Firing Accuracy Analysis of Electromagnetic Railgun Exterior Trajectory Based on Sobol's Method

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Abstract

Firing accuracy is an important index in the performance evaluation of electromagnetic railgun (EMRG). Based on a Six-DOF (degree of freedom) computer model of exterior trajectory, Sobol's method, a global sensitivity analysis approach, is utilized to analyse the influence of multiply model inputs with uncertainty on the strike accuracy of EMRG projectile. The method utilizes the firing data error and the dispersion error as the firing accuracy assessment factors of the projectile, and the input data are sampled based on Latin Hypercube Sampling (LHS). Furthermore, an example is provided, in which Sobol's method is applied in the analysis and calculation of the exterior trajectory. First-order sensitivity and total sensitivity of each factor are obtained, and then we identify the impact mechanism and interaction of different input parameters having on firing accuracy. Finally the results verify that the method is feasible and effective in the process of performance analysis of EMRG exterior trajectory.

Keywords: Sobol's method, firing accuracy, sensitivity analysis, EMRG

1 Introduction

Electromagnetic railgun (EMRG) is a typical representative of the electromagnetic emission weapons. As a new concept of weapon-system, EMRG has the advantages of quick response, hypersonic speed, and high damage efficiency for remote ground target strike mission (Fair, 2009; Ma, 2007) compared with conventional guns. And firing accuracy is an extremely important indicator to measure the operational performance of the EMRG. Nowadays, especially with the rapid development of science and technology, the accurate attack of railguns is strongly stressed by each country's military personnel (Fair, 2005). It's extremely necessary and urgent to analyze the shooting accuracy of the EMRG accurately.

Currently, the domestic and foreign research on the analysis of firing accuracy of EMRG is still in infancy. Shared research about EMRG system firing accuracy analysis is limited. Most of the studies concentrated on the analysis of guns system problems. Guo had in-depth research in detail in artillery weapons system. Reference (Guo, 2001) described the calculation method of gun's firing accuracy and the influence of various factors on the firing accuracy. But for the EMRG system, there are few complete solutions to analyze its firing accuracy precisely. Therefore, figuring out how to analyze the EMRG firing accuracy exactly is essential.

As an important tool for modeling, sensitivity analysis can let the modeler know the influence of the model parameters and inputs to the model outputs. It can be utilized effectively in modeling, model testing, and model calibration. There are a lot of classification methods about the sensitivity analysis, among which the basic idea of Sobol's global sensitivity analysis method is to study the effect of variance of the inputs parameters to the variance of outputs, using integration to describe the sum of progressive increase items factorized by system functions, then calculate ratios of the total variance to each partial variance to get the accuracy after sampling (Sobol, 2013). Compared to other methods of sensitivity analysis, Sobol's method can calculate different order sensitivity more efficiently and accurately, and it can also get the impact index, named total sensitivity which reflects the interaction of all factors. Thereby we can analyze the influence of each factor having on EMRG firing accuracy.

Sobol's method is widely applied in the field of economy, environment and climate (Yu, 2004). In this paper, we analyze the firing accuracy of electromagnetic railgun exterior trajectory by Sobol's method. Taking six factors as the research objects, such as the quality, muzzle and velocity, the first-order sensitivity and total sensitivity of each factor are calculated by the method. In the end, the influence mechanism of various factors on firing accuracy and the interaction among them is gained. The process verifies that Sobol's method is feasible in the process of accurate analysis and provides a foundation for optimizing EMRG's performance.

2 Exterior Trajectory Simulation of Electromagnetic Railgun

The model of EMRG exterior trajectory describes the process of projectiles' movement after leaving the railgun with a high speed in the atmosphere (Keshmiri, 2004; Keshmiri, 2007). It is the foundation of exterior trajectory simulation and the characteristic analysis of trajectory. The paper considers EMRG's shooting range, high altitude flight, the variation of the earth curvature and gravitational acceleration. The Six-DOF model of projectiles is built in the paper. In the model, the projectile of the EMRG is considered as a particle, and Six-DOF motion equations are established. Those are dynamic equations of projectile's centroid, dynamic equations around the centroid, kinematics equations of the centroid, kinematics equations around the centroid and the relevant initial parameters. Run the simulation program, corresponding EMRG exterior trajectory simulation results can be got.

