# Modelling of Transonic and Supersonic Aerodynamics for Conceptual Design and Flight Simulation

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### Abstract

In this paper a comprehensive set of models of aerodynamics for conceptual design that also can be used for flight simulation is presented. In particular these are transonic/supersonic lift coefficient, moment and induced drag. The models are based on a range of models found in literature that have been compiled and reformulated to be more practical to use. Many models in literature exhibit discontinuities or infinities e.g. around Mach one. Care has to be taken to formulate benign expressions that can be used in all parts of the flight envelope. One observation is that models are showing that induced drag have a significant influence on supersonic performance, especially at high altitude and with elevated load factor. The models are implemented in a simulation model so that performance can be evaluated at a mission level already in conceptual design.

Keywords: Supersonic, simulation, conceptual design

# 1 Introduction

Traditionally conceptual design has not been involving flight simulation, although this is gaining in importance. In conceptual design modelling is aimed to predict the behavior and performance of the finished product. As such they should not necessarly show the exact behaviour for the aircraft at the sketchy level of conceptual design. E.g. at the conceptual design stage aerodynamic adjustments such as fillets etc, are not defined and hence a very accurate model such as an advanced CFD model of the aerodynamics will not provide relevant result. At the conceptual design only the important configuration and dimensions of the aircraft, are laid out. Hence models at the conceptual stage are on an entirely different nature than in later part of design where the geometry has a high degree of fidelity. In this paper a comprehensive aerodynamic model with a minimum number of coefficients is established, that is suitable for simulation at the conceptual design stage, in such a way that they can give a "best guess" of the performance of the finished product. Even though the models use a minimum of parameters they can provide a model of high fidelity if they are established e.g. from wind tunnel or flight testing. Therefore the same model can be used well into the design process as better data becomes available.

# 2 Geometric modeling

In order to have a representative model an existing aircraft in this case the F-16, was chosen as a basis for study in this work. In addition the X-29 forward swept experimental aircraft was also used to validate the model since there are data published

in [1].

## **3** Modelling for mission simulation

For evaluating the performance of the aircraft in a realistic scenario a system simulation model was built that could be used in a mission simulation. The flight dynamics model is here based on a 6 degree of freedom rigid body model that is connected to an aerodynamic model. This was presented in Krus et al. [2] and in Abdalla [3]. The aircraft model can have different number of wings, with an arbitrary number of control surfaces, and a body with its inertia characteristics. The aerodynamic model is here based on a static version of the model presented in [4], although the unsteady effects can of course also be included. The control surfaces are modeled both with a linear increase of lift force with deflection and the corresponding increase in induced drag. There is also a cross coupling effect of drag for control surfaces on the same wing e.g. ailerons and flaps. In this way also the effect of trim drag on performance is automatically included, and the effect of reduced weight, as fuel is consumed.

# 4 Modelling transonic/supersonic characteristics

Here, a substantial part of the mission under study is in the trans-sonic and supersonic regimes. Therefore, it is important to have models that capture these characteristics in an adequate way. There are basically three effects that are modeled in the supersonic regime. The first is the *wave drag*. In the



Figure 1: Non-linear aerodynamic model.

model there is a Mach number dependent coefficient that is added to the parasitic drag coefficient.

The second is that the *aerodynamic center* is moved backwards from the quarter cord position for subsonic to approximately half cord for supersonic. The third is that the lift sloop is changed. Furthermore the induced drag is also changed as a consequence of the lift sloop and due to loss of leading edge suction. The transition between subsonic and supersonic is modelled with a logistic (sigmoid) function to produce a soft transition.

### 4.1 Aerodynamic Drag Estimation

Aerodynamic drag in the supersonic range is composed of skin friction drag, form and interference drag, and wave drag. Skin friction drag can be basically considered to be the same for the whole range, while for the form and interference drag it are included in the wave drag in the supersonic region. The wave drag coefficient is, see [5] or [6] roughly proportional to:

$$C_{dw} \propto \frac{1}{\sqrt{M^2 - 1}} \tag{1}$$

This expression, however, has a singularity at M = 1, which is clearly not realistic. In order to remove the singularity (1) is modified into:

$$C_{dw} = C_{dw0} \frac{k_{dw}}{(((M - k_{dwm})^2 - 1)^2 + k_{dw}^4)^{1/4}}$$
(2)

Here  $C_{dw1}$  is the maximum wave drag and  $k_{dw}$  is a shape parameter. A low value of  $k_{dw}$  leads to a quick decay of the wave drag coefficient with Mach number.  $k_{dwm}$  is another shape parameter that can be used to move the Mach number of the maximum drag. Both these values are non-dimensional and are typically less than one. This function only applies to the supersonic region. Therefore it is multiplied with a logistic function to produce a soft step starting at the critical Mach number where the drag rise starts. This function is  $f_M$  is:

$$f_M = \frac{1}{1 + e^{-8\frac{M - (1 - \delta_M/2)}{\delta_M}}}$$
(3)

Here:

$$= 1 - M_{crit} \tag{4}$$

where  $M_{crit}$  is the critical Mach number. Plotting this function for  $\delta_M = 0.2$  yields Fig. 2.

