A Simplified Model of an Activated Sludge Process with a Plug-Flow Reactor

Jesús Zambrano[1] Bengt Carlsson[2] Stefan Diehl[3] Emma Nehrenheim[1]

[1]School of Business, Society and Engineering, Mälardalen University, Västerås, Sweden,
jesus.zambrano@mdh.se
[2]Department of Information Technology, Uppsala University, Uppsala, Sweden.
[3]Centre for Mathematical Sciences, Lund University, Lund, Sweden.

Abstract

The analysis of a simplified activated sludge process (ASP) with one main dissolved substrate and one main particulate biomass has been conducted in steady-state conditions. The ASP is formed by a plug-flow reactor and a settler tank. The biomass growth rate is described by a Monod function. For this process, it is not possible to get an explicit expression for the effluent substrate concentration when the process is subject to a fixed sludge age. However, when the substrate concentration of the influent is much greater than that of the effluent, an approximate and explicit relation between them is obtained. Numerical examples with two models for the settler are presented. One model is the ideal settler, which assumes a complete thickening of the sludge. The other model includes hindered settling and sludge compression. Numerical results show the effectiveness and the limitations of the proposed solution under these scenarios.

Keywords: bioreactor, clarifier, sludge blanket, sludge age

1 Introduction

Steady-state modeling and analysis of ASPs have been extensively studied during the last 50 years. One important reason is that the steady-state analysis of a dynamic model can provide initial values for process operation and optimization.

Generally, the mixing regime in an ASP reactor neither behaves as a completely stirred tank reactor (CSTR) nor as a plug-flow reactor (PFR), but in some sense in between (Tsai and Chen, 2013). In a CSTR, the reactor content is well stirred, so it is assumed that the concentration in the effluent is the same as in the reactor. In a PFR, the key assumption is that the fluid is perfectly mixed in the radial direction and in the axial direction only the transportation of the fluid is considered. Therefore, a PFR can be seen as a series of infinitely thin CSTRs, each with a uniform and different composition than the neighbouring one (Schmidt, 1998). It is expected that a PFR with a volume smaller than several CSTRs in series will give the same performance (Zambrano et al., 2015).

Compared to the classical ASP configuration, i.e. ASP with one CSTR, an explicit (steady-state) solution for an ASP formed by a PFR seems to not be possible to obtain (Diehl et al., 2017). However, some attempts have been made in the analysis of this process. For example, some implicit and approximate expressions for the effluent substrate were presented by San (1989), where the expressions were compared with numerical solutions. Computer techniques to solve the problem of a PFR in an ASP when considering the PFR as a large number of bioreactors in series are shown by Muslu (2000). Design graphs and numerical examples were presented as guidelines to size the process. On the other hand, a study of an ASP formed by a PFR could be seen as an approximation of an ASP with several CSTRs in series (Erickson and Fan, 1968; Zambrano and Carlsson, 2014).

A study of the relationship between the influent and effluent of an ASP formed by one and two CSTRs in series was presented in Zambrano and Carlsson (2014). The study mentions that it does not seem possible to find explicit solutions for the effluent substrate concentration for two or more bioreactors. That work was the motivation that led to the development of Diehl et al. (2017) and the current study.

A steady-state analysis of an ASP formed by using a PFR and a settler was recently studied in Diehl et al. (2017). The study considers and compares two different settler models. One is the ideal settler, which assumes an unlimited flux capacity, i.e. the settler is always considered to be overdimensioned. The other model, recently published by Diehl et al. (2016), here referred to as DZC settler model, includes hindered and compressive settling, which means that a limited flux capacity is modelled. Both numerical and, in some cases, analytical results are obtained. A comparison with an ASP formed by a single CSTR is also shown by Diehl et al. (2017).

In the present work, we continue the analysis of an ASP consisting of a PFR and a settler. For the ideal settler case, the steady-state solution is presented with explicit approximate formulas when the influent substrate concentration is much greater than the effluent substrate concentration. Under the same assumption, we also present an explicit formula for the effluent substrate concentration as a function of the influent substrate concentration when the sludge age is fixed. This formula can be used for both settler models.

