A New Approach to Strategic Periodic Railway Timetabling

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Abstract
One of the criteria to judge a timetable is what passengers think of it, and an operator has to take this into account when designing a timetable. We study this problem in a case study from the Netherlands, where on part of the network the frequency of trains has increased recently. We formulate a model that integrates passenger routing and timetabling in order to find timetables that are good for passengers. This can be used for studies by railway operators, and by infrastructure managers to decide where to invest in new infrastructure.

Keywords
Periodic Timetabling, Periodic Event Scheduling, Passengers

1 Introduction

The so-called 'A2-corridor' between Amsterdam and Eindhoven is one of the most densely used parts of the Dutch railway network: Recently, the frequency on this corridor was increased from four to six intercity trains per hour, to decrease passenger waiting times at their origin station. Ideally, passengers who travel on this corridor come to the station at any time without checking the timetable and should be able to take a train shortly after they arrive at the station.

The demand for travelling on this part of the Dutch railway network is the highest between Amsterdam and Eindhoven (the corridor itself), but there is also a significant number of passengers traveling on the 'branches' of the corridor, that are the parts of the network shown in Figure 1 that are only served by one or two lines and hence by two or four trains per hour. When making a timetable for this network, we thus observe a trade-off between regularity of the trains on the corridor, and regularity of the trains on the branches. Especially if the trains on the branches have a frequency of four per hour, and on the corridor a frequency of six per hour (not a multiple of four), the timetable can only be regular on both corridor and branches if trains wait relatively long on the stations where they enter and leave the corridor. So in this case we trade a regular service and thus short waiting times of passengers at the origin station for longer in-train waiting times.

In this paper, we formulate a model that optimizes a timetable structure. This timetable

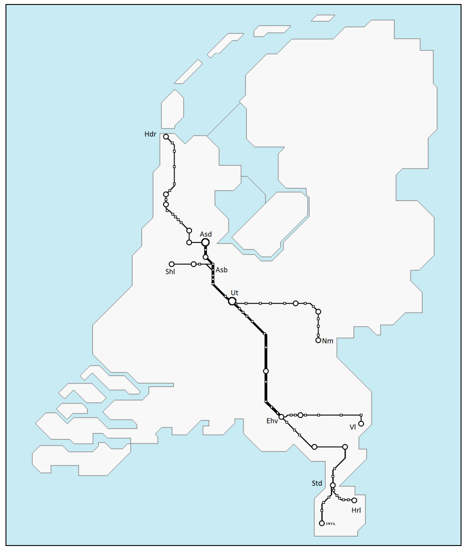


Figure 1: Overview of the geographical network of the A2 corridor instance. The corridor is highlighted.

is based on a input line plan. We build the model by extending the well-known PESP model (Serafini and Ukovich, 1989) for periodic timetabling to include passengers' route choice. In the optimization, we minimize the sum of the perceived travel times for all passengers, which is a weighted sum of waiting time at the origin station and in-train time. Our model can be used as a strategic planning tool, e.g., to evaluate line plans based on the timetable structure they allow. By assuming that we have unlimited infrastructure, we can determine what is an ideal timetable, and hence we can investigate to what extend this timetable fits on the currently existing infrastructure. This can thus also support decision making regarding infrastructure investments.

Our contribution in this paper is twofold. First of all, we propose a quadratic integer programming model to integrate passenger routing and periodic timetabling, where we explicitly take the waiting time at the origin station into account. This model is linearized to a linear mixed integer program. Secondly, we demonstrate the viability of our method on the Dutch A2-corridor instance to advice on the optimal regularity of train lines on this corridor, and the possible benefit of infrastructure investments.

The remainder of this paper is organized as follows. In Section 2 we state the problem we are solving. Section 3 describes background information and literature that is relevant for our study. A quadratic integer programming model is formulated and linearized in Section 4. Computational results on the A2-corridor are provided in Section 5. Finally, we conclude the study and mention future research in Section 6.

2 Problem Statement

In this paper we address a strategic timetabling problem. The goal is to find an ideal timetable structure, that can help us evaluate line plans and advise on infrastructure investments.

The timetable is made based on a line plan. A line plan is a set of train lines that are to be operated on the network. Each of these lines consists of a geographical route through the rail network, a list of stations where the train has to stop, and a frequency by which the train line is to be operated per time period. This line plan serves as input for the timetabling problem.

In this research, we require periodic timetables, i.e., timetables that repeat themselves every time period, say every hour. This type of timetables is often used in European countries.

To find good timetables, we take into account passenger demand. The demand is given in terms of numbers of passengers that want to travel between each pair of stations. We assume that the demand is uniformly distributed over the cycle period, i.e., every minute the same number of passengers want to depart. Often passengers arrive at their origin station shortly before their train departs (Zhu et al., 2017; Ingvardson et al., 2018). However, as the timetable is not yet known, we assume that the demand is evenly distributed over time to find a timetable that best matches this demand assumption, and in the actual operation passengers will adapt their arrival times based on such a timetable.

