Reducing the Adaptation Costs of a Rolling Stock Schedule with Adaptive Solution: the Case of Demand Changes

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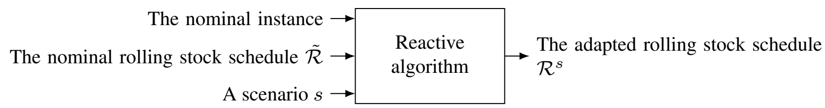
Abstract
In railway scheduling, a nominal traffic schedule is established well in advance for the main resources: train-paths, rolling stock and crew. However, it has to be adapted each time a change in the input data occurs. In this paper, we focus on the costs in the adaptation phase. We introduce the concept of *adaptive nominal solution* which minimizes adaptation costs with respect to a given set of potential changes. We illustrate this framework with the rolling stock scheduling problem with *scenarios* corresponding to increasing demand in terms of rolling stock units. We define adaptation costs for a rolling stock schedule and propose two MILPs. The first one adapts, at minimal cost, an existing rolling stock schedule with respect to a given scenario. The second MILP considers a set of given scenarios and computes an adaptive nominal rolling stock schedule together with an adapted solution to each scenario, again while minimizing adaptation costs. We illustrate our models with computational experiments on realistic SNCF instances.

Keywords
Rolling Stock, Adaptive Solution, Discrete Optimization.

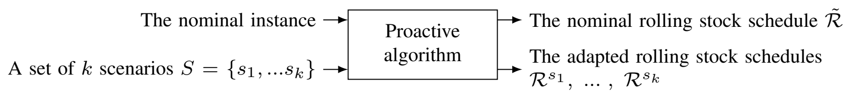
1 Introduction

Railway scheduling is generally divided into different problems, which are solved sequentially. The *line planning* problem computes train lines based on the existing rail network, defining a list of stations and an associated frequency for each line. The *timetabling* problem defines a set of trains with departure and arrival times for each station of the considered lines, with respect to the frequency, providing a complete feasible timetable. The *rolling stock scheduling* problem defines compositions for each train, assigning physical rolling stock units to the given input timetable. The *crew scheduling* problem operates in a similar manner, assigning crew members (*e.g.* train drivers) to each train and each station with respect to specific legal constraints. Finally, the *platforming* problem is solved for each station, assigning a track to each train stopping by or passing through it during the planning horizon.

Ideally, we would like to solve most of these problems together in an integrated manner a few days before the date of operations; in practice, these problems are solved sequentially several years or months in advance for historical, legal and practical reasons. Thus, a complete *nominal* schedule is built some months in advance for each railway resource: train-paths, rolling stock and crew.



(a) The Rolling Stock Adaptation Problem



(b) The Adaptive Rolling Stock Scheduling Problem

Figure 1: The two different methods described in this paper

However, changes may occur afterwards, either prior or during operations. They may either concern the availability of resources, such as infrastructure blockades or rolling stock failures, or some new requirements, such as additional trains to schedule or some changes in their required compositions.

Whenever changes occur, the schedules must be updated. We focus on midterm changes during the *adaptation* phase, which corresponds to rescheduling of resources a few weeks or months before operational time. A schedule can be adapted many times if changes during the adaptation phase are frequent. We describe the changes with the notion of *scenario*, corresponding to a modification in the input data. An *adapted* schedule with regards to this scenario is then computed, in order to satisfy the changes. Our main objective is to reduce the total cost of the adaptation phase.

The rest of this paper is organized as follows. Section 2 is dedicated to a literature review on the main issues discussed in this paper, with a focus on rolling stock resource. We describe in Section 3 the *adaptation costs* in general railway scheduling, and propose a new approach to assessing adaptation costs for the *rolling stock* resource. Besides considering the performance of the new schedule, we also consider *structural* adaptation costs to assess the differences between the nominal and the adapted schedules. We introduce in Section 4 the *Rolling Stock Adaptation Problem* with respect to a given *scenario* in the case of *demand* changes. This is the *reactive* problem appearing during the adaptation phase where a given scenario is revealed (see Figure 1a). A MILP formulation based on the literature review is proposed. In Section 5, we define the notion of *adaptive* nominal solution with regards to a set of scenarios. A nominal solution in the conception phase is said to be adaptive with respect to a set of scenarios if its adaptation cost to each of these scenarios is low. The corresponding *proactive* problem (see Figure 1b) appears in the conception phase where information is available about the probability of occurrence of certain possible scenarios. The *Adaptive Rolling Stock Scheduling Problem* is introduced and a MILP is proposed to solve it. We present in Section 6 computational experiments with realistic instances of SNCF, the major French train operating company. Finally, Section 7 concludes and highlights future perspectives.

2 Literature Review

Models for Rolling Stock Scheduling

There exist a lot of models to schedule rolling stock, with different assumptions.

Fioole *et al.* (2006) introduce the Rolling Stock Circulation Problem, defining a MILP with variables affecting a unique *composition* to each trip. They define a dedicated event graph and obtain a flow formulation. Each trip has one or two successor trips defined as input, which is a strong assumption because it restricts the possibilities.

Cacchiani *et al.* (2010) introduce the *Train Unit Assignment Problem*. They define a graph where each node corresponds to a trip. The authors solve a flow problem with a path formulation, and propose some improvements for the linear relaxation by describing the convex hull of a set of constraints. In this paper, we use a graph similar to this one in order to formulate the MILPs. However, we use a flow formulation, which is more relevant to model adaptation costs.

Giacco *et al* (2014) introduce a rolling stock scheduling problem integrating the maintenance requirements and the empty moves possibility. They define a dedicated graph and propose a MILP to compute a set of hamiltonian paths respecting the maintenance constraints.

Borndörfer *et al.* (2016) introduce a novel approach to schedule railway vehicle rotations. They define a generic *hypergraph* where each train has a departure and an arrival node for each possible composition. Oriented hyperarcs are defined between two set of nodes of two different trains, and indicate the possibility to cover these trains with the same rolling stock units. A MILP formulation is proposed with additional maintenance requirements. It is solved with a dedicated algorithm using column generation and rapid branching heuristics.

Adaptation Costs of a Rolling Stock Schedule

Many papers address real-time disruption management of rolling stock. These *rescheduling* models generally use models deriving from those presented above for rolling stock scheduling. They mostly model the rolling stock adaptation costs by assessing the new performance of the adapted rolling stock schedule and try to minimize the new shunting operations.

Nielsen *et al.* (2012) propose a generic framework for rolling stock rescheduling with rolling horizon approach based on the Rolling Stock Circulation Problem of Fioole *et al.* (2006). They assume major disruptions (infrastructure blockades) and try to reschedule the rolling stock with a dedicated real-time heuristic. Their main objective is to minimize the cancelled trips because of a lack of rolling stock. More recently, Wagenaar *et al.* (2017) propose a MILP formulation based on Fioole's model for the Rolling Stock Rescheduling Problem, while considering dead-head trips (empty moves) possibility and dynamic passenger demands. It allows respectively to decrease the number of cancelled trips and to capture the fact that a cancelled trip will have influence on the passenger demand for the next trip with the same origin and destination. Lusby *et al.* (2017) propose an original approach to solve a rescheduling problem with a dedicated Branch&Price framework. It is based on a path formulation with specific constraints representing operational requirements.