2.1 Simulation Framework

The diagram of the simulation process is shown in Figure 1. By setting the initial projectile conditions and flight termination conditions to limit projectiles flight process, the projectile's flying state in each simulation step is determined by calculating atmospheric parameters and flight parameters of the projectile. After confirming EMRG exterior trajectory simulation process, the MATLAB programming language is utilized to achieve it.

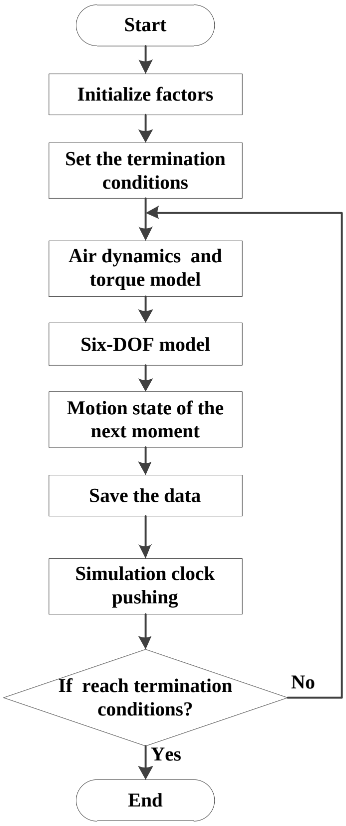

Figure 1. Electromagnetic railgun exterior trajectory simulation process.

2.2 The Experimental Design Method

In order to ensure the reliability of the projectile firing accuracy analysis which is based on the Six-DOF model, reasonable experimental design method should be chosen. Latin Hypercube Sampling (Deng, 2012) is employed in the paper. The method has great one-dimensional projection and stratified distribution characteristics, which can cover upper and lower limits of the probability distribution uniformly and distribute random number to each interval evenly. In this way, sampling frequency declines and the result remains stable. A lot of duplicate sampling work can be avoided, which improves efficiency of sampling (Ding, 2013; Zhong, 2009). The basic steps of LHS are as follows.

Set the objective function

$$y = f(x) \tag{1}$$

where y is a output variable, f is a definite function model, $x = \{x_1, x_2, \cdots, x_k\}^T$ is input variable, k is the number of input variable. Each input variable $x = \{x_1, x_2, \cdots, x_k\}^T$ subjects to a known probability distribution function $F_i(x_i)$. First of all, the input variable x_i should be sampled randomly. When random sampling, M random numbers should be generated between 0 and 1 firstly, and then transform them using the equation as follows

$$U_m = U/M + (m-1)/M \tag{2}$$

where $m = 1, 2, \cdots, M$, U is a random number between 0 and 1. U_m are the random numbers in the m'th interval.

Depending on equation(2), obviously, there is only one generated number in each interval. Because

$$(m-1)/M < U_m < m/M \tag{3}$$

where $(m-1)/M$ and m/M are the lower bound and upper bound of the m'th interval.

LHS strictly ensures the entire area was uniformly sampled, and it is almost impossible to sample repeatedly. Therefore, the method can convergence to a smaller sample size.

3 The Analysis of Electromagnetic Railgun Firing Accuracy

Electromagnetic railgun firing accuracy is influenced by multiple factors, and the influence of various factors is not identical. Meanwhile, there may be a certain coupling relationship between different factors. Thus, it is significant to analyze the impact of various factors on projectiles' firing range and direction, confirm the main factors. This section introduces the definition of firing accuracy, and gives the overall process of electromagnetic railgun firing accuracy analysis.