Nomenclature

A	vertical cross-sectional of PFR $[m^2]$
A_S	horizontal cross-sectional of settler $[m^2]$
B	depth of thickening zone [m]
H	height of clarification zone [m]
K_s	half-saturation constant $[kg/m^3]$
Q	influent volumetric flow rate $[m^3/s]$
S	dissolved substrate concentration $[kg/m^3]$
$U_{z_{sb}}$	function defined in (12) $[kg/m^3]$
V_R	volume of PFR $[m^3]$
X	particulate biomass concentration $[kg/m^3]$
$X^{\infty}_{z_{sb}}$	parameter in (12) $[kg/m^3]$
Y	yield constant $[-]$
h	length of PFR [m]
q	bulk velocity in the thickening zone $[m/s]$
$\hat{q}_{z_{sb}}$	parameter in (12) $[m/s]$
$\check{q}_{z_{sb}}$	parameter in (12) $[m/s]$
r	recycle ratio $[-]$
w	wastage ratio $[-]$
x	horizontal distance from feed in PFR [m]
z	depth from feed level in settler [m]
Greek letters	
μ	Monod function $[1/s]$
μ_{max}	maximum specific growth rate $[1/s]$
θ	sludge age [s]
Subscripts	
$\square_0$	defined constant value
$\square_e$	effluent
$\square_{in}$	influent
$\square_r$	recycle
$\square_{sb}$	sludge blanket
Superscripts	
$\square^*$	PFR steady-state concentration
$\overline{\square}$	PFR influent concentration

The paper is organized as follows. A description of the ASP with a PFR and a settler is presented in Section 2, including the steady-state mass balances and the definitions for the ideal and DZC settler models. In Sections 3 and 4, we review from Diehl et al. (2017) the equations describing the steady-state conditions of the ASP for both settler models. Section 5 contains an approximate explicit expression for the effluent substrate concentration. Numerical examples are shown in Section 6 and conclusions are drawn in Section 7.

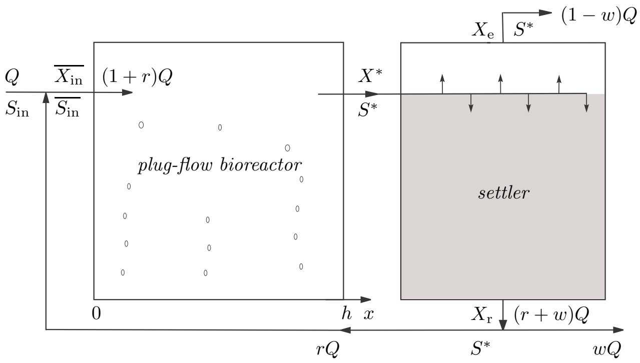

Figure 1. The activated sludge process consisting of a PFR and a settler. The steady-state variables are shown as well as the horizontal x-axis of the PFR.

2 The Activated Sludge Process

For the ASP we consider using a PFR coupled with a settler, see Figure 1, where the recycling flows to the reactor. The PFR has a constant vertical cross-sectional area A and length h, so the volume is $V_R = Ah$. The variable x is used to denote the horizontal axis in the PFR from the inlet $(x = 0)$ to the outlet $(x = h)$. Where the concentrations at location x can be denoted as $S(x)$ and $X(x)$ in the PFR.

We assume two constituents, namely one particulate biomass X and one dissolved substrate S. The influent volumetric flow rate and substrate concentration are denoted by Q and S_{in}, respectively. It is assumed that no biomass is present in the influent $(X_{in} = 0)$. The PFR input concentrations are denoted by $\overline{S_{in}}$ and $\overline{X_{in}}$, and the PFR outputs by S^* and X^*. It is assumed that no reactions are taking place in the settler, so that only particulate biomass is influenced. The substrate concentration is thus unchanged and therefore equal to S^* throughout the settler. The effluent at the top of the settler is X_e and the recycle concentration is X_r. The recycle flow rate is rQ and the waste flow rate is wQ, where $r > 0$ and $0 < w \leq 1$. The kinetics in the PFR are described by using the Monod function (Monod, 1949)

$$\mu(S) = \mu_{max} \frac{S}{K_s + S}, \quad (1)$$

where μ_{max} is the maximum specific growth rate and K_s is the half-saturation constant. It is assumed that the biomass death is negligible.