Some papers deal with changing circumstances in the short-term planning stage. Ben-Khedher *et al.* (1998) describe the *Capacity Adjustment Problem*: considering the number of reservations for each train and some forecasts of a yield management system, they try to adjust the compositions of the scheduled trains in order to maximize the expected profit.

The model computes a feasible schedule with these new compositions, but adaptation costs are not explicitly taken into account.

Lingaya *et al.* (2002) propose a MILP model to schedule locomotives and carriages a few days before operations. They consider a changing (static) demand in terms of cars and specific operational constraints in their problem such as maintenance requirements or minimum connection times. They try to modify the current rolling stock schedule to fit these demand changes and operational constraints. They do not explicitly focus on structural adaptation costs, but they consider it implicitly: they only accept to make changes in the *car cycles*, and do not modify the *locomotive* schedules. Thus, changes are limited, and structural adaptation costs are restricted.

Budai *et al.* (2010) address the rolling stock rebalancing problem. They suppose a lack of units at certain stations at the end-of-day and a surplus at other stations, and try to reduce these off-balances by rolling stock rescheduling. Adaptation costs correspond to the classical nominal performance costs and the changes in shunting plans.

More recently, Borndörfer *et al.* (2017) introduce the re-optimization of rolling stock rotation while considering a reference rotation. They use a hypergraph and define a *template* as a set of trips in the reference rotation such that they are covered by the same rolling stock units. They try to keep these templates unchanged in the adapted rotations. The objective function introduces the notion of *deviation* from the reference rotation. Our definition of adaptation costs is quite similar but is based on a simpler model with an adaptive version that is easier to solve. The authors use different scenarios corresponding to infrastructure constructions where timetables slightly change, which implies to reschedule the rolling stock rotations. In this paper, we focus on demand changes and do not suppose any modification of the timetables.

Adjustable Robustness and Recoverable Robustness

The concept of adaptive solutions is closely related to the concepts of adjustable and recoverable robustness.

The concept of *adjustable* optimization was originally introduced by Ben-Tal *et al.* (2004). Following the context of bi-level stochastic optimization, they consider uncertainty set for some parameters, and solve a mathematical program with two types of variables:

- *here-and-now* variables x must be fixed at the early stage of the optimization process;
- *wait-and-see* variables y must be fixed once a scenario is revealed.

The problem is to assign values to the x variables such there exists y values with (x, y) feasible for any realization of the uncertainty set. For this purpose, the authors introduce variables $y(\xi)$ for each ξ in the uncertainty set, and show that this problem is untractable in the general case.

If we consider wait-and-see variables y corresponding to a *recourse* of the here-and-now variables x, we obtain the concept of recoverable robustness, originally introduced by Liebchen *et al.* (2009). The authors consider an uncertainty set with finite support such that it corresponds to a finite set of *scenarios* S. They describe the recourse variables y as a *recourse algorithm* $\mathcal{A}$. If we consider the generic mathematical program minimizing $f(x)$ subject to a feasibility set for vector x, the associated recoverable robust problem aims to find a solution x and an algorithm $\mathcal{A}$ such that $y = \mathcal{A}(x, s)$ is feasible for each scenario $s \in S$. Algorithm $\mathcal{A}$ must be chosen in a class of algorithms and can have a certain recovery cost to be added to the objective function. Cicerone *et al.* (2009) describe such class for $\mathcal{A}$. For example, $\mathcal{A}$ has to run within a maximal time limit. Our model presented Section 5 is a recoverable robust model where algorithm $\mathcal{A}$ is a MILP, with recourse costs corresponding to the differencies between solutions x and y.

Recoverable robustness was originally applied in railway scheduling. Recoverable robust timetabling was introduced by Liebchen *et al.* (2009) and Cicerone *et al.* (2009), where the uncertainty concerns minimal required time between several pairs of arrival and departure times of a train. The authors compute nominal recoverable robust schedules and propose different classes of algorithms to reschedule trains. These authors also consider applications to platforming and shunting yard problems.

A recoverable robust rolling stock scheduling problem is addressed by Cacchiani *et al.* (2012) with uncertainties corresponding to infrastructure blockade. The authors propose a large MILP based of the model of Fioole *et al.* (2006). They duplicate the nominal variables for each scenario, and minimize both the performance of the nominal solution and the maximal recovery costs among the scenarios. The recovery costs for a given scenario are described with the cancelled trips, the off-balanced units at end-of-days, and the new shunting operations. The authors use Benders decomposition to compute optimal solutions for the relaxed problem, and develop a dedicated Benders heuristic to compute integer solutions to the recoverable robust problem. Our model is based on a different formulation and does not suppose operational disruptions, focusing on demand changes in the adaptation phase. Moreover, our adaptation costs allow to maximize the similarities between the nominal and adapted rolling stock schedules. Another difference is that we minimize the expected adaptation cost instead of the worse one among the scenarios.

3 The Adaptation Costs

In this section, we describe in more details the adaptation phase in railway scheduling. We identify different performance criteria to evaluate the quality of an adaptation, and propose a simple way to evaluate the performance of a rolling stock schedule adaptation.

3.1 Adaptation Costs in General Railway Scheduling

We identify three types of "costs" in the adaptation phase for any railway resource.

1. *Performance cost*
   An adapted schedule has to be assessed with regards to the classical performance criteria. For example, if there is a change in the timetable, the adapted timetable must maximize the passenger satisfaction. However, finding an optimal solution is not crucial in a rescheduling process: we generally look for an acceptable schedule. These performance costs only depend on the adapted schedule, and we can compute them without any information about the nominal one.
2. *Direct costs during the adaptation phase*
   During the adaptation phase, each request of change may impact several departments. They have to look for a new acceptable schedule compatible with the new requirements. It can be difficult and impact different resources, and it implies communication between the departments, which can be interpreted as a direct adaptation cost.
3. *Indirect operational costs*
   Each schedule is generally repeated with a specific horizon (an hour, a day or a week). An adaptation concerns some periods where the schedules are quite different. Thus, an adaptation can have operational consequences. The more different the adapted

schedule from the nominal one, the higher the risk of human error at operational time, which would lead to bad performance, or an increased risks of incidents. We can interpret this as an indirect operational cost of an adaptation.

Let us observe that reducing the first type of cost can lead to an increasing of the two others. Indeed, if we want to have a good performance cost for the adapted schedule, we have to consider the rescheduling of a higher number of resources. It implies a lot of communication between the departments and is responsible for a higher direct cost during the adaptation phase. Moreover, the adapted schedule will probably be very different from the nominal one, implying a higher risk of operational errors at operational time and an increase in the indirect operational costs.

