should be made based on the results according to the practical operation.

(3) Computational results analysis of different type of control time intervals

The control time interval is mainly classified two types, including the control time intervals obtained by Fisher optimal division method and equal control time intervals. This section mainly analyses the influences on the computational results of different type of control time intervals. For the former, the results are presented by (2); for the latter, the length of each control time interval is set as 15min, and the number of the intervals is 6, the determination of the latter is shown in Table 8. And the computational results of different type of control time intervals are as shown in Table 9.

Table 8: The determination of the equal control time intervals

Classification number	**Statistical time interval**	**Control time interval**
1	T1-T3	7:30-7:45
2	T4-T6	7:45-8:00
3	T7-T9	8:00-8:15
4	T10-T12	8:15-8:30
5	T13-T15	8:35-8:45
6	T16-T18	8:45-9:00

Table 9: The influences on the results of different type of control time intervals

Objective function	**The control time intervals obtained by Fisher optimal division**	**The equal control time intervals (15min)**
z1	23190	22967
z2	0	0
z3	909861	880818

In the following analysis, note that the former refers to the result of the model using control time intervals obtained by Fisher optimal division method; the latter refers to the result the model using equal control time intervals.

As seen in Table 9, the number of passengers limited to enter stations for the latter is reduced by about 1% than that of the former while the passenger person-kilometres for the former is increased by about 3.3% than that of the latter. And there are no passengers stranded on platforms for both the former and latter. Combining the Table 7, the number of passengers limited to enter stations for the latter is reduced by about 10.4% than that under the non-collaborative control while its passenger person-kilometres is reduced by roughly 3% than that under the non-collaborative control.

From the above analysis, it is concluded that the former is better than the latter in the trade-off between the service quality and the profits of management enterprise. That is, the results of the model using the control time intervals obtained by Fisher optimal division is better than that using equal control time intervals, which verifies the performance of the proposed approach.

6 Conclusions

In this paper, the problem of the passenger flow cooperative control on an urban rail transit line in peak hours is studied under the condition of limited transportation capacity. The mixed integer linear programming model of the passenger flow cooperative control on the line is developed. Through analysis of the instance, the objective function value, the mean crowding degree on platforms at all of involved stations and the average number of the passengers limited to enter stations are compared under the strategy obtained by solving the model and the non-collaborative control strategy. The results show that the former is better than the latter in the above three aspects, which tests the performance of the proposed approach and provides a theoretical basis and reference on the practical application.

This paper assumes that the trains run well on the line with the fixed schedules. However, a disturbance or disruption always happens inevitably in the daily operation. For example. There is an extension of dwelling time at a station with large passenger flow on the platform, which will impose influence on the train operation, such as delays. Under the circumstance in which a disturbance or disruption happens, train regulation is typically necessary. Therefore, further considering the disturbance or disruption in the train operation process, extending our work to jointly optimize train regulation and passenger flow control can be our future research directions.

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References

Baee, S., Eshghi, F., Hashemi, S.M., Moienfar, R., 2012. "Passenger boarding/alighting management in urban rail transportation", In: *2012 Joint Rail Conference. American Society of Mechanical Engineers*, pp. 823–829.

Chen, S., Liu, S., Xiao, X., Hong, J., Mao, B., 2012. "M/G/c/c-based model of passenger

evacuation capacity of stairs and corridors in metro stations", *Journal of the China railway Society*, vol.34, pp. 7–12.
Cao, S., 2009. *Analysis and modeling on passengers traffic characteristics for urban rail transit*. Ph.D. Dissertation, School of Traffic and Transportation, Beijing Jiaotong University, Beijing, China. (in Chinese)
Davidich, M., Geiss, F., Mayer, H.G., Pfaffinger, A., Royer, C., 2013. "Waiting zones for realistic modelling of pedestrian dynamics: a case study using two major German railway stations as examples", *Transportation Research Part C: Emerging Technologies*, vol. 37, pp. 210–222.
Fernández, R., Valencia, A., Seriani, S., 2015. "On passenger saturation flow in public transport doors", *Transportation Research Part A: Policy and Practice*, vol. 78, pp. 102–112.
Jiang, Z., Fan, W., Liu, W., Zhu, B., Gu, J., 2018. "Reinforcement learning approach for coordinated passenger inflow control of urban rail transit in peak hours", *Transportation Research Part C: Emerging Technologies*, vol. 88, pp. 1-16.
Li, S., Dessouky, M., Yang, L., Gao Z., 2017. "Joint optimal train regulation and passenger flow control strategy for high-frequency metro lines", *Transportation Research Part B: Methodological*, vol. 99, PP. 113-137.
Li, Z., 2011. "Research on dwelling time at Tianfu-Square station for Chengdu Metro Line 1", *Journal of Transportation Engineering and Information*, vol. 9, pp. 20-24. (in Chinese)
Hoogendoorn, S.P., Bovy, P.H.L., 2004. "Pedestrian route-choice and activity scheduling theory and models", *Transportation Research Part B: Methodological*, vol. 38, pp. 169-190.
Niu, H., Zhou, X., Gao, R., 2015. "Train scheduling for minimizing passenger waiting time with time-dependent demand and skip-stop patterns: Nonlinear integer programming models with linear constraints", *Transportation Research Part B: Methodological*, vol. 76, pp. 117-135.
Shi, J., Yang, L., Yang, J., Gao, Z., 2018. "Service-oriented train timetabling with collaborative passenger flow control on an oversaturated metro line: An integer linear optimization approach", *Transportation Research Part B: Methodological*, vol. 110, pp. 26-59.
Xiao, C., Gu, S., Gui, W., Gao, L., Li, Z., 2014. "Application of Fisher Optimal Partition Method in Division of Flood Season in Lixianjiang Basin", *Water Resources and Power*, vol. 32, pp.70-74. (in Chinese)
Xu, X., Liu, J., Li, H., Hu, J., 2014. "Analysis of subway station capacity with the use of queueing theory", *Transportation Research Part C: Emerging Technologies*, vol. 38, pp. 28–43.
Xu, X., Liu, J., Li, H., Jiang, M., 2016. "Capacity-oriented passenger flow control under uncertain demands: algorithm development and real-world case study", *Transportation Research Part E: Logistics and Transportation Review*, vol. 87, pp. 130-148
Wang, L., Yan, X., Wang, Y., 2015. "Modeling and Optimization of Collaborative Passenger Control in Urban Rail Stations under Mass Passenger Flow," *Mathematical Problems in Engineering*, vol. 2015, pp. 8.
Wang, Y., De Schutter, B., van den Boom, T. J. J., Ning, B., Tang, T., 2014. "Efficient bi-level approach for urban rail transit operation with stop-skipping", *IEEE Transactions on Intelligent Transportation Systems*, vol. 15, pp. 2658 – 2670.