Our problem can be stated as follows: Given an input line plan and an estimate of passenger demand, find the timetable structure which minimizes the sum of the perceived travel times for all passengers. The perceived travel time is a weighted sum of waiting time at the origin station and in-train time.

3 Literature Review

In this section, we place the problem we study in the context of existing literature. Section 3.1 describes the problem of periodic timetabling and research that is related to this. Section 3.2 describes how passenger routing can be combined with timetabling and how this is done in existing literature.

3.1 Periodic Timetabling

The periodic timetabling problem is commonly modelled as a Periodic Event Scheduling Problem (PESP) (Serafini and Ukovich, 1989). The task here is to assign event times for all arrivals and departures of the trains in the line plan. As the timetable is periodic, these events are periodic as well, i.e., they re-occur every cycle period, e.g., every hour. This cycle time is denoted by T. The event times have to satisfy several restrictions in order to guarantee a reasonable timetable. These restrictions are generally referred to as *activities*. Each activity is a relation between a pair of events, stating that the time difference between these events should be in a given (periodic) time interval. Examples of these activities are drive, dwell and transfer activities. It is also possible to include headway activities, ensuring a certain time distance between trains. Overviews on how to model timetabling constraints and what can be included in a PESP framework can be found in Odijk (1996); Peeters (2003); Liebchen and Nachtigall and Möhring (2007).

The essence of PESP is to find *any* periodic timetable satisfying all activities. Approaches to find a feasible solution to PESP include constraint programming (Kroon et al., 2008), the modulo-simplex heuristic (Nachtigall and Opitz, 2008; Goerigk and Schöbel, 2013), or using a SAT solver after applying a polynomial transformation from PESP to SAT (Grossmann et al., 2012). If a feasible solution exists, one can be found rapidly.

3.2 Passenger Routing

If many feasible timetables exist, one can distinguish between them by adding an objective function to the PESP model, by which *good* timetables can be found. A definition of a good timetable will consist of several aspects, but at least one of the aspects has to do with *efficiency*. By an efficient timetable we mean that the timetable is optimized with respect to passenger travel times. This is achieved by giving each activity a weight and then minimizing the weighted sum of all the activity durations. To solve such a model, a Mixed Integer Programming formulation can be used. More details about such a modelling approach are provided in Section 4. Examples of successful applications in practice can be found in (Liebchen, 2008).

In the case of efficient timetable, the activity weights are chosen such that they represent the (relative) importance of the activities. For example, the weight for an activity can represent the number of passengers using this activity in their route. In this case, the passenger flows have to be known and the timetable can be found based on these flows. However, as a timetable can be suboptimal for certain passengers, they might choose a new route if the timetable is known, thus changing the weights. This in its turn can again influence the optimality of a timetable, which can be changed based on this. Several approaches exist in which an iterative approach is taken to find good a timetable (Kinder, 2008; Lübbe, 2009; Siebert, 2008; Siebert and Goerigk, 2013; Sels et al, 2016). In these approaches, passenger flows are determined by routing passengers through the network on for example shortest paths. After this, the timetable is optimized (retiming) and passenger are rerouted (reflowing), until a stopping criterion is reached.

Another option is to integrate the passenger routing and timetabling problems, which can provide the optimal timetable for the passengers, although the model is more complex. Here the timetable and the passenger routes are chosen simultaneously. This is the approach we take, where we, additionally to most existing literature, also explicitly take the waiting time at the origin station into account. By assuming that passenger demand follows a uniform distribution, good headway times between trains are found, such that the total experienced travel time of passengers is minimized. The integration of passenger routing and timetabling is not a new field of study, as this already has been applied for the aperiodic case (Schmidt and Schöbel, 2015; Schmidt, 2012) and for the periodic case (Schöbel., 2015; Borndorfer et al., 2017; Gattermann et al., 2016; Schiewe and Schöbel, 2018). However, the approach we take is that we do not assign passengers to a specific departure event before solving the model, but that we allow this freedom in the model. This is most closely related to Schiewe and Schöbel (2018). However, we also assume that demand is uniformly distributed over time and we determine all headway times between consecutive trains based on this, which, to the best of our knowledge, has not yet been applied in literature. Burggraeve et al. (2017) also integrate timetabling and line planning to achieve better results. However, in this approach infrastructure is taken into account in a very detailed manner, while we discard as much of the current infrastructure as possible, in order to determine long term strategic timetables.

In the past, attention has been given to the gap between line planning and timetabling, and that integrating these two problems does not solve everything. Goerigk et al. (2013) propose several evaluations of line plans and determine the influence of a line plan on the resulting timetable. This is done by computing several characteristics of line plans, and by finding a passenger-oriented timetable. However, the passenger routing and timetabling problems are not integrated, but passengers are routed along shortest paths, which is one of the main differences with our approach.