 $\delta_M$ 



Figure 2: The logistic function for  $\delta_M = 0.2$ . Hence for this example  $M_c rit = 0.8$ .

The full expression is then

$$C_{dw} = f_M C_{dw0} \frac{k_{dw}}{(((M - k_{dwm})^2 - 1)^2 + k_{dw}^4)^{1/4}}$$
(5)

With  $C_{dw0} = 0.0264$ ,  $k_{dwm} = 0.05$ ,  $k_{dw} = 0.5$  and  $\delta M = 0.2$  the function in Fig. 3 is obtained.



*Figure 3: Wave drag coefficient as a function of Mach number, using Eq. 5* 

The most critical parameter for the performance prediction is the maximum wave drag contribution, i.e.  $C_{dw}$ .

Under simplified assumptions the Sears-Haack body have the lowest transonic drag. This was shown first suggested by Wolfgang Haack [7].

The theoretical wave drag coefficient for a Sears-Haack body at Mach M = 1 (also the maximum value according to this theory) can be calculated as (with cross-section area as reference area):

$$C_{dw,SH} = 9\pi \frac{S_{max}}{2L^2} \tag{6}$$

For aircraft design it is more usual to use the wing area as reference area. The expression then becomes:

$$C_{dw,SH} = 9\pi \frac{S_{max}}{2L^2} \left(\frac{S_{max}}{S_{ref}}\right)$$
(7)

where  $S_{max} = \pi r_{max}^2$  is the maximum cross section area of the Sears-Haack body, and *L* is the corresponding length.  $S_{ref}$  is another area that should be used as reference are you for  $C_{dw}$ . For an aircraft it is usually the wing area and for a rocket or

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missile the reference area is usually a cross section area and it could be the same as  $S_{max}$ . This value gives a lower limit to the transonic drag. For real supersonic aircraft it is a factor larger. this can be accomodated for by introducing a correction factor  $E_{wd}$  as in [6]. The wave drag can then be calculated as:

$$C_{dw} = E_{wd} C_{dw,SH} \tag{8}$$

The correction factor can be estimated by comparing to values of typical existing designs. Another way is to use a more elaborate method, based on a more detailed geometric analysis based on [8], [9] and [10]. For a more elaborate discussion on practical implementation of such metods, see e.g. in [11] and [12], [13], and in [14].

#### 4.2 Supersonic lift

The liftsloop of a wing changes already for higher subsonic Mach number. For an infinite straight wing the lift sloop can be calculated as:

$$C_{L\alpha,sub} = \frac{C_{L\alpha,0}}{\beta} \tag{9}$$

Here  $C_{L\alpha,sub}$  is the lift-sloop for low Mach numbers, which can be derived eg. from panel code. The factor  $\beta$  is:

$$\beta = \sqrt{1 - M^2} \tag{10}$$

Going into the supersonic region the lift curve changes dramatically. The lift curve can here be calculated from

$$C_{L\alpha,sub} = \frac{4}{\beta} \tag{11}$$

where for this case:

$$\beta = \sqrt{M^2 - 1} \tag{12}$$

However, this is only valid for a wing where the reference area is equal to the actual effective lifting area. For a whole aircraft these are not necessarly the same. therefore a nondimensional lifting area for the whole wing body combination  $S_0$  is introduced as:

$$S_0 = S_{wb} / S_{ref} \tag{13}$$

Eq. 9 can then be rewriten as:

$$C_{L\alpha,sup} = S_0 \frac{4}{\beta} \tag{14}$$

Otherwise the supersonic lift would be dependent on the chosen reference area.  $\beta$  is here changed into an expression that is valid in both ranges:

$$\beta = ((M^2 - 1)^2)^{1/4} \tag{15}$$

However, if (15) is used in (11) or (9) this yields a singularity at M = 1. In reality this does not occur, and the lift curve is smoothed out in the transonic range. Therefore the  $\beta$  is modified into:

$$\beta = ((M^2 - 1)^2 + \varepsilon_M^4)^{1/4} \tag{16}$$

Here  $\varepsilon_M$  is a factor that removes the singularity while the asymptotes are unaffected.

For a swept wing 14 is only valid when the Mach cone, see Fig. 4 has passed the leading edge. This means that there is prolonged transition region for this case. This Mach number can be calculated using geometry. The Mach number where the mach cone coincides with the leading edge of the wing is then.

$$M_1 = \sqrt{1 + \tan^2 \Lambda} \tag{17}$$

Note that for a complete aircraft the notion of Mach cone becomes more complex.



Figure 4: Mach cone touching the leading edge of a delta wing.