Furthermore, if we force the adapted schedule to be *similar* to the nominal one, we find that it reduces indirect operational costs, but it has also a strong positive impact on the direct cost during the adaptation phase. Indeed, if we want the adapted schedule to be similar to the nominal one, we have to look for an adapted schedule in a smaller solution search space, and it reduces both the number of implied departments and the communication between them. Thus, this notion of similarity between the schedules is the relevant criterion to maximize, or in other words, minimizing the changes between the schedules captures both the direct costs in the adaptation phase and the indirect operational costs. Consequently, we define two complementary types of cost in the adaptation phase:

- the *performance* adaptation costs to evaluate the quality of the adapted schedule with regards to classical nominal performance criteria;
- the *structural* adaptation costs to evaluate the similarities and the differences between the adapted schedule and the nominal one.

3.2 Adaptation of a Rolling Stock Schedule

Performance Adaptation Costs

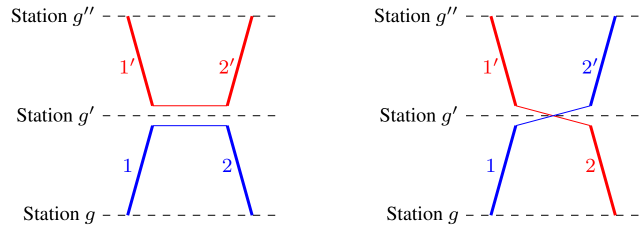
As previously mentioned, the non-optimality of an adapted rolling stock schedule for the classical nominal performance criteria is a first type of adaptation costs. Concerning the rolling stock resource, one generally has to minimize the following criteria:

- The total *lack* of rolling stock units: it is sometimes impossible to propose a schedule with a sufficient number of units for all trains, and we try to minimize the number of missing units;
- The number of *engaged rolling stock units*;
- The number of kilometers of *dead-head trips* for each unit, which correspond to trips between two stations without any passenger (empty moves);
- The number of kilometers of *over-compositions* for each unit, which correspond to trips with higher number of units than required.

Structural Adaptation Costs

In the literature structural adaptation costs are generally defined as the modifications in the shunting plans: if two additional trains have to be combined in the adapted shunting plan, it has indeed a certain operational cost.

We introduce a new definition of structural adaptation costs via the notion of *successions* between trains. Suppose there is a rolling stock unit of type m that covers Train 1, and then covers Train 2 without any train between 1 and 2. In particular, it implies that the arrival



(a) Nominal schedule

(b) Adapted schedule

Figure 2: An example of changes in the successors. In the nominal schedule Figure 2a, Train 2 is the successor of Train 1 and Train 2' is the successor of Train 1'. In the adapted schedule Figure 2b, Train 2 is now the successor of Train 1' and Train 2' the successor of Train 1.

station of Train 1 is the same as the departure station of Train 2. Then, Train 2 is a *successor* of Train 1 for type m, and the *succession* 1-2 exists for this unit type.

More precisely, for each couple of trains i and j and for each unit type m, we define the binary value

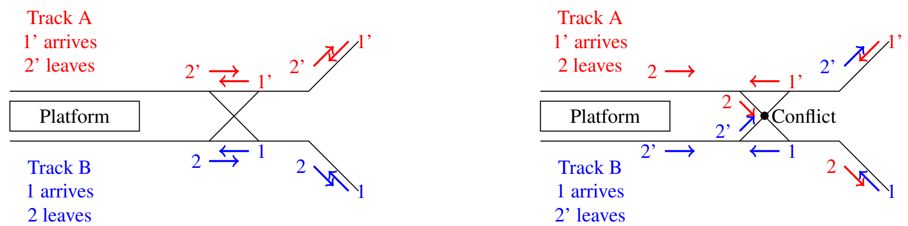
$$Succession(i,j,m) = \begin{cases} 1 & \text{if at least one unit of type } m \text{ is affected to} \\ & i \text{ and } j \text{ without any train between } i \text{ and } j \\ 0 & \text{otherwise} \end{cases} \quad . \tag{1}$$

Let us consider the example of Figure 2 with four trains: 1, 2, 1' and 2'. Suppose Train 2 is the unique successor of Train 1 and Train 2' is the unique successor of Train 1' in the nominal schedule, as shown in Figure 2a. If Train 2 is not anymore a successor of Train 1 in the adapted schedule but a successor of Train 1', as shown in Figure 2b, it will change the *structure* of the rolling stock schedule, and it implies several adaptations.

First, it may change the track-occupation diagram for Trains 1, 2, 1' and/or Train 2'. Trains 1 and 2' (*resp.* 1' and 2) *must* now be scheduled on the same track if there is not enough time to make a shunting movement. Thus, it impacts the passenger information in stations, and is a potential source of bad operational performance. Moreover, it could be difficult to find a new track-occupation diagram with associated paths compatible with the new successions, as shown in Figure 3. It implies more rescheduling effort and is responsible for an increase in adaptation costs.

Second, it possibly modifies the driver schedules in the crew scheduling problem, because they strongly depend on the successors in the rolling stock schedule. If Train 2 is the successor of Train 1, it is convenient that the same driver is assigned to these two trains. Otherwise, the solution is less robust. Indeed, the example of Figure 4 shows that having different successions for the rolling stock and the drivers can lead to a higher number of impacted trains in case of a primary delay, and thus an increase in indirect operational adaptation costs. It is possible to avoid this by changing the drivers schedule, but it increases direct adaptation costs because the rescheduling effort is more important.

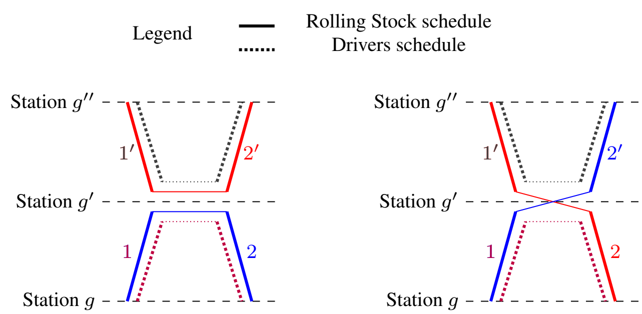
And third, a change in successors may involve changes in the shunting plan, as shown Figure 5. Indeed, if a train has $n \geq 1$ different successors (*resp.* predecessors), it is necessary to make $n-1$ combinations (*resp.* splits) after it arrives (*resp.* before it leaves). Thus, a change in the successors can impact the number of splits and combinations.