The sludge age θ of the process is defined as the amount of biomass in the bioreactor divided by the removed biomass per unit time, and is expressed as

$$\theta = \frac{A \int_0^h X(x)dx}{wQX_r}. \quad (2)$$

2.1 Mass balances and expression for the sludge age

The three mass balances of the process in steady state with $X_{\mathrm{e}} = 0$ are

$$Q(1+r)\overline{S_{\mathrm{in}}} = QS_{\mathrm{in}} + rQS^*, \tag{3}$$
$$Q(1+r)\overline{X_{\mathrm{in}}} = rQX_{\mathrm{r}}, \tag{4}$$
$$Q(1+r)X^* = (r+w)QX_{\mathrm{r}}. \tag{5}$$

Applying the conservation of mass in the PFR we get

$$\frac{Q(1+r)}{A}\frac{\mathrm{d}S}{\mathrm{d}x} = -\mu[S(x)]\frac{X(x)}{Y}, \tag{6}$$
$$\frac{Q(1+r)}{A}\frac{\mathrm{d}X}{\mathrm{d}x} = \mu[S(x)]X(x), \tag{7}$$

where Y refers to the yield constant. The following boundary conditions hold: $X(0) = \overline{X_{\mathrm{in}}}$, $S(0) = \overline{S_{\mathrm{in}}}$, $X(h) = X^*$ and $S(h) = S^*$. Combining Equations (6) and (7) together with the boundary conditions, one gets

$$\frac{Q(1+r)}{A}\frac{\mathrm{d}(YS+X)}{\mathrm{d}x} = 0$$
$$\Longrightarrow \quad Y\overline{S_{\mathrm{in}}} + \overline{X_{\mathrm{in}}} = YS(x) + X(x) = YS^* + X^*. \tag{8}$$

By solving for $X(x)$ in Equation (8) and substituting it into (6), using $V_{\mathrm{R}} = Ah$ and integrating, we get the following equation for the PFR:

$$-Q(1+r)Y\int_{\overline{S_{\mathrm{in}}}}^{S^*}\frac{\mathrm{d}\sigma}{\mu(\sigma)\left[\overline{X_{\mathrm{in}}} + Y(\overline{S_{\mathrm{in}}} - \sigma)\right]} = V_{\mathrm{R}}$$
$$\Longleftrightarrow \quad f(S^*, r, w) = V_{\mathrm{R}}, \tag{9}$$

where

$$f(S^*, r, w) = \frac{Q(1+r)}{\mu_{\max}}\left[P\ln\left(\frac{a(S_{\mathrm{in}} + rS^*)}{S^*(1+r)}\right) + \ln(a)\right], \tag{10}$$
$$P = P(S^*, r, w) = \frac{K_{\mathrm{s}}w(1+r)}{S_{\mathrm{in}}(r+w) - S^*r(1-w)},$$
$$a = a(r, w) = \frac{r+w}{r}.$$

We can obtain the sludge age by rewriting the integral in Equation (2) using Equation (6). Then we have (see Diehl et al. (2017))

$$\theta = \frac{A}{wQX_{\mathrm{r}}}\left[\frac{-Q(1+r)Y}{A}\int_{\overline{S_{\mathrm{in}}}}^{S^*}\frac{(K_{\mathrm{s}} + \sigma)\mathrm{d}\sigma}{\mu_{\max}\sigma}\right]$$
$$= \frac{1}{\mu_{\max}}\left[1 + \frac{(1+r)K_{\mathrm{s}}}{(S_{\mathrm{in}} - S^*)}\ln\left(\frac{S_{\mathrm{in}} + rS^*}{S^*(1+r)}\right)\right]. \tag{11}$$

2.2 The settler

Ideal settler model. For an ideal settler we assume that all the sludge fed to the settler will always pass through the thickening zone, regardless of the amount of incoming sludge and the recycle and waste flows. Although in many cases unrealistic, this model could work well when the settler is over-sized.

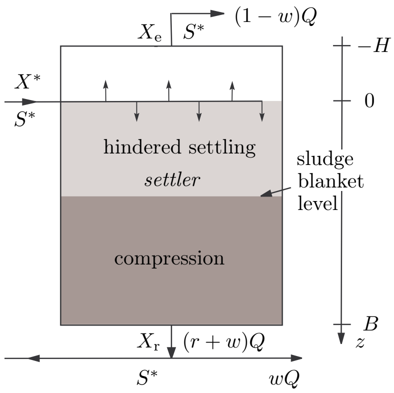

Figure 2. The DZC settler. The steady-state variables are shown as well as the vertical z-axis of the settler.