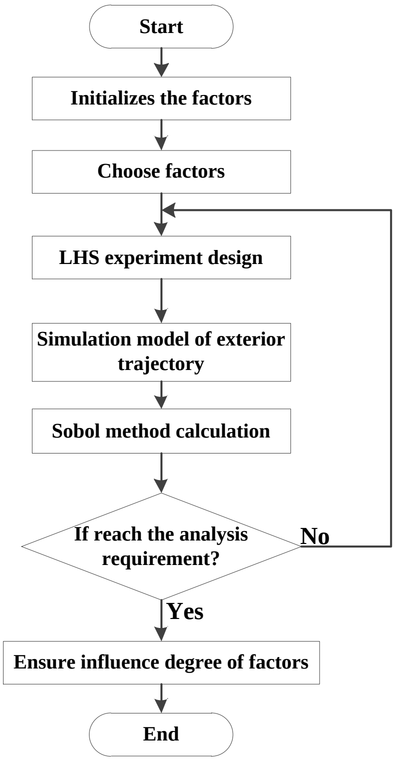

Figure 2. Process of electromagnetic railgun firing accuracy analysis.

3.1 Firing accuracy

In order to facilitate the research of the electromagnetic railgun firing accuracy, suppose that the shooting target of the electromagnetic railgun is on the ground, which is shown in Figure 3.

Assuming that the origin O is the shooting center, the deviation between projectiles average placement and shooting center is $\overline{\Delta}$, the coordinates relative to the shooting center are $(\overline{X}, \overline{Z})$ .The deviation between each projectile placement and shooting center is Δ , the coordinates relative to the shooting center are (x, z) . The deviation of between each projectile placement and their average placement is Δ_r .For each projectile, the deviation between each projectile placement and shooting center Δ can be expressed

$$\Delta = \overline{\Delta} + \Delta_r \tag{4}$$

where Δ is the firing error, $\overline{\Delta}$ is the dispersion error, Δ_r is the firing data error. They can be written as

$$\Delta = \left[x, z\right]^T \tag{5}$$

$$\overline{\Delta} = \left[\overline{x}, \overline{z}\right]^T \tag{6}$$

$$\Delta_r = \Delta - \overline{\Delta} = \left[x_r, z_r\right]^T \tag{7}$$

where x and z are the components of the firing error in x axis and z axis; $\overline{x}$ and $\overline{z}$ are the components of the dispersion error in x axis and z axis; x_r and z_r are the components of the firing data error in x axis and z axis.

In the paper, the firing accuracy of projectiles is described by the components of the firing data error in x axis and z axis Δ_x , Δ_z and the components of the dispersion error in x axis and z axis E_x , E_z .

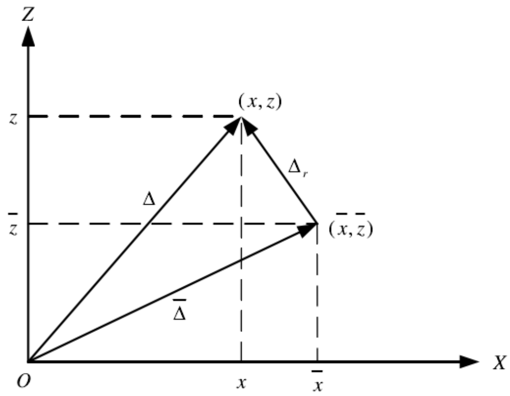

Figure 3. Projectile's firing error.