(a) Nominal track-occupation diagram with associated conflict-free paths

(b) A simple adapted track-occupation diagram with a path-conflict

Figure 3: An example of conflict in the track-occupation diagram in station g' after the change in the successors in Figure 2. Trains 1 and 2’ (*resp.* Trains 1’ and 2) have to be scheduled on the same track in the adapted case 3b. If Trains 2 and 2’ leave g' at the same time, it is impossible to adapt the track-occupation diagram, because the paths for Trains 2 and 2’ are incompatible.



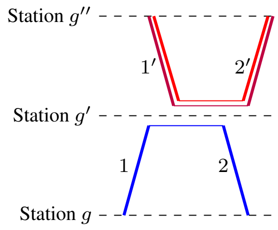
(a) Nominal rolling stock schedule and drivers schedule

(b) Adapted rolling stock schedule and unchanged drivers schedule

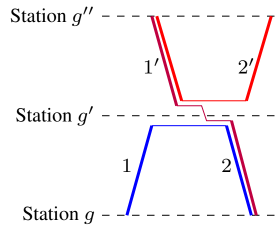
Figure 4: An example of 4 trains with different successions for the rolling stock and the drivers schedules. In Figure 4b, a primary delay of Train 1 may imply a delay propagation to Train 2 because the two trains have the same rolling stock unit. In the adapted schedule Figure 4b, a primary delay of Train 1 may imply both a delay propagation to Train 2 (because they have the same driver) and to Train 2’(because they have the same unit). It is possible to avoid this by changing the driver schedule, which is a source of additional adaptation costs.

Following these observations, we define structural adaptation costs to move from one rolling stock schedule to another as the differences in the successions between them:

$$StructuralAdaptationCosts = \sum_{(i,j,m)}\sum\sum \left|Succession^{adapted}(i,j,m) - Succession^{nominal}(i,j,m)\right| \quad (2)$$



(a) Nominal schedule



(b) An adapted schedule

Figure 5: In the nominal schedule Figure 5a, there is not any split or combination. In the adapted schedule Figure 5b, there is a split after the arrival of Train 1' and a combination before the departure of Train 2. These new shunting operations correspond to the new succession 1'-2.

4 The Rolling Stock Adaptation Problem with respect to a Given Scenario

In this section, we introduce the Rolling Stock Adaptation Problem more precisely, with a detailed description in the case of demand changes, and propose a mathematical formulation to solve it with a MILP.

4.1 Problem Description

In this paper, we focus on one of the main causes for which the rolling stock schedules have to be adapted: the demand changes. Whether it is passenger or freight transportation, there is always an uncertainty about the minimal demand of the trains. Thus, the number of required units for a given train can change during the adaptation phase.

In passenger railway transportation, a forecasted passenger demand is computed in the conception phase for each train. However, if the number of reserved seats is closely monitored, it is possible to update this forecast some weeks or months before the departure of the train and adjust the number of units depending on the evolution of the forecast. In freight railway transportation, the quantity of goods that need to be transported varies slightly from week to week, because of a more or less favourable economical context. Thus, freight transportation is also concerned by the need to adapt the rolling stock schedules because of demand changes.

Let us introduce some notations. The input of the Rolling Stock Adaptation Problem with respect to a given scenario is:

- a set $\mathcal{G}$ of stations where combinations and splits may be allowed: for each station $g \in \mathcal{G}$, we define the parameter $CS(g) \in \{0;1\}$ with value 1 if splits and combinations are allowed in station g, and 0 otherwise;
- a set $\mathcal{M}$ of unit types and, for each $m \in \mathcal{M}$, $K_m \in \mathbb{N}$ is the number of available units of type m;
- a set of trains T^{train}. A train $i \in T^{train}$ is defined by:
  - fixed departure and arrival times;
  - fixed departure and arrival stations in $\mathcal{G}$;

- $\tilde{D}_i > 0$, the nominal demand in terms of rolling stock units, *i.e.* the desired number of rolling stock units for the train i;
- $D_i^{max} \geq \tilde{D}_i$, the maximal number of rolling stock units for the train i;
- $\mathcal{M}_i$, the list of unit types compatible with train i.

- a time horizon H expressed in days, numbered $0, 1, \ldots H - 1$;
- a nominal rolling stock schedule $\tilde{\mathcal{R}}$;
- a *scenario* s, corresponding to a set of updated demands $D_i^s \in [\![\tilde{D}_i \,; D_i^{max}]\!]$ in terms of rolling stock units for each train i.

The Rolling Stock Adaptation Problem is described in Figure 1a. Considering an input instance as described above, find a *feasible* rolling stock schedule $\mathcal{R}^s$ which minimizes both the performance and structural adaptation costs defined in Section 3. A rolling stock schedule is said to be feasible if it respects some particular constraints we will describe below.

4.2 Mathematical Formulation

The Rolling Stock Adaptation Problem with respect to a given scenario is formulated as a multicommodity flow problem in a graph G similar to the one used by Cacchiani *et al.* (2010), where each unit type m has a corresponding flow in the multiflow.

Description of the Graph

We define $G = (T, A)$ as a directed graph in which the nodes correspond to *tasks* that can be performed (see example in Figure 6), which is directly inspired by the graph of Löbel (1998) in vehicle scheduling. The tasks can be decomposed as follows:

$$T = T^{train} \cup T^{depots} \cup \{\alpha,\ \omega\}\,, \tag{3}$$

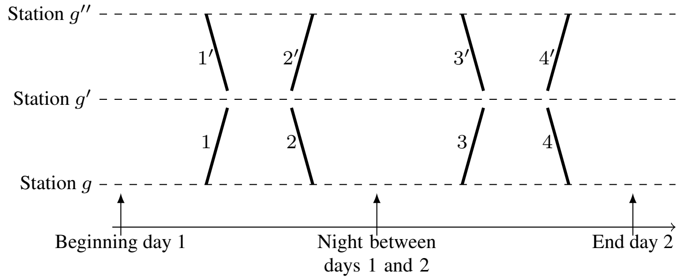
where:

- a node $i \in T^{train}$ corresponds to a train, as defined above;
- a node $i \in T^{depot}$ corresponds to a depot task, which is characterized by a station and two consecutive days. Performing this task means to put some units into the depot during the corresponding night. Consequently, if there is a physical depot at a given station g, we define $H + 1$ nodes for it, with labels g^0, $g^1 \ldots g^H$. They respectively correspond to the depot in station g in the morning of day 0, during the night between day 0 and day 1, … and finally in the evening of day $H - 1$;
- node α is the *source* node, and node ω is the *sink* node.

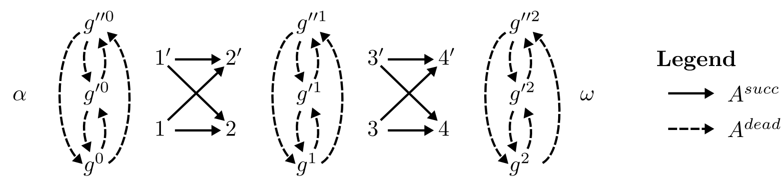
The introduction of the set T^{depot} allows to reduce the number of arcs in the graph and thus the complexity of our formulation.