DZC settler model. The processes in the settler are described by a steady-state approximation of a partial differential equation (PDE) which includes a hindered settling velocity function and a compression function (Bürger et al., 2011). The behavior of a real settler can be divided into three qualitatively different operations: underloaded, overloaded and normal operation. By normal operation we mean that all the biomass fed to the settler is conveyed through the thickening zone and that there exists a sludge blanket in the thickening zone, see Figure 2. In this work we only study the steady-state solutions under normal operation and therefore set $X_{\mathrm{e}} = 0$.

The following simple relationship is a reasonable approximation obtained from the steady-state solutions that have a sludge blanket in the thickening zone (Diehl et al., 2016)

$$X_{\mathrm{r}} = U_{z_{\mathrm{sb}}}(q) := X_{z_{\mathrm{sb}}}^{\infty}\left(1 + \frac{\hat{q}_{z_{\mathrm{sb}}}}{q + \check{q}_{z_{\mathrm{sb}}}}\right), \tag{12}$$

where q is the bulk velocity in the thickening zone, defined as

$$q = q(r, w, Q, A_{\mathrm{S}}) := \frac{Q(r+w)}{A_{\mathrm{S}}}, \tag{13}$$

where $X_{z_{\mathrm{sb}}}^{\infty}$, $\hat{q}_{z_{\mathrm{sb}}}$ and $\check{q}_{z_{\mathrm{sb}}}$ are parameters which depend on the chosen sludge blanket level z_{sb}. A_{S} is the settler constant horizontal cross-sectional area, see model details in Diehl et al. (2016).

3 ASP with ideal settler model

3.1 Steady-state solutions

From the mass balances of the process (5), (8) and (9), the steady-state equations for an ASP with ideal settler can be expressed as (ignoring the variables $\overline{S_{\mathrm{in}}}$ and $\overline{X_{\mathrm{in}}}$; these can be obtained from (3) and (4))

$$S^* = S_{\text{in}} - \frac{w}{Y}X_{\text{r}}, \quad (14)$$

$$X^* = \frac{r+w}{1+r}X_{\text{r}}, \quad (15)$$

$$V_{\text{R}} = f(S^*, r, w), \quad (16)$$

where $f(S^*, r, w)$ is given by (10). Equation (16) is solved for $S^* = S^*(r, w)$, then Equation (14) gives $X_{\text{r}} = X_{\text{r}}(r, w)$ and Equation (15) gives $X^* = X^*(r, w)$. Note that all these variables are two-parameter solutions of the control variables r, w. Note also from Equations (9) and (10) that S^* is expressed implicitly. If $S_{\text{in}} \gg S^*$ is assumed, we have the following results.

Theorem 1. *Given an ASP with an ideal settler described by Equations* (14)–(16)*. If $S_{\text{in}} \gg S^*$ then the solution of Equations* (14)–(16) *can be expressed explicitly as*

$$S^* = S^*(r, w) = \frac{(r+w)S_{\text{in}}}{r\left[(1+r)\exp(\beta) - (r+w)\right]}, \quad (17)$$

$$X_{\text{r}} = X_{\text{r}}(r, w) = \frac{Y}{w}\left(S_{\text{in}} - S^*(r, w)\right), \quad (18)$$

$$X^* = X^*(r, w) = \frac{(r+w)Y}{(1+r)w}\left(S_{\text{in}} - S^*(r, w)\right). \quad (19)$$

where

$$\beta = \frac{S_{\text{in}}(r+w)}{K_{\text{s}}w(1+r)}\left[\frac{V_{\text{R}}\mu_{\max}}{Q(1+r)} - \ln\left(\frac{r+w}{r}\right)\right],$$

and when the denominator in Equation (17) *is positive.*

Proof. The assumption implies that Equation (16) can be expressed as (cf. Equations (9) and (10))

$$\frac{Q(1+r)}{\mu_{\max}}\left[\frac{1}{G}\ln\left(\frac{a(S_{\text{in}}+rS^*)}{S^*(1+r)}\right) + \ln(a)\right] = V_{\text{R}}, \quad (20)$$

where

$$G = \frac{S_{\text{in}}(r+w)}{K_{\text{s}}w(1+r)} \quad \text{and} \quad a = \frac{r+w}{r}.$$