3.2 Sobol's Method

Sobol's method is based on the idea of model decomposition, 1, 2 times and higher sensitivity can be got through it (Sobol, 2013). Usually first-order sensitivity reflects the main influence parameters, and second order and higher sensitivity reflect the sensitivity among the parameters. To describe Sobol's method, firstly, one k dimension unit Ω^k should be defined as the spatial domain of input parameters (Saltelli, 2012). The main idea of Sobol's method is dividing the function $f(x)$ into some progressive increase items.

$$\begin{aligned} f(x_1, x_2, \cdots, x_k) = f_0 + \sum_{i=1}^{k} f_i(x_i) + \sum_{1 \le i < j \le k} f_{i,j}(x_i, x_j) + \cdots \\ + f_{1,2,\cdots,k}(x_1, x_2, \cdots, x_k) \end{aligned} \tag{8}$$

where f(x) can be divided by multiple integral. In equation (8), f_0 is a constant. So we have

$$\int_0^1 f_{i_1,\cdots i_s}(x_{i_1}, x_{i_2}, \cdots, x_{i_s}) \, \mathrm{d}\, x_{i_k} = 0 \tag{9}$$

From equation(8) and equation(9), each item is orthogonal, that is if $\left(i_1, i_2, \cdots, i_s\right) \ne \left(j_1, j_2, \cdots, j_l\right)$, then

$$\int_{\Omega^k} f_{i_1, i_2, \cdots, i_s} f_{j_1, j_2, \cdots j_l} dx = 0 \tag{10}$$

Thus

$$f_0 = \int_{\Omega^k} f\left(x\right) dx \tag{11}$$

The decomposition of equation(8) is unique, and each item can be got from multiple integral

$$f_i\left(x_i\right) = -f_0 + \int_0^1 \cdots \int_0^1 f\left(x\right) dx \sim \left(i\right) \tag{12}$$

$$f_{i,j}\left(x_i, x_j\right) = -f_0 - f_i\left(x_i\right) - f_j\left(x_j\right) + \int_0^1 \cdots \int_0^1 f\left(x\right) dx \sim \left(ij\right) \tag{13}$$

where $x \sim i$, $x \sim \left(ij\right)$ are the variables, except x_i and variables except x_i and x_j , respectively.

The total square deviation of f(x) is D

$$D=\int_{\Omega^k} f^2(x)dx-f_0^2 \quad (14)$$

The partial variance can be got though each items in equation (8).

$$D_{i_1,i_2,\cdots,i_s}=\int_0^1\cdots\int_0^1 f^2_{i_1,i_2,\cdots,i_s}\left(x_{i_1},x_{i_2},\cdots,x_{i_s}\right)dx_{i_1}\cdots dx_{i_s} \quad (15)$$

where $1\le i_1<\cdots<i_s\le k$ and $s=1,2,\cdots,k$ .Calculate the integral of equation (15) squared in the region of Ω^k and from equation, we obtain

$$D=\sum_{i=1}^{k}D_i+\sum_{1\le i<j\le k}D_{i,j}+\cdots+D_{1,2,\cdots,k} \quad (16)$$

Thus, sensitivity $S_{i_1,i_2,\cdots,i_s}$ can be described as

$$S_{i_1,i_2,\cdots,i_s}=D_{i_1,i_2,\cdots,i_s}/D,\left(1\le i_1<\cdots<i_s\le k\right) \quad (17)$$

Therefore, S_i is called the first-order sensitivity, $S_{i,j}$ is the second-order sensitivity, By that analogy total sensitivity S_{Ti} is

$$S_{Ti}=S_i+\sum_{i\ne j_1}S_{i,j_1}+\sum_{\substack{i\ne j_1\\ i\ne j_2\\ j_1\ne j_2}}S_{i,j_1,j_2}+\cdots+S_{1,2,\cdots,k} \quad (18)$$