4 Integer Programming model

In this section, we present a mathematical programming model for passenger routing and timetabling. We start by introducing the necessary notation, after which we present the mathematical model. We conclude the section by linearizing the proposed model.

4.1 Notation

The model that we introduce consists of a timetabling part and a passenger routing part. In the following sections, we introduce these two parts separately.

Periodic Timetabling
First of all, we assume that an Event-Activity network $G = (V, A)$ is given, with events V and activities A. Based on this network, we develop a model to find a periodic timetable with cycle time T. A commonly used model for periodic timetabling is based on the Periodic Event Scheduling Problem (Serafini and Ukovich, 1989). In PESP, next to the network $G = (V, A)$, we are given lower and upper bounds ℓ_{ij} and u_{ij} for each $(i, j) \in A$ and a cycle time T. The task is to find an assignment $\pi : V \to \{0, \ldots, T-1\}$, such that all activities are satisfied. In PESP, each activity $(i, j) \in A$ is of the form

$$y_{ij} = \pi_j - \pi_i + Tp_{ij} \in [\ell_{ij}, u_{ij}], \tag{1}$$

where p_{ij} is an integer variable accounting for the shift from one cycle to another, it acts as a modulo operator. Each activity states that the time difference between events i and j should be within the T-periodic interval $[\ell_{ij}, u_{ij}]$. The additionally introduced variable y_{ij} represents the *activity duration* for activity $(i, j) \in A$.

Without loss of generality, the timetable is planned in full minutes, but any other time grid can be chosen as well. The rationale behind this assumption is that we want to find a timetable for the long future and there is no need for a detailed timetable in this case.

Passenger Routing
Next to timetabling, we have variables and constraints dealing with the routing of passengers. Suppose that passenger demand is given in terms of an OD-matrix. This provides for each origin-destination combination k in the set $\mathcal{OD}$ the number of passengers d_k that want to travel in the cycle period from their origin to their corresponding destination. We assume that this demand is uniformly distributed over the time period.

For each OD-pair $k \in \mathcal{OD}$, we pre-determine a set of possible routes, which we denote by $\mathcal{R}^k$. In our computations, this set consists of all direct travel options for this OD-pair, but this can be extended to routes containing transfers as well. The set of all routes is denoted by $\mathcal{R}$ and is determined as

$$\mathcal{R} = \bigcup_{k \in \mathcal{OD}} \mathcal{R}^k. \tag{2}$$

We assume that these sets are given as input.

A route $r \in \mathcal{R}$ is a path through the Event-Activity Network. It consists of a sequence of trip and dwell activities, so $r \subseteq A$. The total (timetable-dependent) duration Y_r of such a route is determined as the sum of the duration of all activities it uses:

$$Y_r = \sum_{a \in r} y_a. \tag{3}$$

The task in our model is to assign the passengers of every OD-pair to a relevant departure event, and route them all together. For each OD-pair $k \in \mathcal{OD}$, the set of relevant departure events (V^k) can be determined by

$$V^k = \bigcup_{r \in \mathcal{R}^k} j(r), \tag{4}$$

where $j(r)$ is the first event of route $r \in \mathcal{R}$.

In our model, we make the simplifying assumption that every passenger departs with the first train towards his destination. Note that in practice this may not be the best traveling option, since a later train may overtake the first train. However, since in our case study we consider only intercity trains which travel at approximately the same speed, this assumption seems appropriate.

We group all passengers of an OD-pair k for who, due to their arrival time, departure event $v \in V^k$ is the next possible departure event, together and assume they make the same route choice. This assumption is valid since the perceived passenger travel time is minimized and we assume that all passengers have the same perception of travel time, and that there are no capacities on the routes.

In order to compute the number of passengers for who v is the next possible departure event, we determine the time period before event $v \in V^k$, in which no other event $v' \in V^k \setminus \{v\}$ takes place. This is denoted by A_v^k. Note that this variable is timetable dependent, and can be determined by the following set of equations:

$$A_v^k = \min_{v' \in V^k} \{\pi_v - \pi_{v'} + T\alpha_{v',v}\} \tag{5a}$$

$$\alpha_{v,v'} + \alpha_{v',v} = 1. \tag{5b}$$

In the first equation, the time difference between all other relevant departure events is determined, and the minimum is taken. In order to determine an implicit order between events happening at the same time, which is needed to determine how many passengers take a certain train, we add the second set of restrictions.

The number of passengers for event $v \in V^k$ can then be calculated as $A_v^k \cdot d_k/T$. Once event $v \in V^k$ takes place, all these passengers choose a route. The set of all routes, starting at this departure event, is denoted by $\mathcal{R}_v^k \subseteq \mathcal{R}^k$. If a passenger departs with event $v \in V^k$, he will choose exactly one of these routes to use. The duration of the journey, starting from event v, is denoted by Y_v^k, and can be determined as

$$Y_v^k = \min_{r \in \mathcal{R}_v^k} Y_r. \tag{6}$$

Note that this assumes that passengers use shortest paths, which is true since the perceived passenger travel time is minimized.