The arc set A contains several types of arcs:

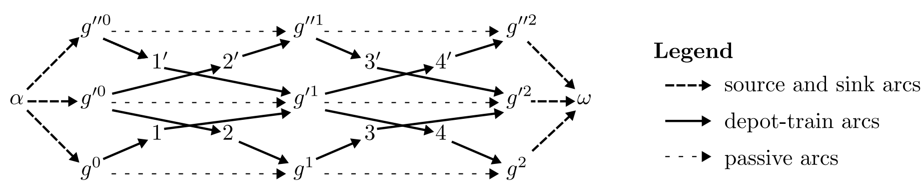
- arcs A^{succ} are the most important arcs, corresponding to *successions* between two trains as defined in Section 3.2.
- arcs A^{dead} between two depot nodes g^0 and g'^0 for two different stations g and g', corresponding to dead-head trips between g and g' during the night. Note that it is impossible to make dead-head trips during a day, or equivalently between two trains operating on the same day;
- arcs between the source node α and a depot node g^0 for a given station g. The value of these arcs in the flow of unit type m corresponds to the number of units of type m



(a) The input timetable



(b) Subgraph with arcs from A^{succ} and A^{dead}



(c) Subgraph with the other arcs, except the fictive sink-source arc (ω, α)

Figure 6: Graph construction for an example with 8 trains and 3 stations over a scheduling horizon of 2 days

starting from the associated depot at the beginning of the horizon. We also define arcs between a depot node g^H for a given station g and the sink node ω, corresponding to units at this physical depot at the end of the horizon;

- arcs between a depot node g^d for a given station g and a train leaving g on day d. The flow value for type m corresponds to the number of units starting with this train on day d. We also define arcs between a train leaving a station g on day d and the depot node g^{d+1}, corresponding to units going into the depot after covering the train;
- passive arcs between two depot nodes of the same station g with consecutive days (g^d,g^{d+1}). They correspond to units staying at the depot during a whole day, without covering any train during this day;
- a fictive arc $(\omega,\ \alpha)$, which is not represented in Figure 6.

In the rest of this paper, we extend the previous definition of $\mathcal{M}_i$ initially defined for all $i \in T^{train}$ to all the nodes $i \in T$, and denote by $\mathcal{M}_i$ the set of unit types compatible with the node $i \in T$. Moreover, for each arc $(i, j) \in A$, we define the set $\mathcal{M}_{ij}$ as $\mathcal{M}_i \cap \mathcal{M}_j$.

Variables Defining a Rolling Stock Schedule

A rolling stock schedule $\mathcal{R}$ can be described with the only integer decision variables x_{ijm} representing flow value of type m for each arc $(i,j) \in A$, and for each unit type $m \in \mathcal{M}_{ij}$.

We introduce the following auxiliary variables in order to get a linear formulation:

- l_i which counts the lack of units for train i if its demand is D_i. More formally,

$$l_i = \max\ (0, D_i - \sum_{\substack{j \in T \\ (i,j) \in A}} \sum_{m \in \mathcal{M}} x_{ijm}); \tag{4}$$

- δ_{ijm}, a binary variable equal to 1 if and only if $x_{ijm} \geq 1$, and 0 otherwise
- δ'_{ij}, a binary variable equal to 1 if and only if at least one variable δ_{ijm} is equal to 1 for all the unit types m.

Let us remark that from variables (x, l, δ, δ') one can complete the description of a rolling stock schedule through path decomposition. However, this partial description is sufficient to describe performance and structural adaptation costs.

Basic Feasibility for a Rolling Stock Schedule

A rolling stock schedule $\mathcal{R} = (x, l, \delta, \delta')$ is said to be *basic-feasible* for the demand vector D if it is feasible with respect to the common strong constraints defined by the following set of inequalities $\mathcal{F}_D$:

$$\sum_{\substack{i \in T \\ (i,h) \in A : m \in \mathcal{M}_i}} x_{ihm} = \sum_{\substack{j \in T \\ (h,j) \in A : m \in \mathcal{M}_j}} x_{hjm} \qquad h \in T,\, m \in \mathcal{M}_h \tag{5}$$

$$\sum_{\substack{j \in T \\ (i,j) \in A}} \sum_{m \in \mathcal{M}_{ij}} x_{ijm} \geq D_i - l_i \qquad i \in T^{train} \tag{6}$$

$$\sum_{\substack{j \in T \\ (i,j) \in A}} \sum_{m \in \mathcal{M}_{ij}} x_{ijm} \leq D_i^{max} \qquad i \in T^{train} \tag{7}$$

$$x_{\omega\alpha m} \leq K_m \qquad m \in \mathcal{M} \tag{8}$$

$$\sum_{\substack{j \in T \\ (i,j) \in A}} \delta'_{ij} \leq 1 \qquad i \in T^{train} | CS(G^{arr}(i)) = 0 \tag{9}$$

$$\sum_{\substack{j \in T \\ (j,i) \in A}} \delta'_{ji} \leq 1 \qquad i \in T^{train} | CS(G^{dep}(i)) = 0 \tag{10}$$

$$x_{ijm} \geq \delta_{ijm} \qquad (i,j) \in A^{succ},\, m \in \mathcal{M}_{ij} \tag{11}$$

$$x_{ijm} \leq M \cdot \delta_{ijm} \qquad (i,j) \in A^{succ},\, m \in \mathcal{M}_{ij} \tag{12}$$

$$\sum_{m \in \mathcal{M}_{ij}} \delta_{ijm} \geq \delta'_{ij} \qquad (i,j) \in A^{succ} \tag{13}$$

$$\sum_{m \in \mathcal{M}_{ij}} \delta_{ijm} \leq M \cdot \delta'_{ij} \qquad (i,j) \in A^{succ} \tag{14}$$

$$x_{ijm} \in \mathbb{N} \qquad (i,j) \in A,\, m \in \mathcal{M}_{ij} \tag{15}$$

$$\delta_{ijm} \in \{0;1\} \qquad (i,j) \in A^{succ},\, m \in \mathcal{M}_{ij} \tag{16}$$

$$\delta'_{ij} \in \{0;1\} \qquad (i,j) \in A^{succ} \tag{17}$$

$$l_i \in \mathbb{R}^+ \qquad i \in T^{train} \tag{18}$$

Constraints (5) are conservative flow constraints for all nodes and all unit types. Constraints (6) force each train to be covered by a sufficient number of rolling stock units or ensure that the variable l_i has the correct value. Constraints (7) prevent a train from being covered by a number of units exceeding the capacity of the train. Constraints (8) check that the number of available rolling stock units for each unit type is respected. If $CS(g) = 0$, Constraints (9) ensure that there is no split at station g. For each train i arriving at g, i must have a unique successor train, with the possibility to have successors of different unit types. For example, a train i can be covered by two units of different unit types but will have a unique successor j, covered by the same units. Constraints (10) ensure that there is no combination after a train j leaves a station g with $CS(g) = 0$ in a similar manner. Constraints (11) – (14) ensure the correct value for the variables δ_{ijm} and δ'_{ij}, where the big-M constant M is arbitrary large. Finally, constraints (15) – (18) restrict the definition set of the variables.