An expression that can be used for the whole range is then:

$$C_{L\alpha} = C_{L\alpha,sub}(1 - f_{ML}) + C_{L\alpha,sup}f_{ML}$$
(18)

Here

$$f_{ML} = \frac{1}{1 + e^{-4\frac{M - (1 + \delta_{ML}/2)}{\delta_{ML}}}}$$
(19)

This is again the logistic function, where

$$\delta_{ML} = M_1 - 1 = \sqrt{1 + \tan^2 \Lambda} - 1 \tag{20}$$



Figure 5: The lift coefficient with respect to  $\alpha$ . The dotted line is without leading edge suction and the dashed line is with leading edge suction. The solid line is a blend of of both for a high Mach cone angle

### 4.3 Supersonic moment

In the supersonic regime the neutral point is moving backwards from approximately the quarter cord position of the wing to approximately the half cord position. This leads to an increased stability of the aircraft and also an increasing negative pitch moment that has to be counteracted by the elevator,

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elevons and/or canard. In the transonic region the moment characteristics can be very complex and show dramatic variations. However, this requires a more detailed analysis of the aerodynamics, and a high fidelity geometric model that is not available in conceptual design, normally the aircraft should not operate in the this region so it is of less importance for performance predictions. A simple model that captures the main effect of moving the neutral point a quarter cord is:

$$C_m = C_{m0} - C_L \frac{MAC}{4} f_M \tag{21}$$

where  $f_M$  is a factor that is typically less than one. For a straight wing  $f_M = 1$  but for e.g. a cranked delta wing  $f_M < 1$ .

### 4.4 Supersonic lift dependent drag

In supersonic flight the lift dependent drag is generally small compared to the wave drag. Hence a very precise model is not needed. It can even be argued that the subsonic model can be used for simulation. However, here a simple expression is shown that at least capture the main effects. In the subsonic regime induced drag can in general be calculated from:

$$C_{Di} = \frac{C_L^2}{\pi e A} \tag{22}$$

This can be rewritten as:

$$C_{Di} = \frac{C_{L\alpha}^2}{\pi e A} \alpha^2 = C_{Di\alpha^2, sub} \alpha^2$$
(23)

For a straight wing in supersonic flow there is no leading edge suction. This means that the lift dependent drag can be found from the lift from trigonometric relations:

$$C_{Di,sup} = C_L \tan(\alpha) \approx C_{L,sup} \alpha = C_{L\alpha^2,sup} \alpha^2 = C_{Di\alpha^2,sup} \alpha^2$$
(24)

Using Eg. (14) yields

$$C_{Di,sup} = S_0 \frac{4}{\beta} \alpha^2 \tag{25}$$

In the firmly supersonic regime Eq. (23) can still be used if the Oswald efficiency factor e is replaced with:

$$e_{sup} = \frac{C_{L,sup}^2}{\pi A C_{Di,sup}} = \frac{C_{L\alpha,sup}^2}{\pi A S_0 \frac{4}{B}}$$
(26)

where  $e_{sub}$  is the subsonic efficiency factor.

For a swept wing the behaviour is more complex. When the trailing edge of the wing is inside the Mach cone there can be some leading edge suction, but this leads to another level of detail that might not be available at the conceptual design stage. However, once the leading edge of the wing is outside the Mach cone, that is when the Mach number  $M > M_1$ , the expression derived here for the supersonic induced drag is valid. Therefore, an approximation suggested here is to assume that the lift function goes from leading edge suction at  $M = M_1$ . When there is leading edge suction the Oswald efficiency factor for the subsonic case is used and then this is gradually moved to the value for supersonic speed.

$$e = e_{sub}(1 - f_{ML}) + e_{sup}f_{ML} \tag{27}$$

In this way Eg. 23 can be used for the whole range.



Figure 6: The coefficient for induced drag with respect to  $\alpha^2$ . The dotted line is without leading edge suction and the dashed line is with leading edge suction. The solid line is a blend of both for a high Mach cone angle

### 5 Comparison of contributions to drag

In order to have an idea of the importance of the different drag components an example with coefficients chosen to loosely resemble the F-16 flying at 10000m with a weight of 12000kg was made. Calculation the corresponding drag contributions using:

$$D = C_D \rho \frac{v^2}{2} \tag{28}$$

where  $\rho = 0.4$ kg/m<sup>2</sup> (corresponding to an altitude of 10000 m) and the speed of sound a = 295 m/s. The mass of the aircraft was set to 12000 kg resulted in a required lift force of L = 117840 N. The lift coefficient was then calculated from

$$L = C_L \rho \frac{v^2}{2} \tag{29}$$

which yields:

$$C_L = \frac{2L}{\rho v^2} \tag{30}$$

The Drag coefficients are shown in Fig. 7 The contributions



Figure 7: The contributions of parasitic, the wave drag and the induced drag as well as the total drag.

of the different drag components are shown in Fig. 8 where also the total drag is shown. It shows that the contribution of induced drag is important at supersonic speed. In e.g. a turn, a load factor for two would increase the induced drag four times, which would make it on par with the parasitic drag.

### 6 Conclusion

In this paper a generic model to model trans-sonic and supersonic aerodynamic characteristics is presented. Different

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*Figure 8: The contributions of parasitic, the wave drag and the induced drag as well as the total drag.* 

models for different parts of the envelope has been combined into continuous functions that can be used for flight simulation. One conclusion is that supersonic induced drag can be substantial and needs to be considered in conceptual design. Even though the velocity is high, the induced drag coefficient goes up when leading edge suction is lost. In addition the neutral point is moved backwards increasing the need for trim that is further increasing the induced drag.

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