The expected waiting time for each group of passengers at the origin station is denoted by W_v^k. As we assume that passengers arrive according to a uniform distribution, this value is calculated as $W_v^k = A_v^k/2$.

To compute the total perceived travel time of a passenger, we weight the waiting time at the origin station with a factor γ_w and add it to the in-train time or duration of the journey. E.g., a factor $\gamma_w = 3$ means that a passengers perceives one minute waiting at the station as bad as three minutes traveling on the train. We can then compute the average perceived travel time of each passenger k for who departure event v is the next possible departure as $\gamma_w W_v^k + Y_v^k$.

4.2 Mathematical Program

Using the notation and constraints introduced above, a Quadratic Integer Program for timetabling with passenger routing, including waiting times, can now be formulated as follows:

$$\text{Minimize} \sum_{k\in\mathcal{OD}} \frac{d_k}{T} \sum_{v\in V^k} A_v^k \cdot \left(\gamma_w \cdot W_v^k + Y_v^k\right) \tag{7a}$$

$$\text{Such that } y_{ij} = \pi_j - \pi_i + Tp_{ij} \qquad \forall\,(i,j)\in A \tag{7b}$$

$$\ell_{ij} \le y_{ij} \le u_{ij} \qquad \forall\,(i,j)\in A \tag{7c}$$

$$Y_r = \sum_{a\in r} y_a \qquad \forall\, r\in\mathcal{R} \tag{7d}$$

$$A_v^k = \min_{v'\in V^k\setminus\{v\}} \{\pi_v - \pi_{v'} + T\alpha_{v',v}\} \qquad \forall\, k\in\mathcal{OD},\ v\in V^k \tag{7e}$$

$$\alpha_{v,v'} + \alpha_{v',v} = 1 \qquad \forall\, k\in\mathcal{OD},\ v\in V^k,\ v'\in V^k\setminus\{v\} \tag{7f}$$

$$W_v^k = \frac{1}{2}A_v^k \qquad \forall\, k\in\mathcal{OD},\ v\in V^k \tag{7g}$$

$$Y_v^k = \min_{r\in\mathcal{R}_v^k} \{Y_r\} \qquad \forall\, k\in\mathcal{OD},\ v\in V^k \tag{7h}$$

$$A_v^k \in [0,T] \qquad \forall\, k\in\mathcal{OD},\ v\in V^k \tag{7i}$$

$$W_v^k \in [0,T/2] \qquad \forall\, k\in\mathcal{OD},\ v\in V^k \tag{7j}$$

$$Y_r, Y_v^k \in [0,\infty) \qquad \forall\, r\in\mathcal{R},\ k\in\mathcal{OD},\ v\in V^k \tag{7k}$$

$$\pi_v \in \{0,\ldots,T-1\} \qquad \forall\, v\in V \tag{7l}$$

$$p_{ij} \in \mathbb{Z}_{\ge 0} \qquad \forall\,(i,j)\in A \tag{7m}$$

$$\alpha_{v,v'} \in \{0,1\} \qquad \forall\, k\in\mathcal{OD},\ v\in V^k,\ v'\in V^k\setminus\{v\}. \tag{7n}$$

The task in this formulation is to minimize the perceived travel time for all passengers (7a). This is composed of waiting time, plus the actual travel time. Constraints (7b) and (7c) are the timetabling constraints. Constraints (7d) determine the length of each route. Constraints (7e) and (7f) determine the time between trains, and define the expected waiting times in (7g). The actual perceived travel time durations are determined in (7h). Constraints

(7i)–(7n) state the domains of the variables. Note that this is a quadratic model. In the next section, we show how this model can be linearized.

4.3 Linearization

The model in (7) contains a quadratic objective and two minima in the formulation. In the following sections, we linearize each part of this.

Objective
The objective in (7) can, by using (7g), be written as

$$\text{Minimize} \sum_{k\in\mathcal{OD}} \frac{d_k}{T} \sum_{v\in V^k} \frac{\gamma_w}{2}\left(A_v^k\right)^2 + A_v^k \cdot Y_v^k. \tag{8}$$

For the linearization we define new variables

$$x_{v,d}^k = \begin{cases} 1 & \text{if } A_v^k \geq d \\ 0 & \text{else} \end{cases} \qquad \forall\, k \in \mathcal{OD},\, v \in V^k,\, d \in \{1,\dots,T\}. \tag{9}$$

Note that these variables satisfy the following restrictions:

$$x_{v,d}^k \leq x_{v,d-1}^k \qquad \forall\, k \in \mathcal{OD},\, v \in V^k,\, d \in \{2,\dots,T\}. \tag{10}$$