In the process of applying Sobol's method, to calculate the first-order sensitivity coefficient and total sensitivity coefficient of factors (Tarantola, 2012), assuming the gained sample $A_{N\times r}$, $B_{N\times r}$ from LHS, where N is sample size, k is the factor number, some equations are given like that

$$f_0\approx\frac{1}{N}\sum_{j=1}^{N}f\left(A\right)_j \quad (19)$$

$$V+f_0^2\approx\frac{1}{N}\sum_{j=1}^{N}f^2\left(A\right)_j \quad (20)$$

$$V_i+f_0^2\approx\frac{1}{N}\sum_{j=1}^{N}f\left(A\right)_j f\left(B_A^{(i)}\right)_j \quad (21)$$

$$V_{-i}+f_0^2\approx\frac{1}{N}\sum_{j=1}^{N}f\left(A\right)_j f\left(A_B^{(i)}\right)_j \quad (22)$$

where under the condition of A is unchangeable, change the i'th column of B into A to get $A_B^{(i)}$, and the same method to get $B_A^{(i)}$ .Thus the first-order sensitivity coefficient S_i and total sensitivity coefficient S_{Ti} of the parameter i can be written as

$$S_i=V_i/V \quad (23)$$

$$S_{Ti}=1-V_{-i}/V \quad (24)$$

4 Case study

In order to verify the effectiveness of the Sobol's method for the analysis of firing accuracy, in this section, the Sobol's method is applied in EMRG exterior trajectory sensitivity analysis. And the influence of each input factor on the firing accuracy is obtained. With the purpose of researching the influence of model output (ballistic range, m) according to the uncertain input factor of the model, six factors, which are mass, initial velocity, initial shooting angle, initial drift angle, y rotational angular velocities, and z rotational angular velocities may have an impact on ballistic range are extracted based on the prior knowledge, which is shown in Table 1.

Since a large number of interference parameters, in order to simulate the real environment, random wind along the positive direction of the Z axis is added to the model. After that 500 groups of testing conditions are designed by LHS. Take the series of testing conditions into the EMRG ballistic model, this series of processes are completed by simulation in MATLAB, scatter diagram is shown in Figure 1.

Table 1. Scope of Each Factor.

No.	Name of factor	symbol	units	value range
1	mass	m	kg	[9.9,10.1]
2	initial velocity	v	m/s	[1990,2010]
3	initial shooting angle	α	rad	[0.8373,0.9769]
4	initial drift angle	β	rad	[-0.001,0.001]
5	y rotational angular velocities	w_y	rad/s	[-0.001,0.001]
6	z rotational angular velocities	w_z	rad/s	[-0.001,0.001]

Table 2. Experimental Result of Longitudinal Dispersion.

Factor Names	m	v	α	β	w_y	w_z
First-order Sensitivity	0.2197	0.5851	0.5016	0.1359	0.1460	0.1440
Total Sensitivity	0.1931	0.2762	0.4432	-10^{-5}	-0.0014	0.0111

Table 3. Experimental Result of Lateral Dispersion.

Factor Names	m	v	α	β	w_y	w_z
First-order Sensitivity	-0.013	-0.012	-0.014	0.9935	-0.013	-0.014
Total Sensitivity	-0.001	0.0046	0.0013	1.0101	0.0013	10^{-5}

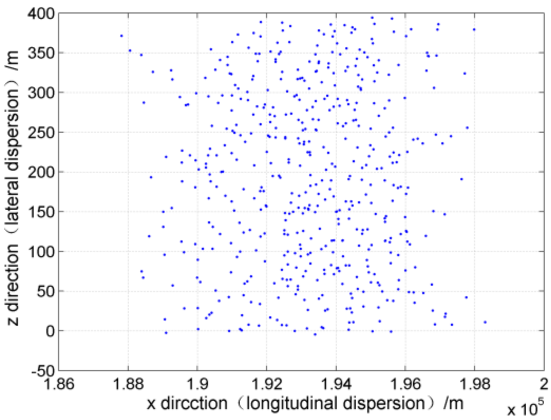

Figure 4. Scatter diagram of experiment.

The influence of each factor on the firing accuracy in two directions cannot be got from the simple scatter diagram. So we need the calculation of Sobol's method. The simulation in MATLAB needs to be run $(6\times2+1)\times500$ times. Analyze longitudinal dispersion and lateral dispersion, respectively. The experimental results are shown in and Table 3.

