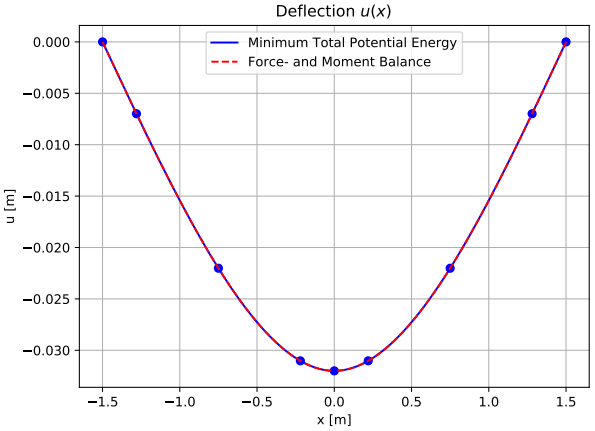

Figure 4. Comparison of $u(s)$ found from (i) the minimum total potential energy principle as derived in this paper, and (ii) from the approach based on force- and moment balances. The blue dots represent the values calculated by the former method, and the blue, solid line represents the polynomial interpolations between these points. $\bar{N} = 5$ is used in the example.

elements of $\mathbf{v}$ are the five Lagrange multipliers defined in Eq. 49. The physical interpretation of each Lagrange multiplier is the derivative of the total potential energy, P, w.r.t. the constraint associated with the multiplier. Hence, λ_1 and λ_3 are the forces from the left and right beam supports, respectively, toward the beam, as shown in Figure 2. Similarly, λ_2 is the force from the piston toward the beam, i.e. the force F in the figure. Multiplier λ_4 can be shown to be the beam's bending moment at $x = 0$.

The last multiplier, λ_5, expresses the total potential energy's sensitivity to discontinuities in the bending curvature at $x = 0$. To the author's knowledge, this formulation does not correspond to a well-known physical variable. λ_5 is a factor 10^{16} smaller than the other multipliers, measured in absolute value. Hence, λ_5 is assumed to be zero in the case of infinite arithmetic precision, which means that the corresponding constraint would have been met even if it was not enforced.

7 Conclusions and Further Work

The present paper derives numerical calculation of the beam equation based on the minimum total potential energy principle under the assumptions of linear material and small deflections. The calculation gives identical results as the derivation based on force- and moment balances which is commonly presented in the literature.

The work presented in this paper is the first milestone towards the author's final goal of modeling subsea power cables and umbilicals during bending stiffness tests. The future milestones are to include the effects of large deflections, nonlinear materials, and shear forces between the cable elements.

It is believed that the minimum total potential energy principle is more suitable for the future extensions than force- and moment balances. It is further believed that it will be impossible to reach an analytical solution of the overall problem due to its complexity, which is the reason for using numerical mathematics.

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