Solving (20) for S^* we get

$$G\left(\frac{V_{\text{R}}\mu_{\max}}{Q(1+r)} - \ln(a)\right) = \ln\left(\frac{a(S_{\text{in}}+rS^*)}{S^*(1+r)}\right).$$

For simplicity we set $\beta = G\left(\frac{V_{\text{R}}\mu_{\max}}{Q(1+r)} - \ln(a)\right)$, then we have

$$\exp(\beta) = \frac{a(S_{\text{in}}+rS^*)}{S^*(1+r)} \iff$$

$$S^*(1+r)\exp(\beta) = a(S_{\text{in}}+rS^*) \iff$$

$$S^* = \frac{aS_{\text{in}}}{(1+r)\exp(\beta) - ar} \iff$$

$$S^* = \frac{(r+w)S_{\text{in}}}{r\left[(1+r)\exp(\beta) - (r+w)\right]},$$

if the denominator is positive.

Once S^* is obtained, X_{r} and X^* are given from Equations (14) and (15), respectively. □

3.2 Substrate input-output relationship for constant sludge age

The two-parameter solution of Equations (14)–(16) (or (17)–(19) in Theorem 1) means that two additional equations can be imposed to define the operation conditions. We are interested in investigating the steady-state solutions of the process for different values of S_{in} for a constant sludge age θ_0. For S_{in} as a variable, we have six variables to take into consideration: $S^*, X^*, X_{\text{r}}, r, w$, and S_{in}. However, to get a one-parameter solution with S_{in} as a parameter, we can add the following to Equations (14)–(16):

$$r = r_0, \quad (21)$$

$$\frac{1}{\mu_{\max}}\left[1 + \frac{(1+r)K_{\text{s}}}{(S_{\text{in}} - S^*)}\ln\left(\frac{S_{\text{in}}+rS^*}{S^*(1+r)}\right)\right] = \theta_0. \quad (22)$$

Since $r = r_0$ is constant, Equation (22) defines implicitly $S^* = S^*(S_{\text{in}})$, then Equation (16) gives $w = w(S_{\text{in}})$, Equation (14) gives $X_{\text{r}} = X_{\text{r}}(S_{\text{in}})$ and Equation (15) gives $X^* = X^*(S_{\text{in}})$.

4 ASP with DZC settler model

4.1 Steady-state solutions

The mass balances of the system considering the DZC settler model have to include Equation (12) in order to get a sludge blanket in the thickening zone. The steady-state equations are then expressed as

$$S^* = S_{\text{in}} - \frac{w}{Y}U_{z_{\text{sb}}}(q), \quad (23)$$

$$X^* = \frac{r+w}{1+r}U_{z_{\text{sb}}}(q), \quad (24)$$

$$X_{\text{r}} = U_{z_{\text{sb}}}(q), \quad (25)$$

$$V_{\text{R}} = f(S^*, r, w). \quad (26)$$

Straightforward calculations give that the expression for the sludge age is the same as for the ideal settler model (cf. Equation (11) and Diehl et al. (2017)).

4.2 Substrate input-output relation for constant sludge age

As in the case of an ASP with ideal settler, we are interested in the solution of the process for a constant sludge age for different values of S_{in}. By imposing $\theta(r, S_{\text{in}}, S^*) = \theta_0$ we get a one-parameter solution. Note that we cannot impose another equation (e.g. $r = r_0$) as we did for the ideal settler model, since we have Equation (12) controlling the sludge blanket.

Hence, Equations (22), (23) and (26) are solved for $S^* = S^*(S_{\text{in}})$, $r = r(S_{\text{in}})$ and $w = w(S_{\text{in}})$. Then, Equation (24) gives $X^* = X^*(S_{\text{in}})$ and Equation (25) gives $X_{\text{r}} = X_{\text{r}}(S_{\text{in}})$.

5 An approximation for S^* given θ_0

Note that S^* is given implicitly in Equation (22), and will depend on S_{in}, r and θ_0. This equation can, however, be solved explicitly for S^* if we make an assumption.