Using these new variables, we can write

$$A_v^k = \sum_{d=1}^{T} x_{v,d}^k \tag{11a}$$

$$\left(A_v^k\right)^2 = \sum_{d=1}^{T} (2d-1)\cdot x_{v,d}^k. \tag{11b}$$

Substituting this in (8) leaves a multiplication of binary variables $x_{v,d}^k$ by bounded variables Y_v^k, which we substitute by $R_{v,d}^k = Y_v^k \cdot x_{v,d}^k$. The objective then becomes

$$\text{Minimize} \sum_{k\in\mathcal{OD}} \frac{d_k}{T} \sum_{v\in V^k} \sum_{d=1}^{T} \left[\frac{\gamma_w}{2}(2d-1)\cdot x_{v,d}^k + R_{v,d}^k\right], \tag{12}$$

with the additional restrictions that

$$R_{v,d}^k \leq u_v^k \cdot x_{v,d}^k \tag{13a}$$

$$R_{v,d}^k \geq l_v^k \cdot x_{v,d}^k \tag{13b}$$

$$R_{v,d}^k \leq Y_v^k - l_v^k \cdot \left(1 - x_{v,d}^k\right) \tag{13c}$$

$$R_{v,d}^k \geq Y_v^k - u_v^k \cdot \left(1 - x_{v,d}^k\right), \tag{13d}$$

where l_v^k and u_v^k are the lowest and highest possible values for Y_v^k respectively.

Minima
Constraints (7e) and (7h) both contain a minimum. We replace (7e) by

$$A^k_v \leq \pi_v - \pi_{v'} + T\alpha_{v',v} \qquad \forall\, k \in \mathcal{OD},\, v' \in V^k \setminus \{v\} \tag{14a}$$
$$\sum_{v \in V^k} A^k_v = T. \qquad \forall\, k \in \mathcal{OD} \tag{14b}$$

(14a) represents the minimum and (14b) ensures that all passengers are assigned to a group.

The linearization of (7h) is done by replacing the following set of restrictions for every $k \in \mathcal{OD}$ and every $v \in V^k$ by the following:

$$Y^k_v \leq Y_r \qquad \forall\, r \in \mathcal{R}^k_v \tag{15a}$$
$$Y^k_v \geq Y_r - M^k_v \cdot \left(1 - z^k_{v,r}\right) \qquad \forall\, r \in \mathcal{R}^k_v \tag{15b}$$
$$\sum_{r \in \mathcal{R}^k_v} z^k_{v,r} = 1. \tag{15c}$$

We introduced new binary variables $z^k_{v,r}$, which correspond to the route chosen. That means, if $z^k_{v,r} = 1$, passengers use route r, starting at event v. The newly introduced constants M^k_v are to be chosen large enough to make the second set of constraints redundant, but still as small as possible. Therefore, we can set

$$M^k_v = \max_{r \in \mathcal{R}^k_v} \{\overline{Y_r}\} - \min_{r \in \mathcal{R}^k_v} \{\underline{Y_r}\}. \tag{16}$$

Here, $\overline{Y}$,$\underline{Y}$ denote respectively the highest and lowest possible value variable Y can take.

To summarize, in the linearization several steps are taken. The objective (7a) is replaced by (12). Here, additional variables $x^k_{v,d}$ and $R^k_{v,d}$ are introduced, with additional restrictions (10) and (13). Next, the minima are replaced by linear restrictions, (7e) is replaced by (14), and (7h) by (15).

5 A Case Study: A2 Corridor

5.1 Description of the Case

The case we consider in our study is the so called 'A2-corridor', which is part of the Dutch railway network between Eindhoven and Amsterdam Centraal. It is named after the highway A2 which runs next to it for a long part of the track. The most interesting feature of this instance is that it is the first corridor in the Netherlands where the frequency of the intercity trains was increased from four to six intercity lines per hour. An overview of the lines in the network is shown in Figure 2. In this Figure, the main stations are shown, with corridors connecting them. For each corridor, the train lines that uses these corridors are shown. For example, it shows that line 3000 travels between Nijmegen and Den Helder. The full names and abbreviations of the involved stations are shown in Table 1.

Each train line is operated in both directions with a frequency of two trains per hour. Note that not all of these trains use the full corridor between Eindhoven and Amsterdam Centraal, line 3000 only uses Utrecht-Amsterdam Centraal, and line 3500 only uses Eindhoven-Utrecht. Line 3100 does not use the corridor itself at all, but it interacts with other lines on the branches of the network as depicted in Figure 1.