Nominal Feasibility

The input nominal rolling stock schedule $\tilde{\mathcal{R}}$ can be described with the variables $(\tilde{x}, \tilde{l}, \tilde{\delta}, \tilde{\delta}')$ and is basic-feasible for the set $\mathcal{F}_{\tilde{D}}$. Moreover, a nominal feasible schedule has to respect the nominal *cyclicity* constraints:

$$\tilde{x}_{\alpha g^0 m} = \tilde{x}_{g^H \omega m} \qquad g \in \mathcal{G},\, m \in \mathcal{M}, \tag{19}$$

to ensure that such a schedule can be followed or preceded by itself. Thus, if there is $\tilde{x}_{g^H \omega m}$ units of a type m at depot node g^H in the schedule, the same number of units is required at depot node g^0.

Feasibility of an Adapted Schedule for Scenario s

A feasible solution for the Rolling Stock Adaptation Problem with respect to a given scenario s is a basic-feasible rolling stock schedule $\mathcal{R}^s = (x^s, l^s, \delta^s, \delta'^s)$ for the demand D^s such that:

- $\mathcal{R}^s \in \mathcal{F}_{D^s}$
- $\mathcal{R}^s$ respects the following constraints:

$$l_i^s \leq \max(0, D_i^s - (\tilde{D}_i - \tilde{l}_i)) \qquad i \in T^{train} \tag{20}$$

$$x^s_{\alpha g^0 m} \geq \tilde{x}_{g^H \omega m} \qquad g \in \mathcal{G},\, m \in \mathcal{M} \tag{21}$$

$$x^s_{g^H \omega m} \geq \tilde{x}_{\alpha g^0 m} \qquad g \in \mathcal{G},\, m \in \mathcal{M} \tag{22}$$

Constraints (20) deal with the quality of service. It bounds the variables l_i^s: if 2 units were affected to a train i in the nominal rolling stock schedule and if the demand is 3 in scenario s, the variable l_i^s cannot exceed the value 1=3-2, because it is unreasonable to reduce the number of units when the demand increases.

Constraints (21) and (22) are side constraints very similar to cyclicity, which are less restrictive. In practice, an adapted schedule is never followed or preceded by itself, because such adaptations are usually limited in time. Thus, it must be preceded or followed by a nominal schedule. If there is $\tilde{x}_{g^H \omega m}$ units of a type m at a depot g^H in the nominal schedule $\tilde{\mathcal{R}}$, there must be at least as many units at depot g^0 in the adapted schedule. This is the purpose of Constraints (21). Constraints (22) are similar and deal with the number of rolling stock units at the end of the horizon in the adapted schedule.

Objective Function

The objective of the Rolling Stock Adaptation Problem represents adaptation costs to move from $\tilde{\mathcal{R}}$ to the adapted rolling stock schedule $\mathcal{R}^s$. As seen in Section 3, it corresponds to the performance and structural adaptation costs.

Performance adaptation costs of a basic-feasible rolling stock schedule $\mathcal{R} = (x, l, \delta, \delta')$ can be described with the following expressions:

- the total lack of rolling stock:

$$Lack(\mathcal{R}) \triangleq \sum_{i \in T^{train}} l_i; \tag{23}$$

- the total number of engaged units:

$$Units(\mathcal{R}) \triangleq \sum_{m \in \mathcal{M}} x_{\omega\alpha m}; \tag{24}$$

- the number of dead-head trips:

$$Dead(\mathcal{R}) \triangleq \left(\sum_{(i,j) \in A^{dead}} \sum_{m \in \mathcal{M}} x_{ijm} \right); \tag{25}$$

- the number of over-compositions:

$$Over(\mathcal{R}) \triangleq \sum_{i \in T^{train}} \left(\left(\sum_{\substack{j \in T \\ (i,j) \in A}} \sum_{m \in \mathcal{M}_{ij}} x_{ijm} \right) - D_i \right). \tag{26}$$

Let us remark that the objectives $Dead(\mathcal{R})$ and $Over(\mathcal{R})$ in Equations (25) and (26) can be easily weighted with the travelled distance in kilometers.

Structural adaptation costs to move from a feasible nominal rolling stock $\tilde{\mathcal{R}}$ to a feasible adapted rolling stock schedule $\mathcal{R}^s$ are defined with the following equation:

$$Struct(\tilde{\mathcal{R}}, \mathcal{R}^s) \triangleq \sum \sum_{\substack{(i,j) \in A^{succ} \\ i,j \in T^{train}}} \left((1 - 2\tilde{\delta}_{ijm}) \cdot \delta^s_{ijm} + \tilde{\delta}_{ijm} \right), \tag{27}$$

where the expression inside the sum corresponds to a rewriting of $\left| \delta^s_{ijm} - \tilde{\delta}_{ijm} \right|$, which is true because δ are binary variables.

The objective of the Rolling Stock Adaptation Problem can be written as a sum of the different previous objectives with relevant coefficients $(\beta, \Gamma, \Delta, \zeta, \eta)$:

$$\begin{aligned} \min_{\mathcal{R}^s} \quad & \beta \cdot Lack(\mathcal{R}^s) + \Gamma \cdot Units(\mathcal{R}^s) + \Delta \cdot Dead(\mathcal{R}^s) \\ & + \zeta \cdot Over(\mathcal{R}^s) + \eta \cdot Struct(\tilde{\mathcal{R}}, \mathcal{R}^s). \end{aligned} \tag{28}$$

5 The Adaptive Rolling Stock Problem with respect to a Given Set of Scenarios

5.1 Problem Description

As described in Figure 1b, the Adaptive Rolling Stock Scheduling Problem is a recoverable robust problem for rolling stock scheduling.

A solution is said to be *adaptive* for a set of scenarios S if its expected adaptation costs are low for that set of scenarios. In the following, we suppose without lost of generality that the scenarios have the same probability of occurrence.

5.2 Mathematical Formulation

Our mathematical formulation for the Adaptive Rolling Stock Scheduling Problem is based on that of the Rolling Stock Adaptation Problem in Section 4. Following Cacchiani *et al.* (2012) in a different setting, we duplicate the nominal variables $(\tilde{x}, \tilde{l}, \tilde{\delta}, \tilde{\delta}')$ for every scenario $s \in S$ and obtain a MILP with a higher dimension.