Transform the tables into histograms, as shown in Figures 5 and 6. From Table 2 and Figure 5, we can draw the conclusion that each factor has influence on the longitudinal dispersion and the influence degree is different. First-order sensitivity and total sensitivity of initial velocity, initial shooting angle and mass are large. Though comparing the first-order sensitivity and total sensitivity, it is explicit that influence of velocity on longitudinal dispersion decreases obviously under the comprehensive effect of multiple factors. On the contrary, the influence caused by the initial shooting angle and mass is mainly not affected by other factors. Furthermore, the sensitivity of the latter three factors is close and small. And there is poorly impact on longitudinal dispersion after they interact with other factors. In conclusion, there are three main influencing factors, initial velocity, initial shooting angle and mass under the designed experimental condition.

It is clear from Table 3 and Figure 6 that the first-order sensitivity and total sensitivity of initial drift angle all are largest (close to 1), and the sensitivity of other factors is very little (close to 0). It turns out that for the lateral dispersion, the impact of the initial drift angle is extremely significant and basically there is no effect on other factors. So in the process of testing the firing accuracy on lateral dispersion, initial drift angle should be taken into account particularly.

The first-order sensitivity in the table reflects the individual effect of each factor and the total sensitivity reflects the interaction among factors. It describes the effect of one factor on the firing accuracy under the interaction of other factors. Furthermore, the first-order sensitivity and the total sensitivity of each factor are different, which shows the interaction between the six factors is disparate.

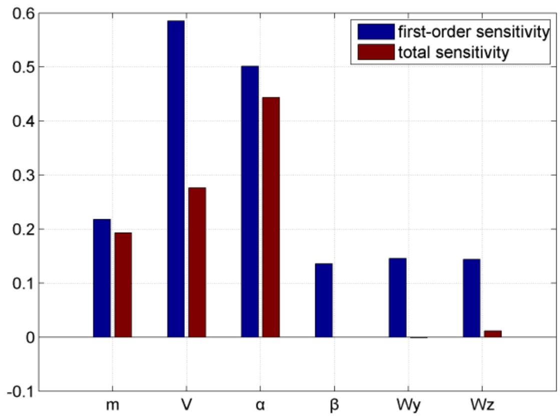

Figure 5. Result of longitudinal dispersion.

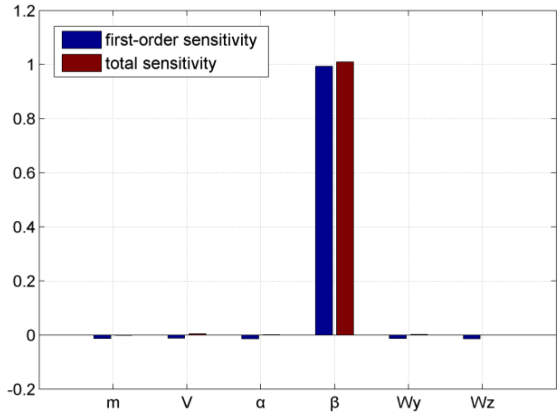

Figure 6. Result of lateral dispersion.

5 Conclusions

In this paper, the Sobol's method, a global sensitivity analysis approach, is applied to analysis of EMRG exterior trajectory firing accuracy. Single influence and interaction among each factor can be analyzed using the method, especially in the analysis which has much uncertain impact factors, a wide range of design parameters and obvious interactions between each factor. Simulation analysis results using Sobol's method show that various factors have different impact on firing accuracy of EMRG exterior trajectory, which verifies that Sobol's method is feasible and effective for firing accuracy of it. Finally the impact mechanism of each input factor on the firing accuracy and the interaction among them is obtained. In this paper, the simulation of EMRG exterior trajectory, data collection and application of Sobol's method are all implemented in MATLAB. It is easy to achieve, but the simulation needs to take a long time. In order to improve it and make firing accuracy analysis faster and more accurate, EMRG exterior trajectory simulation and data analysis tools based on C++ and Visual Studio 2010 will be designed and achieved in the future.

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