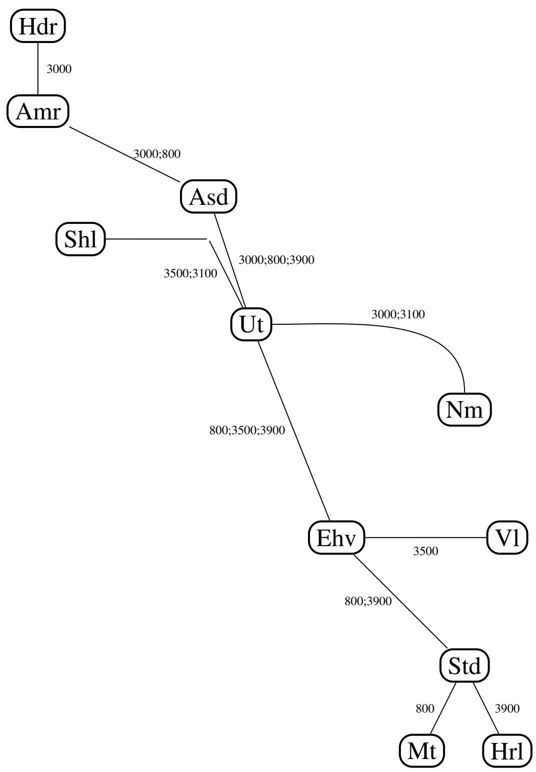


Figure 2: Overview of the train lines in the A2-corridor

Abbreviation	Name	Abbreviation	Name
Hdr	Den Helder	Sgn	Schagen
Amr	Alkmaar	Asd	Amsterdam Centraal
Shl	Schiphol	Asb	Amsterdam Bijlmer ArenA
Ut	Utrecht Centraal	Ah	Arnhem
Nm	Nijmegen	Ht	’s Hertogenbosch
Ehv	Eindhoven	Vl	Venlo
Std	Sittard	Hrl	Heerlen
Mt	Maastricht		

Table 1: Abbreviations of the stations

In Section 5.2, we investigate how regular the train lines should run on the corridor and on the branches of the network, if we assume unlimited infrastructure. We also investigate how this result changes when we adjust the weighting parameter γ_w which controls the impact of the waiting time at the origin.

As a reference, we also compute a timetable in Section 5.3, where we assume that we have the current infrastructure available.

In our computations, we only consider direct travel options for all passengers, so the set of possible routes can be easily determined. For the OD-matrix, we make use of an OD-matrix containing the number of all passengers that travelled between two stations in the year 2015. Hence, this is based on historic data and is an aggregated matrix. The flows will be different in peak and off-peak hours, and in the weekends. For our study that is no problem, as we are doing strategic studies and only the estimated magnitude of the flows matter.

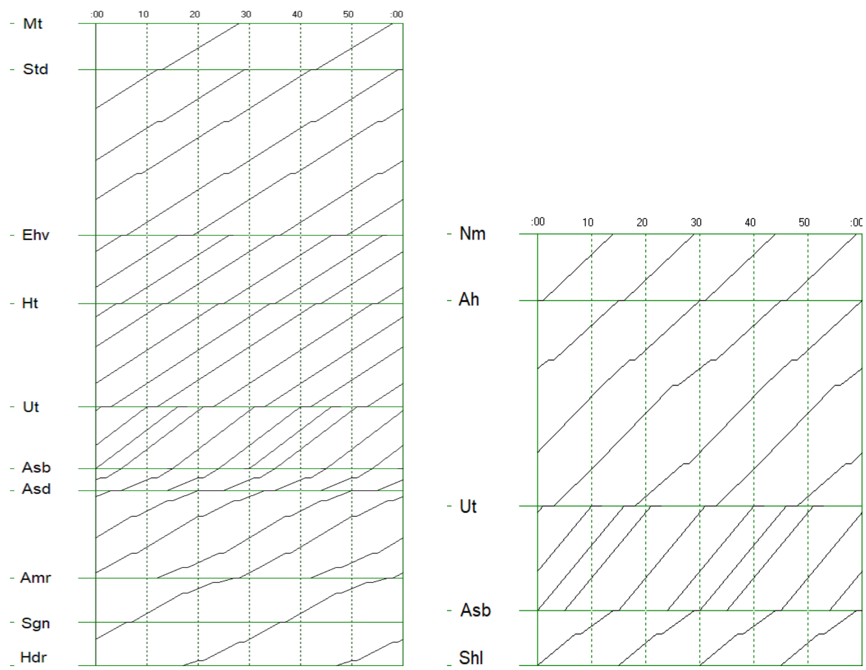
5.2 Unlimited Infrastructure

As the main goal of this research is to find out how an optimal timetable from a passengers perspective looks like, we assume there is sufficient infrastructure available to operate any timetable. We compute a timetable by solving (7) with a time limit for the computation of one hour. As travel options we only consider direct connections for all passengers. We distinguish between two cases. First, we assume that waiting at the origin station is considered to be less pleasant than sitting in a train. Second, we compare this to the situation where waiting time is equally pleasant to in-train time.

Waiting Time is Less Pleasant

When passengers travel, they often arrive shortly before their train departs, in order to minimize their waiting time. However, as no timetable is available yet, these arrival times can not be determined, and therefore we assume that passengers arrive according to a uniform distribution. In order to deal with the situation that passengers generally do not like to wait, we set the coefficient for waiting time relatively high. In-train time has a coefficient of 1, so we set the waiting time coefficient higher to $\gamma_w = 3$.