Feasible Solution
A feasible solution for the adaptive Rolling Stock Scheduling Problem is composed of:

- a nominal feasible rolling stock schedule $\tilde{\mathcal{R}} = (\tilde{x}, \tilde{l}, \tilde{\delta}, \tilde{\delta}') \in \mathcal{F}_{\tilde{D}}$ that satisfies Equations (19);
- a collection of adapted feasible schedules $(\mathcal{R}^s)_{s \in S} = (x^s, l^s, \delta^s, \delta'^s)_{s \in S}$, where each $\mathcal{R}^s$ is in $\mathcal{F}_{D^s}$ and satisfies Constraints (20) – (22) for scenario s.

Thus, the corresponding MILP contains variables $(\tilde{x}, \tilde{l}, \tilde{\delta}, \tilde{\delta}')$ for the nominal rolling stock schedule $\tilde{\mathcal{R}}$ and variables $(\mathcal{R}^s)_{s \in S} = (x^s, l^s, \delta^s, \delta'^s)_{s \in S}$ for each adapted schedule $\mathcal{R}^s$ to scenario s.

Objective Function
The main objective of the adaptive Rolling Stock Scheduling Problem is to minimize expected adaptation costs of the adapted solution $\mathcal{R}^s$, which corresponds to the following objective function:

$$\min_{\tilde{\mathcal{R}}, \mathcal{R}^s\, s \in S} \quad \beta \cdot \sum_{s \in S} Lack(\mathcal{R}^s) + \Gamma \cdot \sum_{s \in S} Units(\mathcal{R}^s) + \Delta \cdot \sum_{s \in S} Dead(\mathcal{R}^s) + \zeta \cdot \sum_{s \in S} Over(\mathcal{R}^s) + \eta \cdot \sum_{s \in S} Struct(\tilde{\mathcal{R}}, \mathcal{R}^s), \quad (29)$$

where the objective $Struct$ is now quadratic and can be rewritten with a simple linearization.

Controlling the Nominal Performance
We have to ensure a good performance for the rolling stock schedule $\tilde{\mathcal{R}}$. For this purpose, one may minimize the following additional objective, which corresponds to the performance criterion for the nominal rolling stock schedule:

$$\min \quad \tilde{\beta} \cdot Lack(\tilde{\mathcal{R}}) + \tilde{\Gamma} \cdot Units(\tilde{\mathcal{R}}) + \tilde{\Delta} \cdot Dead(\tilde{\mathcal{R}}) + \tilde{\zeta} \cdot Over(\tilde{\mathcal{R}}). \quad (30)$$

In practice, we have no information about the rate of time periods H without any demand changes. We just know these demand changes are quite rare. In other words, although we know the occurrence probability of a scenario s, we have no information about the probability $\tilde{p}$ of a fictive scenario without any demand changes. Thus, we prefer to introduce the nominal performance criteria in the constraints of our MILP:

$$Lack(\tilde{\mathcal{R}}) \leq (1 + \varepsilon^{Lack}) \cdot Lack^* \tag{31}$$

$$Units(\tilde{\mathcal{R}}) \leq (1 + \varepsilon^{Units}) \cdot Units^* \tag{32}$$

$$Dead(\tilde{\mathcal{R}}) \leq (1 + \varepsilon^{Dead}) \cdot Dead^* \tag{33}$$

$$Over(\tilde{\mathcal{R}}) \leq (1 + \varepsilon^{Over}) \cdot Over^*. \tag{34}$$

Parameters $Lack^*$, $Units^*$, $Dead^*$ and $Over^*$ correspond to the optimal associated performance cost for a (non adaptive) nominal schedule without any scenario. They can be computed by solving a MILP, looking for a schedule $\tilde{\mathcal{R}} \in \mathcal{F}_{\tilde{D}}$ respecting Constraints (19) while minimizing the objective (30).

Parameters ε^{Lack}, ε^{Units}, ε^{Dead} and ε^{Over} are non-negative and control the optimality gap between the adaptive nominal rolling stock $\tilde{\mathcal{R}}$ and an optimal non adaptive rolling stock schedule. The larger ε is, the more the adaptive nominal schedule is allowed to degrade the performance criteria. However, a larger value for any ε implies a larger solution space for the rolling stock schedules $\tilde{\mathcal{R}}$ and $(\mathcal{R}^s)_{s \in S}$, and thus to reduce adaptation costs of the objective function (29).

6 Computational Experiments

Description of the instances

We illustrate the relevance and efficiency of the adaptive model by computational experiments on two realistic nominal instances inspired by SNCF instances.

Table 1: Characteristics of the two instances

	Instance 1	**Instance 2**
Context	Passengers	Freight
Horizon	7 days	7 days
Trains	819	339
Stations	9	29
Number of unit types	1	3
Total number of units	23	35
Nominal demands	1	between 1 and 2
Maximal demands	2	between 1 and 4
Split/combination restrictions	no	yes

Table 1 shows the main characteristics of the two instances. The first one is derived from a set of regional trains, while the second one represents a pool of freight trains. The first instance has a lot of trains but has a simple structure, with few stations, only one unit type, homogeneous demands and no split and combinations restrictions. On the other hand, the second instance has less trains but is more complex, with a lot of stations and three unit types.

We complete each of these 2 instances with a set of 3 arbitrarily generated scenarios $S = \{s_1, s_2, s_3\}$ with equal probabilities. Each $s \in S$ has an updated demand $D_i^s = \tilde{D}_i + 1$ for about 10% of randomly chosen trains i. The demand of the other trains is unchanged.

Parameters $(\beta, \Gamma, \Delta, \zeta, \eta)$ for the objective function
From an industrial point of view, the lack of rolling stock units is the main objective to minimize. The second one is the number of engaged units, the third one concerns the dead-head trips and the fourth one the over-compositions. Thus, we can use our formulation with $\beta \gg \Gamma \gg \Delta \gg \eta$, which corresponds to a lexicographical order.

With regard to structural adaptation costs, we assume that they are more important than those of over-compositions, but less important than those of dead-head trips. Indeed, scheduling additional dead-head trips often has an impact on drivers schedules. Thus, it is not reasonable to schedule unnecessary dead-head trips to reduce the adaptation costs. Moreover, a surplus in over-compositions has no impact on drivers schedules and it seems reasonable to increase them to reduce structural adaptation costs.

The MILP formulation with these 5 parameters is not suitable for an optimization tool as typical values of the objective function are too large which may lead to floating errors. Thus, the MILPs are solved for the first objective, after which a constraint is added so that this objective does not exceed its obtained value, and the second criterion is minimized. We proceed in the same way for each of the objectives. This process also enables to understand which of the objectives are the most difficult.