Theorem 2. *Given an ASP described by Equations* (14)–(16) *(for an ideal settler) or by* (23)–(26) *(for a DZC settler), and where* θ *is given by* (11). *Assume that* θ *is fixed to* θ_0*, i.e. Equation* (22) *is imposed. If* $S_{\text{in}} \gg S^*$*, then the following simple expression for* S^* *holds:*

$$S^* = \frac{S_{\text{in}}}{(1+r)\exp(\alpha S_{\text{in}}) - r}, \tag{27}$$

where

$$\alpha = \frac{\theta_0 \mu_{\max} - 1}{K_s(1+r)}.$$

Proof. Assuming $S_{\text{in}} \gg S^*$, Equation (22) can be written as

$$\frac{1}{\mu_{\max}}\left[1 + \frac{(1+r)K_s}{S_{\text{in}}}\ln\left(\frac{S_{\text{in}} + rS^*}{S^*(1+r)}\right)\right] = \theta_0,$$

solving for S^* gives

$$S^* = \frac{S_{\text{in}}}{(1+r)\exp(\alpha S_{\text{in}}) - r},$$

where $\alpha = (\theta_0\mu_{\max} - 1)/(K_s(1+r))$. □

6 Numerical example

We assume that the ASP has the following constants and parameters: $V_R = 3000\,\text{m}^3$, $Q = 1000\,\text{m}^3/\text{h}$, $\mu_{\max} = 0.17\,\text{h}^{-1}$, $K_s = 0.05\,\text{kg/m}^3$, $Y = 0.7$. For the DZC settler model we let: $B = 3\,\text{m}$, $z_{\text{sb}} = 1\,\text{m}$, $X_1^{\infty} = 6.52\,\text{kg/m}^3$, $\hat{q}_1 = 0.32\,\text{m/h}$, $\check{q}_1 = 0.45\,\text{m/h}$. The latter constants were obtained with standard parameters for the hindered settling and compression functions and the procedure in Diehl et al. (2016).

Numerical solutions of the model equations will now be compared with the approximate solutions given by Theorems 1 and 2. The numerical solutions are obtained with `fsolve`, a function in Matlab which solves systems of non-linear equations.

6.1 Theorem 1

This case deals with an ASP with an ideal settler model. Figure 3 shows the numerical (without approximation, i.e. without the assumption $S_{\text{in}} \gg S^*$) and approximated solutions given by Theorem 1 for S^*, X^* and X_r. That is, we compare the results from Equations (14)–(16) with results from Equations (17)–(19). The influent substrate concentration is set to $S_{\text{in}} = 0.1\,\text{kg/m}^3$. The results are shown for an interval of values of r and for some values of the wastage ratio w.

Note that in plot (a), for higher values of S^*, the difference between the values given by Theorem 1 and the values from the solution with no approximation becomes larger. The same effect can be seen in plot (b) for X^* and in plot (c) for X_r. S^* starts to decrease for higher values in r. Then, the difference between values with and without approximation starts to decrease.

6.2 Theorem 2

For the ideal settler model, Figure 4 shows numerical and approximated solutions given by Theorem 2. Equation (22) is solved for $S^* = S^*(S_{\text{in}})$ at an interval of values for S_{in}. The recirculation is set to $r = r_0 = 1$ (cf. Equation (21)), and we set $\theta_0 = 16\,\text{h}$. Note that, for higher values of S_{in}, the values for S^* given by Theorem 2 are closer to those given by the solution of the process with no approximation.

For the DZC settler model, Equations (22), (23) and (26) are solved for $S^* = S^*(S_{\text{in}}), r = r(S_{\text{in}})$ and $w = w(S_{\text{in}})$ at an interval of values for S_{in}. Figure 5 shows the numerical and approximated solutions for $S^* = S^*(S_{\text{in}})$. We set $\theta_0 = 6.5\,\text{h}$ and show some results for some values of the settler area A_S.

Note that, for a given S_{in} and when a higher A_S is used, the solution with no approximation gives a higher recycle concentration X_r (see Equations (12) and (13)). This means that we have a more thickened sludge, which gives a lower S^* (see Equation (23)). Therefore, S^* becomes much lower compared to S_{in} as A_S increases. Hence, the values from Theorem 2 are much closer to the solution of the model equations without approximation.

7 Conclusions

An ASP formed by a PFR and a settler has been studied in steady-state operation. It is shown that explicit, but approximate, solutions can be obtained for the case of an ideal settler under the assumption that the influent substrate concentration is much greater than the effluent one. With this assumption it is also possible to obtain an explicit expression for the effluent concentration as a function of the influent one under the operating condition that the sludge age should be maintained at a specific value. Numerical examples show the performances of the simpler explicit expressions under two different models for the settler and hence when the simpler formulas can be used. Further research might be focused on considering the decayed particulate biomass as an additional constituent in the process.