The results of our computations are shown in Figure 3, showing the results in terms of two time-space diagrams. Only the southbound trains are shown to make the picture more clear. The first picture shows the tracks between Maastricht and Den Helder (and hence includes the full corridor), the other shows the tracks between Nijmegen and Schiphol. The vertical axes denotes space and the labels show the different stations where a trains stops. The horizontal axis shows time, so the timetable is depicted for one cycle period of one hour.



(a) Maastricht-Den Helder

(b) Nijmegen-Schiphol

Figure 3: Timetable with unlimited infrastructure, $\gamma_w = 3$.

The objective value and a lower bound for this timetable can be found in Table 2. The objective is split in two parts to better distinguish what the contribution is of the individual components.

Interesting to note in this Figure, is that the trains run very regular between the large stations on the main corridor, i.e., between Eindhoven and Utrecht, and between Utrecht and Amsterdam. Between Amsterdam and Alkmaar, the pattern becomes slightly less regular. Here a choice has to be made for a service between these stations and hence a longer waiting time at the Amsterdam Central station at the border of the corridor (because the frequency decreases from 6 to 4 trains per hour), or a less regular service and short waiting times. Apparently it seems to be better not to stop too long at the border of the corridor to obtain the regular pattern between Amsterdam and Alkmaar.

Another interesting thing to note here, is that the train paths of lines 3000 and 3500 coincide. It is hard to see in Figure 3, but can be deduced together with Figure 2. Line 3000 comes from Amsterdam Central and passes Amsterdam Bijlmer Arena (Asb) at .45. Also line 3500, coming from Schiphol, leaves Amsterdam Bijlmer ArenA at .45, and it travels to

Utrecht at the same time as line 3000. From Utrecht onwards, line 3000 leaves the corridor for Nijmegen, while line 3500 takes over the position of line 3000 in the pattern on the corridor and drives towards Eindhoven. So we can say that these trains replace each other in the pattern on the corridor. Passengers traveling from Amsterdam to Eindhoven could hence use also this route with a transfer. This happens even though we do not allow passengers to transfer in our model - it is a consequence of the fact that regular train services are preferred by the model due to the high impact of the waiting time.

Waiting Time and In-Train Time is Equal

We now compare the findings of Section 5.2 with a setting of $\gamma_w = 1$, i.e., the in-train time and waiting time are perceived equally. The resulting timetable for the corridor is shown in Figure 4.

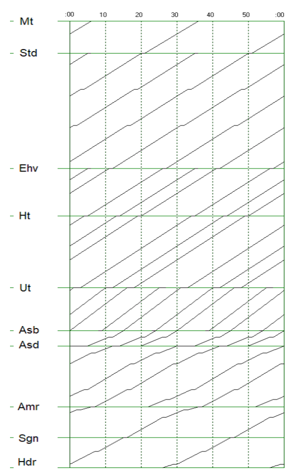


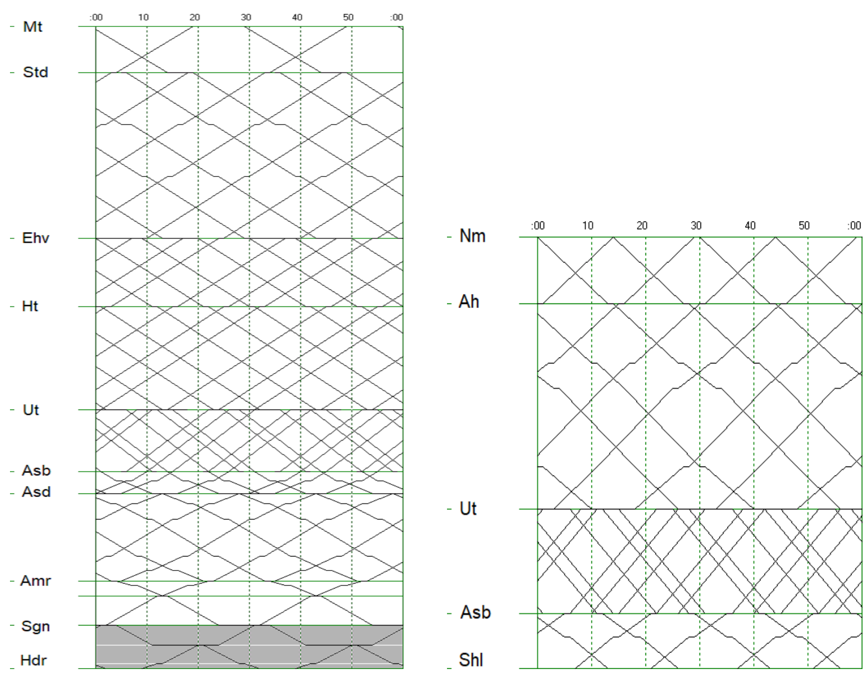
Figure 4: Maastricht-Den Helder with unlimited infrastructure and $\gamma_w = 1$

In this timetable, the trains have a less regular pattern. As the waiting time is less important, a smaller focus is on the regularity of the trains. Also in Table 2, this effect can be seen. The contribution of in-train time decreases, whereas the contribution of waiting time is increased.