Comparison with the traditional approach
We want to compare the efficiency of the *traditional approach* used at SNCF and an *adaptive* process based on the problem that we described in Section 5. The traditional approach is simulated using the two following steps:

1. We solve the MILP (5)–(19) with objective (30) and we obtain the nominal rolling stock schedule $\tilde{\mathcal{R}}^{tr} = (\tilde{x}^{tr}, \tilde{l}^{tr}, \tilde{\delta}^{tr}, \tilde{\delta}'^{tr}) \in \mathcal{F}_{\tilde{D}}$;

2. For each scenario $s \in S$, we solve a Rolling Stock Adaptation Problem and obtain mean adaptation costs to move from $\tilde{\mathcal{R}}^{tr}$ to $\tilde{\mathcal{R}}^{tr,s}$.

We test three different adaptive processes with different values for parameters ε^{Lack}, ε^{Units}, ε^{Dead} and ε^{Over}. We set ε^{Lack} and ε^{Units} to 0 to prevent any deterioration of these objectives and test three different values $\varepsilon \in \{0, 0.1, 0.25\}$ for $\varepsilon^{Dead} = \varepsilon^{Over}$.

In the following, we use the notation

$$\overline{Lack}(\mathcal{R}^S) \triangleq \frac{1}{\text{card}(S)} \cdot \sum_{s \in S} Lack(\mathcal{R}^s) \tag{35}$$

to represent the mean expected adaptation costs for objective (23) with regard to S, and we proceed in the same way for the other objectives. The objectives $Dead$ and $Over$ are expressed in kilometers.

First instance
The performance of the nominal rolling stock schedule is summarized in Table 2. As expected, when $\varepsilon = 0$, the nominal adaptive solution has exactly the same (optimal) performance as the solution in the traditional approach. The same applies when $\varepsilon = 0.1$, but when $\varepsilon = 0.25$ the numbers of dead-head trips kilometers and over-compositions kilometers are no more optimal.

This nominal near-optimality allows to reduce the mean expected adaptation costs as represented in Table 3, especially when it comes to the structural adaptation costs, passing

Table 2: Performance costs of $\tilde{\mathcal{R}}$ for Instance 1

	$Lack(\tilde{\mathcal{R}})$	$Units(\tilde{\mathcal{R}})$	$Dead(\tilde{\mathcal{R}})$	$Over(\tilde{\mathcal{R}})$
Traditional approach	0	22	228	678
Adaptive process with:				
$\varepsilon = 0$	0	22	228	678
$\varepsilon = 0.1$	0	22	228	678
$\varepsilon = 0.25$	0	22	265	833

Table 3: Mean adaptation costs to move from $\tilde{\mathcal{R}}$ to an adapted schedule for Instance 1

	$\overline{Lack}(\mathcal{R}^S)$	$\overline{Units}(\mathcal{R}^S)$	$\overline{Dead}(\mathcal{R}^S)$	$\overline{Struct}(\tilde{\mathcal{R}}, \mathcal{R}^S)$	$\overline{Over}(\mathcal{R}^S)$
Traditional approach	1.6	23	69.3	30	1985.6
Adaptive process with:					
$\varepsilon = 0$	1.6	23	69.3	5.3	4490.3
$\varepsilon = 0.1$	1.6	23	69.3	5.3	4490.3
$\varepsilon = 0.25$	1.6	23	69.3	4.6	5021

from 30 in the traditional approach to about 5 in the adaptive processes, even if $\varepsilon = 0$. This means that the nominal solution of the traditional approach has a very high mean expected structural adaptation costs which can be reduced without any deterioration of the performances. However, the mean expected number of kilometers for the over-compositions is increased by a factor of 2.5 which represents a big price to pay in term of energy consumption.

All the MILPs are solved to optimality within a few minutes, except those concerning the objectives $Struct$ and $Over$ during the adaptive processes. After an hour of computations, MILPs with objective $Struct$ have an optimality gap between 7% (for $\varepsilon = 0$) and 44% (for $\varepsilon = 0.25$), while those for $Over$ have an optimality gap of about 60%. These large gaps could explain why the objective $Over$ has very high values compared to the optimal values of the traditional approach.

Second instance

Table 4 summarizes the performance costs of the nominal rolling stock schedule. The objectives $Dead$ and $Over$ are not optimal for $\varepsilon = 0.1$ or $\varepsilon = 0.25$. They have significantly higher values than in Instance 1 as there are significantly more stations which makes it more difficult to respect the cyclicity constraints without doing dead-head trips and over-compositions. The objective $Over$ is better with $\varepsilon = 0.25$ than with $\varepsilon = 0.1$, since the objective $Dead$ is optimized before and has a larger value with $\varepsilon = 0.25$.

Table 5 summarizes the mean expected adaptation costs for the 3 scenarios. Resolution times are similar to those of Instance 1, and the optimality gaps after an hour of computa-

Table 4: Performance costs of $\tilde{\mathcal{R}}$ for Instance 2

	$Lack(\tilde{\mathcal{R}})$	$Units(\tilde{\mathcal{R}})$	$Dead(\tilde{\mathcal{R}})$	$Over(\tilde{\mathcal{R}})$
Traditional approach	0	34	1529	15226
Adaptive process with:				
$\varepsilon = 0$	0	34	1529	15226
$\varepsilon = 0.1$	0	34	1653	16616
$\varepsilon = 0.25$	0	34	1851	16383

Table 5: Mean adaptation costs to move from $\tilde{\mathcal{R}}$ to an adapted schedule for Instance 2

	$\overline{Lack}(\mathcal{R}^S)$	$\overline{Units}(\mathcal{R}^S)$	$\overline{Dead}(\mathcal{R}^S)$	$\overline{Struct}(\tilde{\mathcal{R}}, \mathcal{R}^S)$	$\overline{Over}(\mathcal{R}^S)$
Traditional approach	1.3	35	2359.3	25.6	11913.3
Adaptive process with:					
$\varepsilon = 0$	1.3	35	2067.6	11	11843.3
$\varepsilon = 0.1$	1.3	35	2017	5.3	11791
$\varepsilon = 0.25$	1.3	35	2017	5	12036

tion reach about 70% for the objective $Struct$ and 10% for $Over$ in the adaptive processes. However, except the objective $Over$ for $\varepsilon = 0.25$ with a small increase of about 100 kilometers, all the objectives have a better value in the adaptive processes. These results can be explained by the fact that Instance 2 is much more complicated than Instance 1. As a consequence, any demand change is hard to satisfy if it has not been properly anticipated which is precisely the main interest of an adaptive process.

7 Conclusion and Perspectives

In this paper, we developed a new way to model the adaptation costs in rolling stock railway scheduling. We introduced the concept of adaptive solution to reduce the adaptation costs of a rolling stock schedule. Two MILPs were proposed, the first one is solved in the adaptation phase while the second one is designed to compute adaptive solution in the conception phase. Our first results on realistic instances are promising. They show that the adaptation costs can be significantly reduced with an adaptive process while keeping good performance criteria for the nominal solution, especially for instances with complex structures.

In future research, we want to address this issue with a more sophisticated multi-objective optimization. We want to find adaptive solutions with balanced structural and performance costs. In addition, we want to improve the resolution of the MILPs with decomposition techniques.

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