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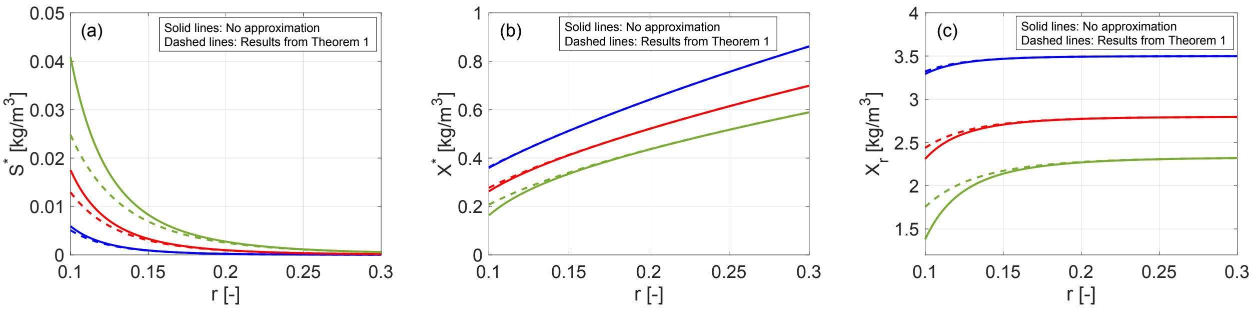

Figure 3. PFR with ideal settler model. Comparison between the numerical (no approximation) and the approximated solutions given by Theorem 1 as functions of r. The results are shown for three values of the wastage: $w = 0.02$ (in blue), $w = 0.025$ (in red), $w = 0.03$ (in green). The influent substrate concentration is fixed to $S_{\text{in}} = 0.1$ kg/m^3.

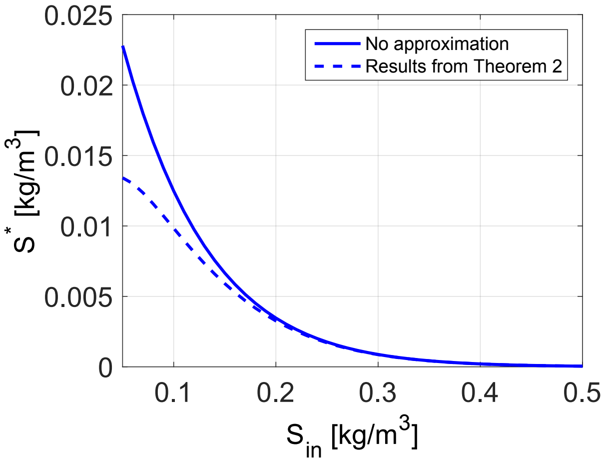

Figure 4. PFR with ideal settler model. Comparison between the numerical and the approximated solutions given by Theorem 2 as functions of S_{in}. The recirculation is fixed to $r_0 = 1$ and the sludge age is kept to $\theta_0 = 16$ h.

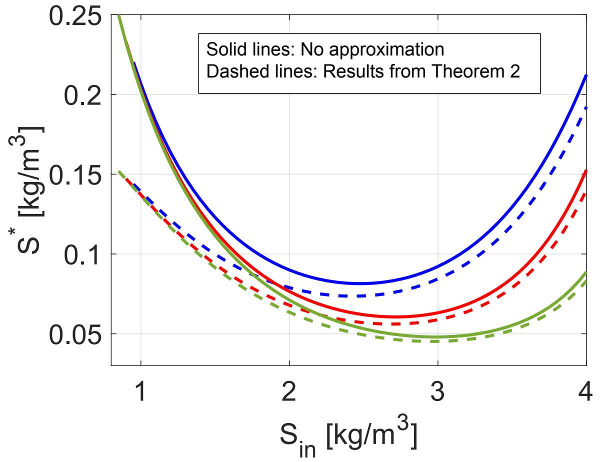

Figure 5. PFR with DZC settler model. Comparison between the numerical (no approximation) and the approximated solutions given by Theorem 2 as functions of S_{in} for some values of the settler area A_S [m^2]: 500 m^2 (in blue), 1500 m^2 (in red), 3000 m^2 (in green). For every curve, the sludge age is kept to $\theta_0 = 6.5$ h.

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