5.3 Current Infrastructure

If we compare the timetables determined in the previous sections with the current infrastructure on the A2 corridor and its branches, we observe that the current infrastructure is not sufficient. In particular, between Den Helder (Hdr) and Schagen (Sgn), the network is currently single-track, so crossings of trains from two directions, as we see them in Figure 3a, are not possible with the current infrastructure. Furthermore, currently the trains from Utrecht to Amsterdam Centraal (Asd) and from Utrecht to Schiphol (Shl) use the same infrastructre until Amsterdam Bijlmer Arena (Asb), thus a headway of three minutes has to be respected between them if no additional infrastructure would be provided here. To see the benefit of providing extra infrastructure, we now compute the ideal timetable structure taking current infrastructure restrictions into account by adding them as headway constraints to our optimization model, and comparing the timetable achieved in this way to the timetable from Section 5.2.

More precisely, the headways we consider state that between every pair of trains running in the same direction, a headway time of 3 minutes has to be respected. Furthermore, between trains entering and leaving a station in opposite directions on single track regions, a time difference of at least 1 minute has to be respected. Finally, on single track regions trains have to wait for each other before the single track can be entered. Again, we set $\gamma_w = 3$. Within one hour of computation time, several solutions can be found. However, due to the increased complexity caused by the added headway activities, these solutions are not very good. Therefore, we set a time limit of three hours for these computations. The resulting timetable is shown in Figure 5 and the objective values are shown in Table 2. The shaded area in Figure 5a denotes that this part has only 1 track, and trains can only pass each other at the stations.



(a) Maastricht-Den Helder

(b) Nijmegen-Schiphol

Figure 5: Timetable with current infrastructure, $\gamma_w = 3$.

A few differences can be noted with respect to the case with unlimited infrastructure and $\gamma_w = 3$. First of all, trains dwell for a long time on stations around the single-track area. Especially the northbound trains have long dwell times. These trains come from a region that has a very regular timetable, and then they have to wait for the southbound trains because these are using the tracks. Based on passengers' demand, a choice can be made whether the southbound of the northbound trains have to wait. Also at other stations trains dwell for a longer time. As an example, trains coming from Amsterdam and going to Nijmegen have to wait at Utrecht from .58 to .03, in order have a regular service on the branch Utrecht-Nijmegen.

For this line plan, the cost of taking infrastructure into account in terms of experienced travel time is not much more than in the case with unlimited infrastructure, the objective increases by only 1.18% (Table 2).

	γ_w	In-train time	Waiting Time	Objective value	Lowerbound
Unlimited infrastructure	3	2047.004	592.576	3824.733	3808.588
Unlimited infrastructure	1	2042.174	597.926	3835.953[1]	
Current infrastructure	3	2083.382	595.510	3869.912	3808.547

Table 2: Objective values for the timetables

6 Conclusion and Future Research

In this paper, we have developed a mathematical model that allows us to determine timetable structures based on passenger demand, explicitly taking into account waiting time of passengers. By applying the model to the A2 corridor in the Dutch railway network, we could observe that the inclusion of passenger waiting time leads to regular timetable structures, although regularity was not imposed as a constraint in our model. This effect is particularly strong when the waiting time at the origin station plays a prominent role in the objective function. When the weight of waiting time at the origin is smaller, the regularity diminishes to improve dwell times in stations. By applying our model also to a case with infrastructure restrictions, we could furthermore quantify the impact the added value of infrastructure investments for passengers.

In the future, we will apply our approach to other cases from the Dutch Railway network to see whether these findings are instance-specific or can be generalized. In both cases, we think that our model is a valuable tool for strategic timetabling, in the sense that it can help to evaluate line plans and infrastructure and help to make improvements.

At this moment, we have several ideas for extensions to our model and to our solution approach. Currently the model deals with direct travel options for passengers only. This is a limitation that we are improving, such that any travel option can be included. Furthermore, we want to include the option of not taking the first departing train, but to take a later train instead, and to investigate to what extend this improves the solutions. Thirdly, as PESP is NP-complete, computation times rapidly grow when more activities are taken into account. Especially headway restrictions can make it hard to find a feasible solution, let alone a good solution. Therefore we have set a higher computation time limit for these experiments. In order to deal with this increasing complexity, we developed an algorithm that first finds a good timetable as is done in this paper, with the assumption of unlimited infrastructure. In a

[1] In order to compare the objective value of this timetable with the other timetables, we have evaluated this timetable with $\gamma_w = 3$.

second step, we use an algorithm based on Cacchiani et al. (2013) that updates this timetable considering a given infrastructure network, such that all trains can be safely operated on this network, and if necessary cancels trains.

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